

THEORETICAL AND EXPERIMENTAL INVESTIGATION ON PREDICTING LONGITUDINAL AND TANGENTIAL ELASTIC CONSTANTS AND RATIOS OF WOOD

Shaghayegh Kurzinski¹, Paul L. Crovella²

ABSTRACT: Among all the elastic constants of wood that are required for the engineering design of wood structures, only the modulus of elasticity in the longitudinal (L) direction (E_0 or E_L) is readily available or can be readily measured for the majority of species. Longitudinal and tangential moduli of elasticity and shear moduli are required to predict the structural performance of any composite timber product such as mass timber products. This study selected a low-density local softwood; eastern white pine, and a high-density local hardwood; black locust, to investigate their longitudinal and tangential elastic properties. The eastern white pine and black locust defect-free specimens were prepared and tested under centre-point bending loading (ASTM D198-2015 [1]) alternately on true longitudinal and tangential surfaces. Also, linear regressions with respect to the longitudinal modulus of elasticity were performed on the tangential elastic modulus, and the longitudinal-radial and tangential-radial shear moduli of the softwood and hardwood species with available mechanical properties ratios published in the USDA Wood Handbook [2]. These elastic constants and their ratios from the experiment have been compared to the predicted values and ratios from the regressions using the published values of the species in the USDA Wood Handbook. This provides data on the accuracy of assumed elastic ratios in published references for elastic and shear moduli.

KEYWORDS: Shear modulus, Modulus of elasticity, Centre-point bending test, Elastic ratios of wood, Elastic constants of wood.

1 INTRODUCTION

The arrangement and geometry of wood cells in the tree define three local symmetric axes in the material which are referred to as longitudinal (L, parallel to the grain direction and tangent to the growth rings), tangential (T, perpendicular to the grain) and radial (R, perpendicular to the grain direction and normal to the growth rings) [3]. These three axes determine the mechanical properties of wood by nine elastic constants and related compliances typically used in the engineering design of wood structures. Characterization of these constants in classical mechanics is done in reference to orthogonal axes. However, it should be noted that wood is not a truly orthotropic material because of growth ring curvature. Plain-sawn timber boards are generally used as typical layers of composite engineered timber products when the majority of the surface perpendicular to the major load direction is the tangential surface rather than the radial surface. Therefore, as shown in **Figure 1** the longitudinal and tangential elastic moduli (E_L and E_T) can be referred to as elastic moduli parallel to the grain (E_0) and perpendicular to the grain (E_{90}) respectively. Similarly, the longitudinal-radial and tangential-radial shear moduli can also be referred to as shear moduli parallel to the grain (G_0) and perpendicular to the grain (G_{90}) respectively. The modulus of elasticity in the longitudinal (L) direction (E_0 or E_L) is readily available or can easily be measured for the majority of wood species. The other elastic

constants, such as elastic moduli in the radial (R) and tangential (T) directions, the three Poisson's ratios, and the three shear moduli associated with the three major orthotropic planes, have not been thoroughly examined for most species due to the difficulty of making appropriate experimental measurements.

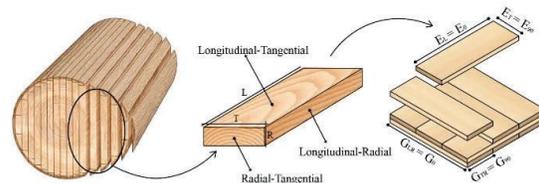


Figure 1: Plain-sawn Cut Board Details and Usage in a Cross-laminated Timber Panel, multi-layered mass timber product [4]

To make up for the lack of engineering data on a large number of species, there have been a few methods proposed to predict the elastic constants and their correlations in different species. Adamopoulos [5] studied the difference between the shear modulus in two orientations of longitudinal-radial (G_{LR}) and longitudinal-tangential (G_{LT}), in juvenile and mature black locust wood. They performed centre-point bending tests on 66 small clear specimens and measured the shear modulus in each load direction. Their study showed that the shear moduli in longitudinal-radial and longitudinal-tangential for black locust samples were not significantly different.

