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DAMAGE MODEL FOR A BIOSSOURCED HETEROGENEOUS MATERIAL: APPLICATION TO TIMBER

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ABSTRACT: The purpose of this study is to investigate the damage mechanisms of timber, by developing a numerical model based on the concept of damage mechanics. The first stage of this work is to study the existing damage models used for quasi-brittle materials and then propose a fitting to timber. Then a new model is proposed which allows to take into account the orthotropy of timber, the local variation of the mechanical properties due to heterogeneity induced by the defects of timber and the behaviour of timber according to the mode of loading in relation to the longitudinal axis of the grain. Finally, the model calibration is carried out through different experimental tests: fracture characterisation tests in mode I and II and bending tests integrating the heterogeneity of timber.

Keywords: Numerical model, damage mechanics, fracture, timber, quasi-brittle, ductility.

1 INTRODUCTION

To achieve the goals of sustainable development, the use of renewable materials is a response to the climate change. Timber has many advantages allowing it to be more attractive than materials used mainly in the field of construction (steel and concrete). Indeed, it is the only one totally renewable, natural and biological building material. Due to its advantages, research is focusing on improving its mechanical characterization. This means that researchers are focused on better understanding the mechanical properties of wood and finding ways to optimize its performance in terms of strength, stiffness and durability.

The design of wood structures is carried out according to the Eurocode 5 standard to guarantee their durability. It essentially implies a macroscopic characterization of the timber which is regarded as a homogenous material integrating the intrinsic its variability by semi-probabilistic analyses. In fact, wood is a highly anisotropic material, its properties are highly variable and are sensitive to loading types and environmental conditions in relation with the moisture content of the material.

Its mechanical behaviour is strongly influenced by the singularities (like knots) and also by the mode of solicitation with respect to the orthotropy directions. In fact, the bending failure tests show that the fracture is due, in more than 90% of the cases, to the presence of knots and the corresponding deviation of the grain slope. The influence of these defects has been the subject of several studies [1, 4, 6, 7, 10, 11] which show that most of the mechanical properties are lower in sections with knots than free defect wood. Longitudinal tension strength is the most affected property, followed by bending strength, compressive strength parallel to the grain and elasticity modulus. This reduction is due to a stress concentration caused by the presence of knots and the change in the orientation of the local frame along the fibres, thus changing the complacency matrix as a function of the grain slope. In view of this material complexity due to the variation of its mechanical properties, the development of mathematical laws presents an interesting challenge to establish reliable predictive models.

In order to improve the mechanical grading of timber and to limit qualification tests. New numerical approaches need to be developed to study its mechanical behaviour at failure, considering the anisotropy and the influence of its heterogeneities induced mainly by knots.

The majority of existing approaches are based on failure criteria associated with elastic behaviour [1, 3, 9...]. they used elastic failure criteria such as the Tsai-Hill criterion and the Tsai-WU criteria. The load that corresponds to the initiation of damage is determined, but these models are not able to correctly predict the strength and describe the shape of the post-peak portion of the load-displacement curve.

The simulation of timber behaviour requires a model that takes into account the defects of wood, the variation of the grain slope and that integrates the damage of the quasi-brittle anisotropic material to predict the initiation and the progressive damage up to the failure. This work is part of this framework, we propose a new non-linear approach based on the Mechanics of Continuous Damage (MCD) to predict the mechanical behaviour of timber.

MCD is a non-linear elastic approach that was developed in 1958 by Rabotnov and Kachanov [8] to study creep rupture and was subsequently used by several researchers to describe metal damage and failure. This

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concept is part of the thermodynamics of irreversible processes [9]. It has become a promising tool for describing material degradation. The damage is presented by scalar or tensorial variables depending on the objective to be achieved during the modelling. In our case, the damage is anisotropic and several variables are defined according to each type of stress (tension, compression or shear). The MCD is based on the concept of effective constraints which consists in associating to the damaged real space a fictional space for which the material is healthy (Figure 1). New quantities are defined in this space which must be related to the real quantities according to the principle of equivalence used (principle of equivalence in strain, principle of equivalence in stress or principle of equivalence in energy) [13].

