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ANALYTICAL STUDY ON P- Δ EFFECT ON MEDIUM-RISE WOODEN BUILDINGS

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ABSTRACT: In recent years, medium to high rise buildings up to about 10 stories of wooden structure have become a reality in Japan. However, analytical research is still insufficient. In this report, we aim to clarify the influence of the P- Δ effect by pushover analysis, capacity spectrum method, and time history response analysis for wooden buildings of 3 to 5 stories, and to obtain basic data for seismic design method. The result was that the influence of the P- Δ effect on the seismic performance of the medium-rise wooden buildings was small according to the capacity spectrum method, but the increase rate may reach 22 % in time history response analysis, so future examination is required.

KEYWORDS: Wood frame construction, Capacity spectrum method, Time history response analysis

1 INTRODUCTION

1.1 BACKGROUND

In recent years medium to high rise buildings up to about 10 stories of wooden structure have become a reality in Japan. In addition, by using wood as a structural material for sustainable forest management, we can expect to reduce climate change. In other words, wooden structures are good for the environment. Demand for medium to high rise wooden buildings that use a large amount of wood is increasing. However, analytical research is still insufficient.

For conventional low-rise wooden buildings up to about three stories, the safety limit deformation angle against large earthquake motion is considered to be about 1/30. On the other hand, the problem is pointed out whether large deformation is allowed even in medium to high rise wooden buildings such as 5-10 stories.

Among the discussion on whether or not the safety limit deformation angle should be reduced for medium to high rise wooden buildings, there is an opinion that the influence of the P- Δ effect cannot be ignored in medium to high rise wooden buildings because the vertical load is large. However, in theory, there should be no big difference between low-rise wooden buildings and high-rise wooden buildings as far as the ratio to the seismic force is concerned.

1.2 PURPOSE

In this study, we examine the P- Δ effect of medium-rise wooden buildings, for which analytical research is insufficient. The P- Δ effect is the effect of additional bending moment due to the axial force of vertical members and horizontal displacement of the story. In this report, we aim to clarify the influence of the P- Δ effect for wooden buildings, and to obtain basic data for seismic design method. Specifically, 3- to 5-story building models by wood frame construction are created and are analysed using an analysis software, wallstat [1]. And the influence of the P- Δ effect is discussed from the difference of earthquake response between the models with and without P- Δ effect.

2 ANALYSIS MODEL

The analysis models are 3- to 5-story building models by wood frame construction. The floor plan is shown in Fig. 1, and the appearance of 3- to 5- story models are shown in Figs. 2 to 4.

Any construction method can be used for the purpose of this study as the P- Δ effect only depends on the vertical load and the deformation angle. The reason for using wood frame construction method is to make the models simple as the main structural elements against seismic force is shear walls in wood frame construction, and deformation state of one floor does not affect the performance of other floors. The height of each floor was set to 3 maters. We prepared two types of analysis models, the standard model and the soft first story model. For the standard models, the sufficiency rate, the ratio of the allowable strength of a story to the requirement by allowable stress design, is 2 for all the story. For the soft first story models, the sufficiency rate of 1st story is 3, and that of 2nd story or higher is 6.

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Figure 1: Floor plan of the model



Figure 2: 3 story model



Figure 3: 4 story model



Figure 4: 5 story model

3 ANALYSIS METHOD

3.1 ANALYSIS SOFTWARE

The analysis model is created using wallstat ver.4.3.13. Originally, it was an analysis software for the conventional wooden framework construction method, but we created a model by replacing it with the wood frame construction. At that time, not all studs and joists were modelled. The studs were modelled at the position of columns of the conventional wooden framework construction method. We use the presence or absence of gravity to determine whether the P- Δ effect should be considered in the design.