¹ Shaghayegh Kurzinski, Ph.D., shaykurzinski@gmail.com

² Paul L. Crovella, Ph.D., Assistant Professor, Construction Management, SUNY ESF, Syracuse, NY, USA. plcrovella@esf.edu

The shear moduli values in juvenile wood, however, were slightly lower than the values in mature wood. Divos et al. [6] used three different methods: static bending tests, torsional vibration, and the variation of span method to determine the shear modulus of construction-size timber. They found that between these methods, the static bending test and torsional techniques provided a more precise prediction of the shear constant in different directions.

In research done by Sliker [7], longitudinal-tangential and longitudinal-radial shear moduli of a variety of softwood and hardwood species were determined by two methods: the two-plate shear test and the off-axis tensile test at a 20° angle of load to the grain. In the longitudinal-tangential plane, the shear modulus (G_{LT}), calculated from the off-axis tensile test ranged from 0.87 to 1.25 times the shear modulus (G_{LT}), from the plate test. In the longitudinal-radial plane, the shear modulus (G_{LR}), from the off-axis tensile test ranged from 0.98 to 1.26 times the shear moduli G_{LR} , from the plate test. For the tested species, the linear regressions done for Poisson's ratios in correlation with shear and elastic moduli did not show any significant relationships for either of Poisson's ratios ν_{lr} or ν_{tr} as functions of density, E_L , or E_T .

Yoshihara et al. [8] examined the applicability of Timoshenko's theory and proposed an empirical equation to derive the shear modulus. To validate this empirical approach, three softwoods and three hardwoods were tested. First, the Young's and longitudinal-radial shear moduli were measured by free-free flexural vibration tests. Then three-point static bending tests were undertaken, varying the depth/span ratios. Additionally, the bending tests were simulated by the finite element method (FEM). The longitudinal-radial shear moduli obtained by these methods were then compared. They concluded a new prediction method for determining the longitudinal-radial shear modulus by modifying Timoshenko's theory.

The longitudinal modulus of elasticity to shear modulus ratios of different lumber species is not constant. It varies between 5 and 24 depending on the softwood and hardwood species and the grain direction (L, T, and R). There are published references that assume a typical ratio for longitudinal-radial and tangential-radial shear and tangential elastic moduli regarding the longitudinal elastic modulus (E_L/E_T , E_L/G_{LR} , E_L/G_{TR}). For instance, the North American and European Cross-laminated Timber (CLT) standards; ANSI/APA PRG 320-2019 [9] and EN 16351-2021 [10] assume these ratios to be: $E_0/E_{90}=30:1$, $E_0/G_0=16:1$, and $E_0/G_{90}=160:1$ for softwood species. The timber standards for Australia/New Zealand AS1720-2010 [10] and the Brazilian Timber standard [12] assume the ratio of E_0/G_0 to be around 15:1 for softwoods [13]. This study has selected one local low-density softwood and one local high-density hardwood species to investigate the shear and elastic moduli in two grain directions, longitudinal and tangential. Also, using linear regressions, the study evaluates the elastic ratios of E_0/E_{90} , E_0/G_0 , E_0/G_{90} for softwoods and hardwoods with available elastic ratios in the USDA Wood Handbook [2]. The values and ratios of the elastic constants from the experimental testing have been compared to those from

the regressions. This study can provide insight into predicting elastic ratios for species with no available published or tested elastic constant values.

2 MATERIALS AND METHODOLOGY

2.1 SPECIMEN PREPARATION AND DIMENSIONS

A total of 20 small black locust specimens and 20 eastern white pine specimens, 19-mm thick and 19-mm wide, with different lengths over a range from 89-mm to 280-mm (as shown in **Table 1**) were cut (respectively) from two defect-free, ungraded, rough-sawn black locust boards obtained from a local sawmill in Newfield, NY and one defect-free NeLMA [14] finish grade board purchased from a local lumber supplier near Syracuse, NY. The specimens' cross-section and lengths have been prepared according to the standard's requirement [1] for the centre-point bending test explained in **Section 2.2**. The boards were surfaced and planed to the desired thickness before the cutting process. The final cut specimens were straight-grained and free from any visible defects. The specimens were conditioned in an environmental chamber set at 20°C and 65% relative humidity until a constant weight was achieved. The final measured moisture content of the boards was approximately in the range of 12±1%. The specimens were prepared to be tested in two different directions relative to the wood grain orientation.