Stress increments are calculated as a function of strain increments via an effective stiffness matrix determined as a function of the initial stiffness matrix and damage variables.

Validation of the approach is carried out in two stages. The purpose of the initial modelling is to verify the model parameters by applying it to mode I and II fracture tests on Maritime Pine wooden (free defects) specimens. Then the model is applying to study the bending behaviour of timber beams, integrating the local variation of mechanical properties due to defects.

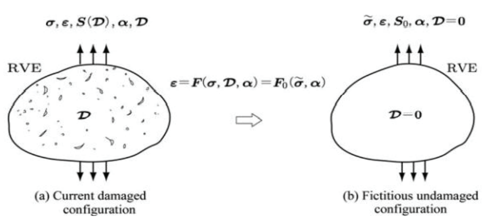

Figure 1: *Assumption of mechanical equivalence and effective stress [13]*

2 DEVELOPMENT OF THE BEHAVIOUR LAW

Before introducing the model, it is necessary to define some indices used for the wood material. Because of its mode of growth, the wood is characterized according to its three natural directions: longitudinal (L, axis of the tree), radial (R) and tangential (T) and its three planes: radial (LR), tangential (LT) and transverse (RT) (Figure 2)

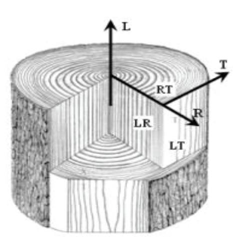

Figure 2: *orthotropy of wood [12]*

In the elastic field, the behaviour is determined by the generalized Hooke's law (Equation (1)), which shows the symmetrical elastic flexibility matrix, the components of this matrix are modified during loading according to the evolution of the damage variables.

$$\begin{bmatrix}\varepsilon_{RR}\\ \varepsilon_{TT}\\ \varepsilon_{LL}\\ \tau_{TL}\\ \tau_{LR}\\ \tau_{RT}\end{bmatrix} = \begin{bmatrix} \frac{1}{E_R} & \frac{-\vartheta_{RT}}{E_T} & \frac{-\vartheta_{RL}}{E_L} & 0 & 0 & 0\\ \frac{-\vartheta_{TR}}{E_T} & \frac{1}{E_T} & \frac{-\vartheta_{TL}}{E_L} & 0 & 0 & 0\\ \frac{-\vartheta_{LR}}{E_R} & \frac{-\vartheta_{LT}}{E_T} & \frac{1}{E_L} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{TL}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{RL}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{RT}} \end{bmatrix} \cdot \begin{bmatrix}\sigma_{RR}\\ \sigma_{TT}\\ \sigma_{LL}\\ \sigma_{TL}\\ \sigma_{LR}\\ \sigma_{RT}\end{bmatrix} \quad (1)$$

2.1 Initiation and evolution of damage

Existing damage laws developed for isotropic materials are generally applied even if the damage is anisotropic.

The proposed method to implement a damage model for an orthotropic material is based on laws often used for quasi-brittle isotropic materials [5]. These are adapted according to the mechanical and geometric characteristics of wood. In addition, the model must have a ductile behaviour relative to the compressive stresses.

For a complete description of timber as an orthotropic material, different strain and stress-based failure criteria have been defined in tension and compression (Figure 3).

Wood has a quasi-brittle behaviour in tension and shear and a ductile behaviour in compression.

