

The Zakon Series on Mathematical Analysis

Mathematical Analysis

VOLUME I

Elias Zakon

University of Windsor

The Trillia Group



West Lafayette, IN

Printed from e-media with permission by:

Curran Associates, Inc.
57 Morehouse Lane
Red Hook, NY 12571



Some format issues inherent in the e-media version may also appear in this print version.

Anyone purchasing this book is granted a single-user license to the associated e-book; see <http://www.trillia.com/zakon-analysisI.html> for details.

Mathematical Analysis I
© 1975 Elias Zakon
© 2004 Bradley J. Lucier and Tamara Zakon

All rights reserved.
Printed by Curran Associates, Inc. (2010)
ISBN 978-1-61738-647-3

Published by the Trillia Group, West Lafayette, Indiana, USA.
First published: May 20, 2004. This version released: July 11, 2011.
Technical Typist: Betty Gick. Copy Editor: John Spiegelman. Logo: Miriam Bogdanic.
The phrase "The Trillia Group" and the Trillia Group logo are trademarks of the Trillia Group.

This book was prepared by Bradley J. Lucier and Tamara Zakon from a manuscript written by Elias Zakon. We intend to correct and update this work as needed. If you notice any mistakes in this work, please send e-mail to lucier@math.purdue.edu and they will be corrected in a later version.

For any additional requests please contact the Trillia Group at info@trillia.com.

Contents*

Preface	ix
About the Author	xi
Chapter 1. Set Theory	1
1–3. Sets and Operations on Sets. Quantifiers	1
Problems in Set Theory	6
4–7. Relations. Mappings	8
Problems on Relations and Mappings	14
8. Sequences	15
9. Some Theorems on Countable Sets	18
Problems on Countable and Uncountable Sets	21
Chapter 2. Real Numbers. Fields	23
1–4. Axioms and Basic Definitions	23
5–6. Natural Numbers. Induction	27
Problems on Natural Numbers and Induction	32
7. Integers and Rationals	34
8–9. Upper and Lower Bounds. Completeness	36
Problems on Upper and Lower Bounds	40
10. Some Consequences of the Completeness Axiom	43
11–12. Powers With Arbitrary Real Exponents. Irrationals	46
Problems on Roots, Powers, and Irrationals	50
13. The Infinities. Upper and Lower Limits of Sequences	53
Problems on Upper and Lower Limits of Sequences in E^*	60
Chapter 3. Vector Spaces. Metric Spaces	63
1–3. The Euclidean n -space, E^n	63
Problems on Vectors in E^n	69
4–6. Lines and Planes in E^n	71
Problems on Lines and Planes in E^n	75

* “Starred” sections may be omitted by beginners.

7. Intervals in E^n	76
Problems on Intervals in E^n	79
8. Complex Numbers	80
Problems on Complex Numbers	83
*9. Vector Spaces. The Space C^n . Euclidean Spaces	85
Problems on Linear Spaces	89
*10. Normed Linear Spaces	90
Problems on Normed Linear Spaces	93
11. Metric Spaces	95
Problems on Metric Spaces	98
12. Open and Closed Sets. Neighborhoods	101
Problems on Neighborhoods, Open and Closed Sets	106
13. Bounded Sets. Diameters	108
Problems on Boundedness and Diameters	112
14. Cluster Points. Convergent Sequences	114
Problems on Cluster Points and Convergence	118
15. Operations on Convergent Sequences	120
Problems on Limits of Sequences	123
16. More on Cluster Points and Closed Sets. Density	135
Problems on Cluster Points, Closed Sets, and Density	139
17. Cauchy Sequences. Completeness	141
Problems on Cauchy Sequences	144
Chapter 4. Function Limits and Continuity	149
1. Basic Definitions	149
Problems on Limits and Continuity	157
2. Some General Theorems on Limits and Continuity	161
More Problems on Limits and Continuity	166
3. Operations on Limits. Rational Functions	170
Problems on Continuity of Vector-Valued Functions	174
4. Infinite Limits. Operations in E^*	177
Problems on Limits and Operations in E^*	180
5. Monotone Functions	181
Problems on Monotone Functions	185
6. Compact Sets	186
Problems on Compact Sets	189
*7. More on Compactness	192

- 8. Continuity on Compact Sets. Uniform Continuity.....194
 - Problems on Uniform Continuity; Continuity on Compact Sets.200
- 9. The Intermediate Value Property.....203
 - Problems on the Darboux Property and Related Topics.....209
- 10. Arcs and Curves. Connected Sets.....211
 - Problems on Arcs, Curves, and Connected Sets.....215
- *11. Product Spaces. Double and Iterated Limits.....218
 - *Problems on Double Limits and Product Spaces.....224
- 12. Sequences and Series of Functions.....227
 - Problems on Sequences and Series of Functions.....233
- 13. Absolutely Convergent Series. Power Series.....237
 - More Problems on Series of Functions.....245

Chapter 5. Differentiation and Antidifferentiation 251

- 1. Derivatives of Functions of One Real Variable.....251
 - Problems on Derived Functions in One Variable.....257
- 2. Derivatives of Extended-Real Functions.....259
 - Problems on Derivatives of Extended-Real Functions.....265
- 3. L'Hôpital's Rule.....266
 - Problems on L'Hôpital's Rule.....269
- 4. Complex and Vector-Valued Functions on E^1271
 - Problems on Complex and Vector-Valued Functions on E^1275
- 5. Antiderivatives (Primitives, Integrals).....278
 - Problems on Antiderivatives.....285
- 6. Differentials. Taylor's Theorem and Taylor's Series.....288
 - Problems on Taylor's Theorem.....296
- 7. The Total Variation (Length) of a Function $f: E^1 \rightarrow E$300
 - Problems on Total Variation and Graph Length.....306
- 8. Rectifiable Arcs. Absolute Continuity.....308
 - Problems on Absolute Continuity and Rectifiable Arcs.....314
- 9. Convergence Theorems in Differentiation and Integration.....314
 - Problems on Convergence in Differentiation and Integration....321
- 10. Sufficient Condition of Integrability. Regulated Functions.....322
 - Problems on Regulated Functions.....329
- 11. Integral Definitions of Some Functions.....331
 - Problems on Exponential and Trigonometric Functions.....338