The centre-point bending test has been used to determine the apparent modulus of elasticity by measuring the uniform strain state at the centre of the specimens which is a combination of shear and bending deflection. The relative contribution of bending and shear deflections to the total deflection vary with span-to-depth ratio of the specimen. The measured apparent modulus of elasticity can help determining the shear-free modulus of elasticity by excluding the shear deflection to provide a true bending stiffness.

Table 1 shows the matrix of samples of each species divided into two groups tested under centre-point bending tests in the longitudinal and tangential directions for measuring the apparent modulus of elasticity (E_{app}) in both longitudinal and tangential directions and (respectively) the shear-free longitudinal elastic modulus (E_L) with longitudinal-radial shear moduli (G_{LR}) and shear-free tangential elastic modulus (E_T) with the tangential-radial shear modulus (G_{TR}). **Figure 2** shows the final specimens with different span lengths.

Table 1: Specimens and Testing Details

Species	Sample (Test Direction)	Sample Size	Span to Depth (l/d)
Eastern White Pine	Longitudinal-Radial Specimen	10	14:1, 11.3:1, 10.6:1 10:1, 9.3:1, 8.6:1, 8:1, 7.3:1, 6.6:1, 6:1
	Tangential-Radial Specimen	10	10:1, 9.3:1, 8.6:1, 8.3:1, 8:1, 7.6:1, 7.3:1, 7:1, 6.6:1
Black Locust	Longitudinal-Radial Specimen	10	14.8:1, 14.1:1, 13.5:1, 12.8:1, 12.1:1, 9.1:1, 8.4:1, 7.7:1, 7:1, 6:1
	Tangential-Radial Specimen	10	7.6:1, 7.3:1, 7.1:1, 7:1, 6.6:1, 6.5:1, 6.3:1, 6:1, 5.6:1, 5.3:1

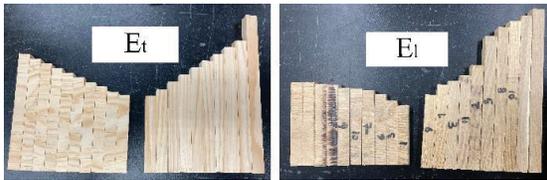


Figure 2: Specimen Samples with Different Span Lengths Prepared for Centre-point Bending Test (Eastern White Pine: Left, Black Locust: Right)

2.2 CENTRE-POINT BENDING TEST

A non-destructive centre-point bending test as described in ASTM D198-2015 [1] (section 7.3.2.2) was performed on the specimens to provide the shear modulus and modulus of elasticity. The standard requires at least four different span lengths and the same depth such that the span-to-depth ratio (l/d) is between 20:1 and 5:1 ($0.0025 < (d/l)^2 < 0.035$). Using this method, the apparent modulus of elasticity and the shear modulus were predicted. The centre-point bending test was performed using a tabletop Shimadzu Precision Universal Tester 10KN AGX model machine set to a loading rate of 0.889 mm per minute. The test was performed until the proportional limit in the elastic range was reached and before the total failure of the specimen. The data was recorded with Trapezium X Software and a video extensometer. **Figures 3 and 4**, respectively, show illustrative and actual details of the test configuration and the loading direction in each group with respect to the wood grain orientation.

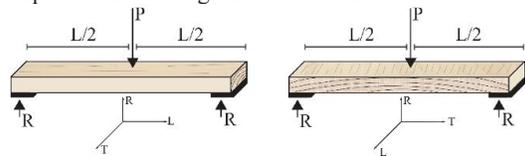


Figure 3: Center-point Bending Test Details with Respect to Specimens' Grain Orientation



Figure 4: Actual Center-point Bending Tests to Determine Shear and Elastic Moduli

According to ASTM D198-2015 [1] to calculate the shear modulus, first, the value for the apparent modulus of elasticity should be acquired using **Equation 1**:

$$E_{app} = \frac{PL^3}{4bd^3\Delta} \quad (1)$$

where E_{app} is the apparent elastic modulus at a specific span-to-depth ratio and grain orientation, P is the load at the proportional limit, b is the width of the specimen, d is the depth of the specimen and Δ is the deflection that occurs up to the proportional limit.