3.2 INPUT EXTERNAL FORCE

There are three types of external forces to be input. The first is a pushover analysis so that the deformation angles between the layers are equal in each layer. This is done to confirm the validity of the analysis using wallstat by comparing with the load-displacement relationships of input shear wall data. The second is a pushover analysis using distribution coefficient Ai which is provided in the Building Standard Law (BSL) of Japan as seismic shear force distribution. Using the result of second push over, the maximum response displacement against strong ground motion is calculated using equivalent linearization method or the method known as capacity spectrum method [2, 3] and the response spectrum of type 2 ground stipulated in the BSL. The third is time history response analysis. Five artificial seismic waves conforming to the response spectrum of type 2 ground stipulated in the BSL are input, and the response displacement is calculated.

4 DATA OF ANALYSIS MODEL

4.1 BUILDING MATERIALS

The Young's modulus and flexural strength of wood were set to 1.24×10^4 N/mm² and 40.0 N/mm², respectively, referring to the material for design [4].

Table 1: Young's modulus and flexural strength of wood

Young's modulus	$1.24 \times 10^4 [N/mm^2]$
Flexural strength	40.0 [N/mm ²]

4.2 BUILDING WEIGHT

Weights were set as shown below with reference to the material of trial design [5].

Table 2: Weight of	
3story model	

240

WR

W3

 W_2

 W_1

 weight[kN]
 W_R

 275
 W_4

 481
 W_3

 481
 W_2

	weight[kN]
WR	275
W_4	481
W3	481
W_2	481
W_1	273

Table 3: Weight of4 story model



4.3 JOINTS OF BUILDING MATERIALS

Tension & compression springs and rotational springs are set between stud and beam and between beams. The loaddisplacement relationships of these joints are set as shown in Tables 5 and 6. The tension spring is assumed to be a joint with an allowable strength of 100 kN. The joint at the top or the bottom of each stud is assumed to have no moment resistance, so the stiffness of the rotational spring is close to 0 Nmm.

Table 5: Tension & compression springs

	K _{s1} [kN/mm]	K _{s2} [kN/mm]	K _{s3} [kN/mm]	D ₁ [mm]	D ₂ [mm]
Tension springs	50.0	13.6	-10.0	4.00	15.0

Table 6: Rotational springs



Tension & compression springs

*K*_{s1}: Primary stiffness of slip spring

K_{s2}: Secondary stiffness of slip spring

K_{s3}: Tertiary stiffness of slip spring

 D_1 , D_2 : Inflection point of curve of slip spring



Figure 5: Restoring force characteristics of tension springs



Figure 6: Restoring force characteristics of rotational springs



Figure 7: Restoring force characteristics of stud's rotational springs

4.4 SHEAR WALLS AND FLOOR STRUCTURES

The structure was set to be shown in Table 7 and 8 based on References [6] and [7]. The values of viscous dumping ratio of all the shear walls and floor structures are set to be 2%. The stiffness values in the table are those for the wall length of 0.91 meters and are converted to the stiffness of the wall length in the model automatically in the software. Floor structures are assumed rigid enough. In the models, the stiffness and the strength are the same as those of the shear walls at the first story. Fig. 8 shows the hysteresis characteristics of the shear wall.

Table 7: Shear walls and floor structures of standard model settings

		Ka	Ka	Ka	Ku	Kia	D.	D ₂	D ₂	D,
		[N/mm]	[N/mm]	[N/mm]	[N/mm]	[N/mm]	[mm]	[mm]	[mm]	[mm]
	3rd story	230	21.9	-32.9	318	11.0	4.00	20.0	70.0	420
2	2nd story	484	46.1	-69.2	668	23.1	4.00	20.0	70.0	420
5 atomiaa	1st story	660	62.9	-94.3	912	31.4	4.00	20.0	70.0	420
stories	Floor structure	660	62.9	-94.3	912	31.4	4.00	20.0	70.0	420
	4th story	266	25.3	-38.0	367	12.7	4.00	20.0	70.0	420
	3rd story	553	52.6	-79.0	763	26.3	4.00	20.0	70.0	420
4	2nd story	765	72.9	-109	1057	36.4	4.00	20.0	70.0	420
stories	1st story	917	87.3	-131	1266	43.7	4.00	20.0	70.0	420
	Floor structure	917	87.3	-131	1266	43.7	4.00	20.0	70.0	420
	5th story	301	28.7	-43.0	416	14.3	4.00	20.0	70.0	420
	4th story	619	59.0	-88.5	855	29.5	4.00	20.0	70.0	420
5	3rd story	864	82.3	-123	1193	41.2	4.00	20.0	70.0	420
5 stories	2nd story	1054	100	-151	1455	50.2	4.00	20.0	70.0	420
	1st story	1208	115	-173	1669	57.5	4.00	20.0	70.0	420
	Floor structure	1208	115	-173	1699	57.5	4.00	20.0	70.0	420