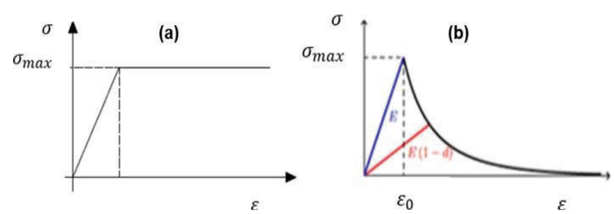

Figure 3: *Stress-strain: (a) ductile behaviour curve in compression, (b) softening behaviour in tension*

2.1.1 Tension behaviour

In tension parallel to the grain direction ($\sigma_L > 0$), damage initiation is governed by strain in the longitudinal direction ($\varepsilon_{t,L}$) and the elastic limit of tensile strain ($\varepsilon_{dt,L}$), the damage criterion ($F_{t,L}$) is given by the Equation (1):

$$F_{t,L} = \frac{\varepsilon_{t,L}}{\varepsilon_{dt,L}} - 1 \qquad (2)$$

In tension perpendicular to the grain direction ($\sigma_n > 0$) (n=R or T), damage is caused by tensile stresses (σ_n) and/or shear stresses (σ_{Ln} and σ_{RT}), the damage criterion in both directions perpendicular ($F_{t,n}$) is given by the Equation (2):

$$F_{t,n} = \frac{{\sigma_n}^2}{{f_{t,R/T}}^2} + \frac{{\sigma_{Ln}}^2}{{f_{Ln}}^2} + \frac{{\sigma_{RT}}^2}{{f_{RT}}^2} - 1 \quad (3)$$

with $f_{t,n}$, f_{Ln}, f_{RT} are respectively the elastic limit of tensile stress in n direction (n=R or T), the maximum shear stresses in Ln plane and the maximum shear stresses in RT plane.

In order to take into account, the quasi-brittle behaviour of wood in tension as well as the parameters relating to failure, we are interested in existing laws on the damage of isotropic materials such as the Fichant and Mazars law. These laws use fracture energy and strains as the main parameters for predicting material behaviour.

In this quasi-brittle case, damage variables ($d_{t,n}$) have an exponential shape determined through the following relationship based on the Fichant law [5]:

$$\begin{cases} d_{t,n} = 1 - \frac{\varepsilon_{dt,n}}{\varepsilon_{t,n}} \exp\left(B_n\left(\varepsilon_{dt,n} - \varepsilon_{t,n}\right)\right) & if\ F_{t,n} > 0 \\ d_{t,n} = 0 & if\ F_{t,n} \leq 0 \\ 0 \leq d_{t,n} \leq 1 \end{cases} \quad (4)$$

with $F_{t,n}$ is the damage initialisation criterion, $\varepsilon_{dt,n}$ is the elastic limit in tension in the direction n (n=L, R, T), $\varepsilon_{t,n}$ is the tensile strain in the direction n at the current time increment and B_n is a damage parameters depending on the elastic modulus, strain limit, cracking energy in n direction ($G_{f,n}$) and finite element size (h) by the following relation :

$$B_n = \frac{hE_n\varepsilon_{dt,n}}{G_{f,n} - \frac{1}{2}hE_n(\varepsilon_{dt,n})^2} \quad (5)$$

2.1.2 Compressive behaviour

For compressive strains parallel or perpendicular to the grain direction, timber has a ductile behaviour, in this case the damage is controlled by the compression deformations, the damage criterion ($F_{c,n}$) is given by the Equation (4):

$$F_{c,n} = \left|\frac{\varepsilon_{c,n}}{\varepsilon_{dc,n}}\right| - 1 \quad (6)$$

with $\varepsilon_{dc,n}$ is the compression threshold deformation in the direction n and $\varepsilon_{c,n}$ is the deformation at time t.

The evolution of the damage variable ($d_{c,n}$) is determined from the following relationship:

$$\begin{cases} d_{c,n} = 1 - \frac{1}{F_{c,n}} & if\ F_{t,n} > 0 \\ d_{c,n} = 0 & if\ F_{t,n} \leq 0 \\ 0 \leq d_{c,n} \leq 1 \end{cases} \quad (7)$$

Wood has a quasi-brittle shear behaviour, which is in the proposed approach, related to its tensile and compressive behaviour. For example, a failure parallel to the LR plane can be caused by a tension perpendicular to the mode I, a shear (mode II) or a combination of both. Therefore, it is not possible to define separate failure modes for each direction of stress, a coupling is proposed. Therefore, shear damage is determined by the tensile and compression damage.