After determining the values for apparent elastic modulus for different span lengths, their inverses were plotted versus the span-to-depth ratios. The slope of this linear curve is a factor in predicting the shear modulus according to **Equation 2**,

$$G = \frac{6}{5K_1} \quad (2)$$

where G is the shear modulus with respect to the loading direction on the specific grain orientation, and K_1 is the slope of the inverses of apparent elastic moduli versus the span-to-depth ratio (**Error! Reference source not found. 5**).

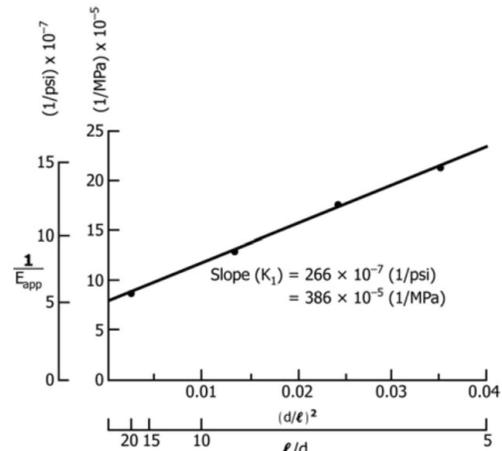


Figure 5: Shear Modulus Determination based on the Apparent Elastic Modulus and Span-to-Depth Ratio [13]

After the shear modulus is found for each group of specimens, the shear-free modulus of elasticity can be calculated using **Equation 3**.

$$E_{sf} = \frac{PL^3}{4bd^3\Delta(1 - \frac{3PL}{10bd\Delta G})} \quad (3)$$

where E_{sf} is the shear-free modulus of elasticity, P is the load at the proportional limit, b is the width of the

specimen, h is the depth of the specimen and Δ is the deflection that occurs up to the proportional limit, and G is the shear modulus with respect to the loading direction on the specific grain orientation.

2.3 THEORETICAL INVESTIGATION OF SHEAR AND ELASTIC MODULI CORRELATION

One of the most easily measured physical properties of wood is the Young's modulus or modulus of elasticity (MoE) in the longitudinal direction (E_0 or E_L). This quantity is available for the majority of the wood species in technical publications, or it can also be obtained on a piece-by-piece basis from non-destructive testing. The USDA Wood Handbook [2] provides ratios for shear moduli in different grain directions (RT, LT, LR) and elastic moduli in tangential and radial directions with respect to the elastic modulus in the longitudinal direction for a few hardwood and softwood species. To predict tangential elastic modulus and longitudinal-radial and tangential-radial shear moduli of eastern white pine and black locust, the Handbook's softwood and hardwood species with available elastic ratios were selected (include the number). The shear moduli in longitudinal-radial and tangential-radial and elastic modulus in tangential directions were calculated for these softwoods and hardwoods using the available elastic ratios. Linear regressions for each elastic constant from the selected species were performed using Microsoft Excel [15] to provide a predicted ratio with respect to the longitudinal elastic modulus. Since the available reference [2] shows a true linear relation of $Y=AX$, for each elastic constant regarding the longitudinal elastic modulus, an intercept-free linear model was selected for predicting the elastic ratios. These ratios were used to theoretically estimate values for shear moduli in longitudinal-radial and tangential-radial directions (G_{LR} and G_{TR}) and tangential elastic modulus (E_T) for eastern white pine and black locust with respect to their published longitudinal elastic modulus (E_L).

3 RESULTS AND DISCUSSION

3.1 MEASURED ELASTIC CONSTANTS AND RATIOS BY CENTER-POINT BENDING TESTS

The curves showing the correlations between $1/E_{app}$ and span-to-depth ratios (l/d) obtained from the centre-point bending tests for longitudinal-radial and tangential-radial specimens are presented in **Figures 6** and **7**, for eastern white pine, and **Figures 8** and **9** for black locust. The shear moduli values have been calculated from these curves using **Equation 2**. The final values of shear moduli (G_{LR} and G_{TR}) and the measured apparent and shear-free elastic moduli in each loading direction (E_L and E_T) are presented in **Table 2**.