 Table 8: Shear walls and floor structures of soft first story model settings

		K _{s1}	Ks2	K _{s3}	K _{b1}	K _{b2}	D_1	D_2	D_3	D_4
		[N/mm]	[N/mm]	[N/mm]	[N/mm]	[N/mm]	[mm]	[mm]	[mm]	[mm]
	3rd story	690	65.7	-98.6	953	32.9	4.00	20.0	70.0	420
2	2nd story	1452	138	-207	2005	69.2	4.00	20.0	70.0	420
5 atomiaa	1st story	990	94.3	-141	1368	47.2	4.00	20.0	70.0	420
stories	Floor structure	990	94.3	-141	1368	47.2	4.00	20.0	70.0	420
	4th story	797	75.9	-114	1101	38.0	4.00	20.0	70.0	420
	3rd story	1658	158	-237	2290	79.0	4.00	20.0	70.0	420
4	2nd story	2295	219	-328	3170	109	4.00	20.0	70.0	420
stories	1st story	1375	131	-196	1899	65.5	4.00	20.0	70.0	420
	Floor structure	1375	131	-196	1899	65.5	4.00	20.0	70.0	420
	5th story	904	86.1	-129	1248	43.0	4.00	20.0	70.0	420
	4th story	1858	177	-265	2566	88.5	4.00	20.0	70.0	420
5	3rd story	2593	247	-370	3580	123	4.00	20.0	70.0	420
stories	2nd story	3161	301	-452	4365	151	4.00	20.0	70.0	420
	1st story	1813	173	-259	2503	86.3	4.00	20.0	70.0	420
	Floor structure	1813	173	-259	2503	86.3	4.00	20.0	70.0	420



 $K_{s1} \sim K_{s3}$: Slip stiffness[N/mm] $K_{b1} \sim K_{b2}$: Bilinear stiffness[N/mm] $D_1 \sim D_4$: Displacement of break point[mm]

Figure 8: Hysteresis characteristics of the shear wall

5 PUSHOVER ANALYSIS RESULTS

5.1 SUMMARY

Pushover analysis is performed so that the story deformation angle is equal in each layer. This is to confirm the validity of the analysis using wallstat.

5.2 STANDARD MODEL ANALYSIS RESULTS

Figs. 9 and 10 shows the result of pushover analysis on 5story models as the samples. The "assumed wall" with gravity is theoretical solution considering the P- Δ effect. The results of the pushover analysis were generally in agreement with that, and it was confirmed that the P- Δ effect was included in the same degree as the theory.



Figure 9: The results of pushover analysis in case of 5-story without gravity model



Figure 10: The results of pushover analysis in case of 5-story with gravity model

5.3 SOFT FIRST STORY MODEL ANALYSIS RESULTS

Figs. 11 and 12 shows the result of the pushover analysis on 5-story models as the samples.