Finally, there are six damage variables, three in tension ($d_{t,n}$) and three in compression ($d_{c,n}$) (n=R, T or L).

Using Macaulay's mathematical operator to differentiate damage variables that are activated by the same stress component, but are sensitive to the sign of the component, i.e. tensile or compressive stresses.

So in each direction of orthotropy the damage variable is given by the following relation:

$$D_n = d_{t,n}\frac{\langle\sigma_n\rangle}{|\sigma_n|} + d_{c,n}\frac{\langle-\sigma_n\rangle}{|\sigma_n|}\ ;\ \text{with}\ \langle a\rangle = \frac{a+|a|}{2} \quad (8)$$

We can define the diagonal tensor which represents the orthotropic damage due to the reduction of the effective bearing surface:

$$D = \begin{bmatrix} D_R & 0 & 0 \\ 0 & D_T & 0 \\ 0 & 0 & D_L \end{bmatrix} \quad (9)$$

3 NUMERICAL SIMULATION OF FAULTLESS WOOD MECHANICAL BEHAVIOUR

In order to carry out the validation of the developed model, the experimental results of Perez [15] in mode I and II on perfect Maritime pine specimens are compared to the numerical results. The geometry of the specimen and the proposed limit conditions for the three experimental configurations (mode I and II) are shown in Figure 4. First, we used the mean experimental elastic and mechanical properties and mean experimental cracking energies.

Table 1: *Elastic and mechanical model parameters*

Parameter	Value
E_L (MPa)	10900
E_T(MPa)	1050
ϑ_{LT}	0.43
ϑ_{TR}	0.39
ϑ_{LR}	0.32
G_{LT} (MPa)	1030
$f_{t,L}$(MPa)	68
$f_{t,T}$(MPa)	3
$f_{c,L}$(MPa)	38
$f_{c,T}$(MPa)	4
$G_{f,I}$(J/m²) (Mode I cracking energy)	550
$G_{f,II}$(J/m²) (Mode I cracking energy)	1739

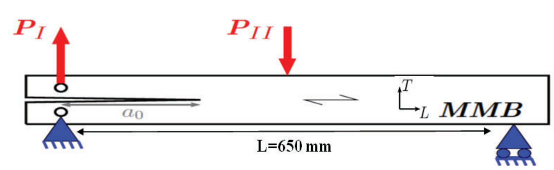

Figure 4: *Mode I DCB test (for $\boldsymbol{P_{II} = 0}$) and Mode II test (for $\boldsymbol{P_I = 0}$) (same specimen size for all tow failure modes)*

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The numerical results are compared with the experimental ones by analysing the load-displacement curves and the corresponding resistance curves to verify the cracking energies declared in the model. The resistance curves (R-curve) are determined from the load-displacement curves and on the basis of the equivalent elastic fracture mechanics (MLER*eq*). They provide an estimate of the fracture properties and also information on the extent of the fracture development zone. Bazant and Kazemi [2] introduced the notion of equivalent elastic crack length (a_{eq}) in their expression of MLER*eq*. This approach defines the equivalent elastic crack length to that which, in a perfect elastic model, produces the same complacency as the actual crack with its development zone. Furthermore, the increase in complacency depends only on the extent of the potential fracture zone (FPZ) or the progression of the macro-crack with its development zone. On this basis, the crack propagation resistance can be estimated using the following equation proposed by Morel et al. (2005) [14].

$$G_R(a) = \frac{P^2}{2b}\frac{d\lambda(a)}{da} \tag{10}$$

with λ(a) is the compliance such that λ(a) = δ(a)/P(a) with δ(a) and P(a) are the displacement and load for an equivalent elastic crack length a The equivalent elastic crack length is numerically determined from a linear elastic finite element model.