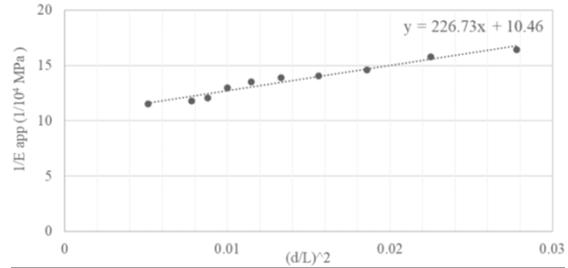


Figure 6: Apparent Elastic Modulus vs. Span-to-Depth Ratio Curves of Eastern White Pine Specimens in Longitudinal-Radial Direction

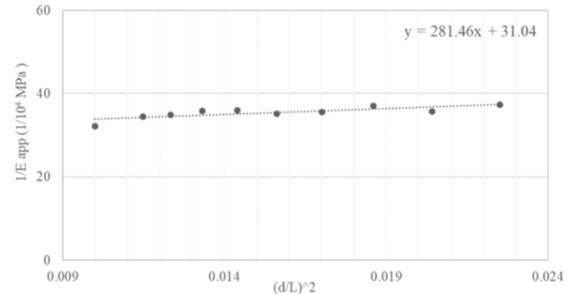


Figure 7: Apparent Elastic Modulus and Span-to-Depth Ratio Curves of Eastern White Pine Specimens in Tangential-Radial Direction

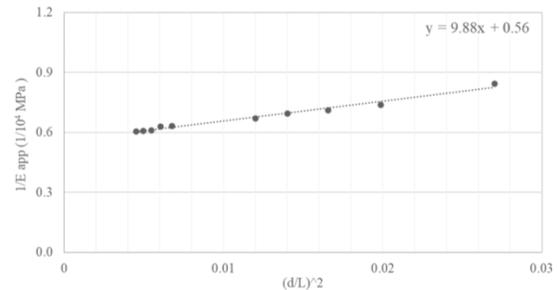


Figure 8: Apparent Elastic Modulus and Span-to-Depth Ratio Curves of Black Locust Specimens in Longitudinal-Radial Direction

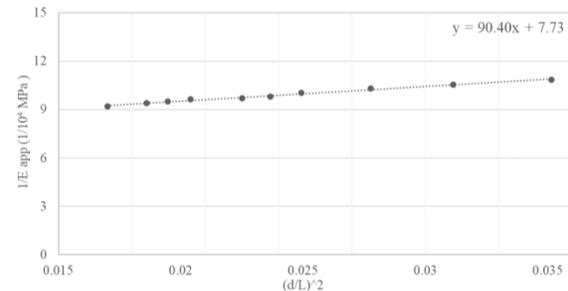


Figure 9: Apparent Elastic Modulus and Span-to-Depth Ratio Curves of Black Locust Specimens in Tangential-Radial Direction

Table 2: Final Measured Values of Apparent Elastic Modulus of Small Specimens Under Centre-point Bending Test

Species	Properties	Longitudinal	Tangential
Eastern White Pine	Apparent Modulus of Elasticity (MPa)	$E_{app} = 7,418$	$E_{app} = 283$
Black Locust	Apparent Modulus of Elasticity (MPa)	$E_{app} = 14,993$	$E_{app} = 1,013$

3.2 THEORETICAL PREDICTED ELASTIC CONSTANTS AND RATIOS FROM THE ELASTIC REGRESSIONS

The published longitudinal modulus of elasticity (E_L) values of eastern white pine and black locust have been found from the USDA Wood Handbook [1] to be respectively 8,555 MPa and 14,134 MPa. To theoretically predict the values of longitudinal-radial and tangential-radial shear and tangential elastic moduli of these two species, the estimated elastic ratios from the regressions on the USDA Wood Handbook's [2] softwoods and hardwoods were used. These linear regressions of tangential elastic modulus and shear moduli with respect to longitudinal elastic modulus are presented in **Table 3**. The final theoretical elastic ratios and values of eastern white pine and black locust with respect to the predicted elastic ratios from the regressions are shown and compared to the measured ratios and values, respectively in **Table 4** and **Table 5**.