The R-curve for a quasi-brittle material can be divided into two zones. The first regime corresponds to the expansion of the elaboration zone until it reaches its critical size. The second regime is the plateau, which occurs when the macro-crack propagates in auto-similarity with its zone of development. The plateau value of the R-curve supplies the critical resistance to crack propagation, which is the cracking energy declared in the model to describe the mechanical behaviour of the material.

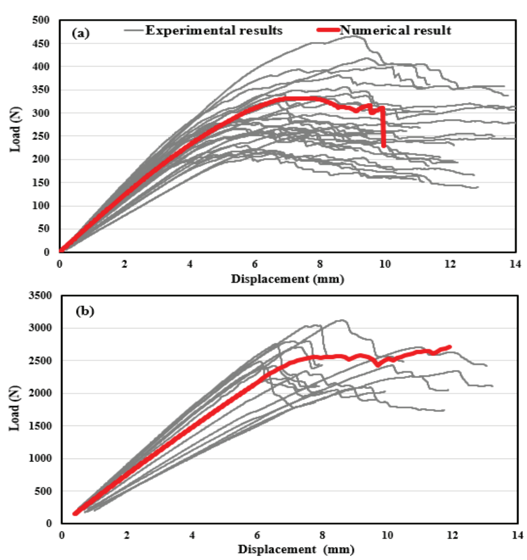

Figure 5: *Load -displacement curves: (a) mode I; (b) mode II; (c) mixed mode*

Using the average elastic and fracture properties of Perez, we observe a good agreement of the model in mode I and II (Figures 5 and 7).

Then, to verify each test, we performed a curve-to-curve comparison using the failure properties of each test (test 1: G_{fI}=400 J/m² ; test 2 : G_{fI}=520 J/m² ; test 3: G_{fI}=690 J/m²). The Figure 6 shows a comparison between experimental and numerical results for three tests performed in mode I.

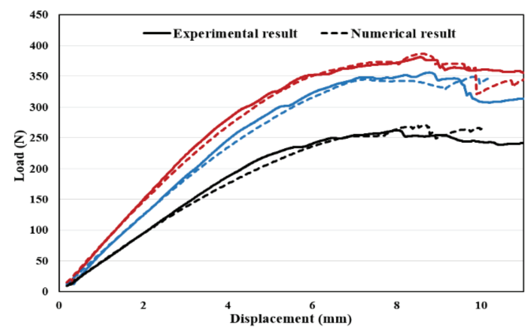

Figure 6: *Curve to curve comparison between numerical and experimental result (Mode I Mode II Mixed Mode colours)*

The numerical and experimental results are close and agree significantly, the numerical methods are reliable and can be used to predict the experimental results with some accuracy.

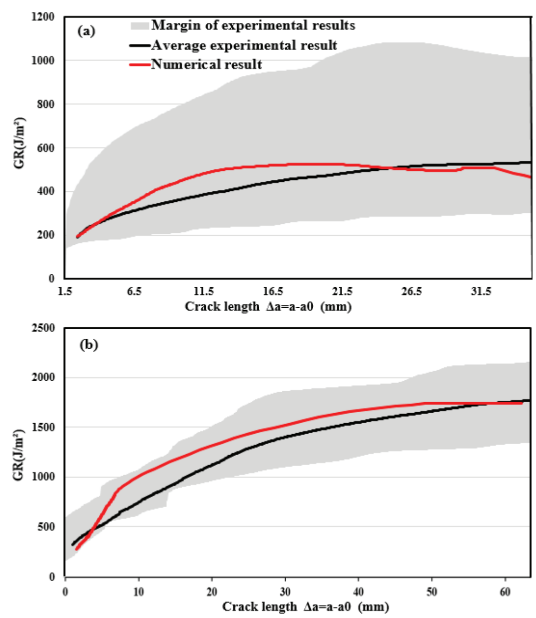

Figure 7: *Resistance curves: (a) mode I; (b) mode*

4 NUMERICAL SIMULATION OF THE MECHANICAL BEHAVIOUR OF HETEROGENEOUS WOOD

The model is then tested on heterogeneous wood by under bending tests. Eleven beams were tested in three-point bending of 1200 mm length and a section of 100×50 mm².