Table 3: Linear Regression Components of Tangential Elastic and Shear Moduli of USDA Wood Handbook Softwood and Hardwood Species with Respect to their Longitudinal Modulus Species

Species	Regression	R ² Value	Regression Model
Softwood	$E_L - E_T$	0.9157	$Y=0.0547X$
	$E_L - G_{LR}$	0.8631	$Y=0.073X$
	$E_L - G_{TR}$	0.7426	$Y=0.0057X$
Hardwood	$E_L - E_T$	0.9386	$Y=0.061X$
	$E_L - G_{LR}$	0.9403	$Y=0.0921X$
	$E_L - G_{TR}$	0.941	$Y=0.0194X$

Table 4: Predicted and measured Shear and Elastic Moduli Values with Respect to Longitudinal Elastic Modulus (E_L) from Regressions on the USDA Wood Handbook Species

Species	Properties	Longitudinal	Tangential
Eastern White Pine	Predicted Modulus of Elasticity (MPa)	$E_L = 8,550$	$E_T = 468$
	Predicted Shear Modulus (MPa)	$G_{LR} = 624$	$G_{TR} = 49$
	Measured Modulus of Elasticity (MPa)	$E_L = 9,569$	$E_T = 322$
	Measured Shear Modulus (MPa)	$G_{LR} = 529$	$G_{TR} = 43$
Black Locust	Predicted Modulus of Elasticity (MPa)	$E_L = 14,134$	$E_T = 862$
	Predicted Shear Modulus (MPa)	$G_{LR} = 1,302$	$G_{TR} = 274$
	Measured Modulus of Elasticity (MPa)	$E_L = 17,925$	$E_T = 1,294$
	Measured Shear Modulus (MPa)	$G_{LR} = 1,215$	$G_{TR} = 133$

Table 4: Measured and Predicted Ratios of Shear and Elastic Moduli (LR: Longitudinal-Radial, TR: Tangential-Radial)

Species	Ratio	Measured	Predicted
Eastern White Pine	E_L/E_T	29.6	18.3
	E_L/G_{LR}	18.0	13.7
	G_{LR}/G_{TR}	12.4	12.8
Black Locust	E_L/E_T	13.8	16.4
	E_L/G_{LR}	14.8	10.85
	G_{LR}/G_{TR}	9.1	4.7

According to the experimental and theoretical results, the longitudinal shear-free elastic moduli for eastern white pine and black locust from the centre-point bending tests are respectively 11% and 26% higher than the published longitudinal elastic modulus in the USDA Wood Handbook [2]. The predicted shear-free tangential elastic modulus from regressions is approximately 45% higher and 33% lower than the measured tangential elastic modulus for eastern white pine and black locust respectively. The predicted longitudinal-radial shear modulus is 15% and 6% higher than the measured longitudinal-radial shear modulus for eastern white pine and black locust respectively. Also, the predicted tangential-radial shear modulus is 14% and 106% higher than the measured tangential-radial shear modulus for

eastern white pine and black locust respectively. Between the elastic ratios from measured constants, the ratio of eastern white pine longitudinal-radial shear modulus to tangential-radial shear modulus is almost the same (3% difference percentage) as the one predicted from regressions. However, the same ratio (G_{LR}/G_{TR}) for black locust is almost half of what is found based on the measured constants from the centre-point bending tests. Also, the elastic ratio of predicted longitudinal to the tangential modulus of elasticity for eastern white pine is approximately 30% of the measured one while the other predicted elastic ratios from the regressions have a percentage prediction error between 10% and 20% of the measured ratios from the tests.

4 CONCLUSIONS

This research investigated the values and ratios of shear and elastic moduli of black locust and eastern white pine experimentally and empirically. The results showed that longitudinal and tangential shear moduli and tangential elastic modulus can be predicted using longitudinal elastic modulus. The measured elastic ratios of longitudinal to the tangential modulus of elasticity (E_L/E_T) for eastern white pine from the tests were in agreement with the suggested ratios of modulus of elasticity parallel to the grain to the modulus of elasticity perpendicular to the grain (E_0/E_{90}) for softwoods in North American Cross-laminated Timber standard, ANSI/APA PRG 320 [9] to be 30:1. The measured longitudinal elastic modulus to the longitudinal-radial shear modulus ratio ($E_L/G_{LR}=18:1$) of the eastern white pine was approximately 17% higher than the suggested elastic ratio of shear modulus parallel to the grain to the modulus of elasticity parallel to the grain ($E_0/G_0=16:1$) for softwoods in ANSI/APA PRG 320 [9]. Therefore, it can be concluded that ANSI/APA PRG 320 [6] provides a conservative prediction of softwoods' elastic ratios compared to the regressions from the USDA Wood Handbook [2] elastic values.

The regression between hardwoods' tangential-radial shear modulus to the longitudinal modulus of elasticity showed a non-conservative agreement with the measured value. The reason for this difference may be the small sample size for the regression. The USDA Wood Handbook [2] provides the tangential-radial shear modulus (G_{TR}) for only six hardwood species. Except for the predicted longitudinal-radial shear modulus of black locust to its published longitudinal modulus of elasticity, the other predicted elastic ratios for this species with respect to the hardwoods' regressions, have predicted a ratio with a percentage prediction error approximately between 10% and 20% of the measured ratios from the tests.

This study provided methods for predicting the shear and elastic moduli in different grain directions of the species without available published elastic constants values. To predict the final stiffness value of any composite or mass timber product (e.g. CLT), the shear and elastic values of the species used in the layers are required. There are several non-standard species that might benefit the timber industry due to density, resource availability, locality, etc. but are not being used for the fabrication since there is no

available published value for their elastic constants. Therefore, predicting the elastic constants of different wood species with unknown elastic values can help in using them as individual layers in several mass timber products such as Cross-laminated Timber panels. Further research is recommended to experimentally investigate the elastic values of other softwoods and in particular hardwoods to validate the predicted elastic ratios from the regressions provided in this study.

REFERENCES

- [1] Forest Products Laboratory, USDA Wood handbook—Wood as an engineering material. General Technical Report FPL-GTR-190., Madison, WI: U.S., 2021. <https://doi.org/10.1161/01.RES.39.4.523>.
- [2] A.R. de Geus, S.F. da Silva, A.B. Gontijo, F.O. Silva, M.A. Batista, J.R. Souza, An analysis of timber sections and deep learning for wood species classification, *Multimedia Tools and Applications*. 79 (2020) 34513–34529. <https://doi.org/10.1007/s11042-020-09212-x>.
- [3] Xavier J, Jesus, A, Morais, J, Pinto, J, On the determination of the modulus of elasticity of wood by compression tests parallel to the grain, *Revista da Associação Portuguesa de Análise Experimental de Tensões (APAET)*, 20 (2012), 59-66.
- [4] Adamopoulos.S, Flexural Properties of Black Locust (*Robinia pseudoacacia*) Small Clear Wood Specimens in Relation to the Direction of Load Application, *Holz Als Roh-Und Werkstoff*, Springer-Verlag. 60 (2002) 325–327.
- [5] F. Divos, T. Tanaka, H. Nagao, H. Kato, Determination of shear modulus on construction size timber, *Wood Science and Technology*. 32 (1998) 393–397.
- [6] A. Sliker, A Method for Predicting Non-shear Compliances in the RT Plane of Wood, *Wood and Fiber Science*. 20 (1988) 44–45.
- [7] H. Yoshihara, Y. Kubojima, K. Nagaoka, M. Ohta, Measurement of the shear modulus of wood by static bending tests, *Journal of Wood Science*, 44 (1998).
- [8] American National Standard Institute, ANSI-APA/PRG 320 - Standard for Performance-Rated Cross-Laminated Timber, APA – The Engineered Wood Association, New York, NY, 2019.
- [9] CEN National Members., EN 16351-2021, Timber Structures, Cross laminated timber, Requirements., (2021).
- [10] Australian Standard Committee TM-001, AS 1720.1-Timber structures - Part 1: Design methods, Council of Standards Australia, Sydney, Australia, 2010.
- [11] Brazilian Committee on Civil Construction, ABNT NBR 7190, Brazilian Standard: Design of Timber Structures, ABNT/ CB-02, 2011.
- [12] S. Kurzinski, P. Crovella, P. Kremer, Overview of Cross-Laminated Timber (CLT) and Timber

- Structure Standards Across the World, Mass Timber Construction Journal. 5 (2022)
- [13] American Lumber Standard Committee (ALSC), Northeastern Lumber Manufacturers Association, Standard Grading Rules for Northeastern Lumber, Northeastern Lumber Manufacturers Association, Maine, USA., 2021.
 - [14] American Society for Testing and Materials Committee, ASTM D198- 15, (2015).
 - [15] Microsoft Corporation., Microsoft Excel, (2021).