Before the test, the variation of the grain slope and the geometric parameters of the knots is obtained experimentally and is taken into account in the modelling to determine the local elastic and fracture properties of the wood. The measurement of the grain slope is performed by a device named Xyloprofile. It is based on the projection of a line of laser points perpendicular to the longitudinal axes of the wood and by moving the beam longitudinally, it is possible to measure the fibre angle every 5 mm (Figure 8).

Figure 8: *Xyloprofile wood scan and measurement of the wire slope variation*

The modelling of the grain slope is performed from a numerical procedure that is able to read the experimental data of the slope as a function of the coordinates of each point of the beam and interpolate it in the mesh to build the field of variation of the slope. Then, the local elastic and mechanical parameters are determined by pass matrices calculated as a function of the slope value in each mesh element (Figure 9).

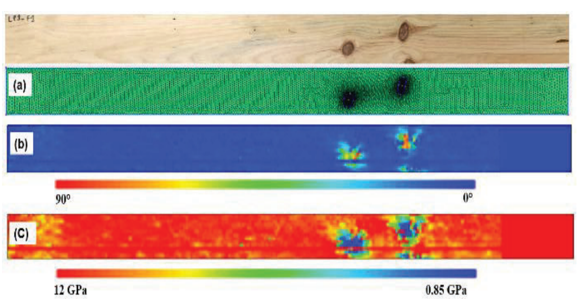

Figure 9: *Numerical modelling of grain slope: (a) mesh by EF, (b) variation of the grain angle in each point of the mesh, (b) variation of the longitudinal modulus of elasticity as a function of the grain slope*

Once we have all the necessary information on the variation of fibre inclination in the surfaces of each beam, on the position of knots and their geometry, the beams are then tested in three-point bending to determine their actual behaviour in the event of failure (Figure 10).

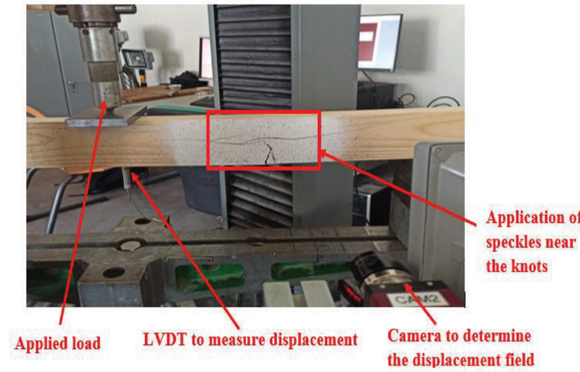

Figure 10: *three-point bending test*

Numerical modelling (2D) of these beams is performed using the average mechanical properties of perfect wood and experimental data of the slope variation. Then the failure stress is determined by taking the average of the numerical results found for the two load of the beam. The figure (Figure 11) shows the load-displacement curves found experimentally and by numerical simulation, as well as the cracking pattern at the fracture time of the three-points bending beams.

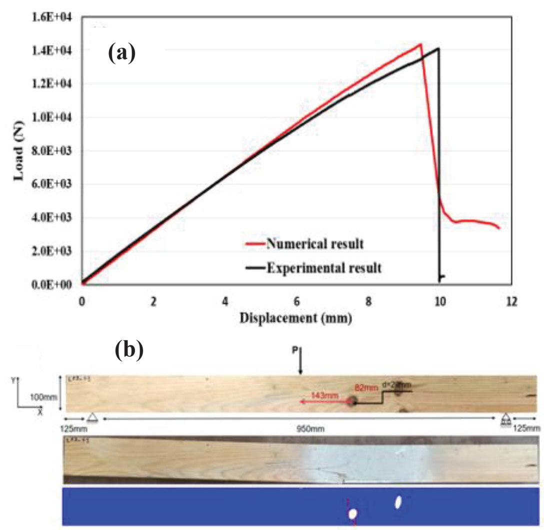

Figure 11: *Comparison between numerical and experimental results for a heterogeneous wood beam subjected to bending loading: (a) prediction of breaking load, (b) fracture facies.*

With the data of the variation of the grain slopes and the average mechanical properties of the perfect wood, we found a good correlation between the numerical and experimental results of the breaking load and displacement which corresponds to the breaking stress (Figure12).

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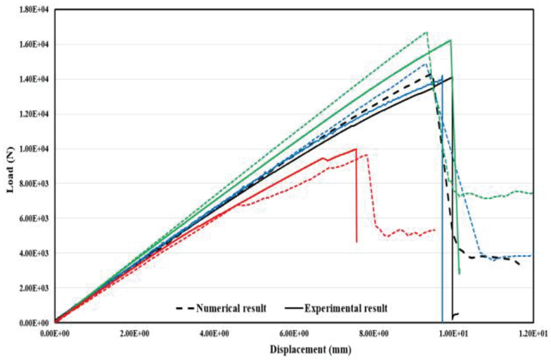

Figure 12: Comparison between experimentally and numerically obtained fracture loads for four test cases

In order to determine the displacement and strain fields that occur near the knots, a non-contact measurement method based on digital image correlation (2D) is used. This method allows for local observation of the material behaviour on the pre-nodal surface during the test. This can be very useful in understanding how the wood reacts to applied load and how the knots affect the stability of the structure in general. The results are then used to validate the developed numerical model by comparing the experimental and numerical deformation field values for the same loading times.

The Figure 13 shows the displacement fields (ε_{xx}) obtained from numerical modelling and by image correlation for the same imposed displacement. Comparing the two methods, it can be seen that the strain distribution is very similar around the knots. Furthermore, the strain values obtained from both methods are close in scale.

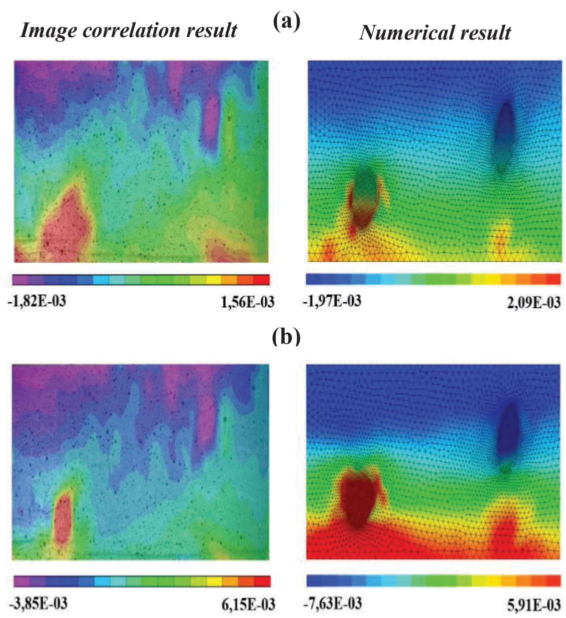

Figure 13: Comparison between the displacement field obtained by numerical modelling and by image correlation: (a) ID= 3 mm; (b) ID = 6 mm (ID= imposed displacement)

5 CONCLUSIONS

In this work, an anisotropic damage model for wood has been developed. The proposed model is able to model the fracture behaviour of wood with its heterogeneities, by implementing the damage criteria of wood according to the loading mode: a softening behaviour of wood in tension and shear and a ductile behaviour in compression. The satisfactory agreement obtained between the simulation results and the experimental results confirms the validity of the approach and the damage model proposed for the experimental conditions considered in this work. The integration of this mechanical model is a way of improvement. The next work consists in studying different timber species.

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