Figure 11: The results of pushover analysis in case of 5-story without gravity model



Figure 12: The results of pushover analysis in case of 5-story with gravity model

6 RESPONSE PREDICTION BY CAPACITY SPECTRUM METHOD

6.1 OVERVIEW

Capacity spectrum method is a simple prediction method for elasto-plastic response proposed in a 1976 paper by Akenori Sibata et al. The idea of capacity spectrum method is to consider a steady vibration at a certain displacement, replace it with an equivalent linear response, and predict the response using the response spectrum. In the case of a multi-story building, it is necessary to replace it with an equivalent one-degree-of-freedom system. We calculated acceleration response spectrum and displacement response spectrum according to the Building Standard Law of Japan [8]. We used Equations (1) and (2).

$$S_a = \frac{\sum_{i=1}^{N} m_i \cdot \delta_i^2}{(\sum_{i=1}^{N} m_i \cdot \delta_i)^2} \cdot Q_B \tag{1}$$

$$S_d = \frac{\sum_{i=1}^N m_i \cdot \delta_i^2}{\sum_{i=1}^N P_i \cdot \delta_i} \cdot S_a \tag{2}$$

 m_i : Mass of i-th story (kg) δ_i : Displacement relative to the ground on i-th story (m) Q_B : Base shear (N) P_i : Load on i-th story (N)

6.2 STANDARD MODEL ANALYSIS RESULTS

Table 9 shows the values of maximum relative story displacement and story deformation angle by capacity spectrum method. The rate of increase in interlayer displacement due to the P- Δ effect of the standard model was -4.64 % to 8.68 %. Figs. 21 to 26 show the analysis results of the standard models.

Table 9: Comparison of response values by capacity spectrum

 method in a standard model

		Without	Gravity	With C	iravity	Increase rate due to P-∆ effect
		Relative story displacement	story deformation angle	Relative story displacement	story deformation angle	Relative story displacement
		$_1\delta[mm]$	$\theta[\times 10^{-3} rad]$	₂ δ[mm]	$\theta[\times 10^{-3} rad]$	(2δ-1δ)/1δ×100[%]
	3rd story	73.6	24.5	72.2	24.1	-1.88
3 stories	2nd story	73.5	24.5	78.3	26.1	6.55
	1st story	71.4	23.8	73.5	24.5	2.95
	4th story	66.8	22.3	63.7	21.2	-4.64
4	3rd story	66.4	22.1	70.1	23.4	5.57
stories	2nd story	65.2	21.7	68.9	23.0	5.73
	l st story	62.9	21.0	61.8	20.6	-1.77
	5th story	59.2	19.7	56.5	18.8	-4.53
	4th story	62.5	20.8	62.8	20.9	0.493
5 stories	3rd story	63.0	21.0	67.1	22.4	6.54
	2nd story	58.9	19.6	64.0	21.3	8.68
	1st storv	54.2	18.1	53.5	17.8	-1.26



Figure 21: The results of capacity spectrum method in case of 3-story without gravity model



Figure 22: The results of capacity spectrum method in case of 3-story with gravity model



Figure 23: The results of capacity spectrum method in case of 4-story without gravity model



Figure 24: The results of capacity spectrum method in case of 4-story with gravity model



Figure 25: The results of capacity spectrum method in case of 5-story without gravity model



Figure 26: The results of capacity spectrum method in case of 5-story with gravity model

6.3 SOFT FIRST STORY MODEL ANALYSIS RESULTS

Table 10 shows the values of maximum relative story displacement and story deformation angle by capacity spectrum method. The rate of increase in interlayer displacement due to the P- Δ effect of the soft first story model was 2.22 % to 8.97 %. Figs. 27 to 32 show the analysis results of the soft first story models.

Table 10: Comparison of response values by capacity spectrum method in a soft first story model

\backslash		Without	Gravity	With C	Gravity	Increase rate due to P-∆ effect
		Relative story displacement	story deformation angle	Relative story displacement	story deformation angle	Relative story displacement
		₁ δ[mm]	$\theta[\times 10^{-3} rad]$	₂ δ[mm]	$\theta[\times 10^{-3} rad]$	(2δ-1δ)/1δ×100[%]
	3rd story	16.2	5.40	16.9	5.63	4.25
3 stories	2nd story	15.4	5.13	16.2	5.40	4.75
	l st story	56.7	18.9	61.5	20.5	8.58
	4th story	19.0	6.33	19.4	6.47	2.31
4	3rd story	19.3	6.43	19.9	6.63	3.41
stories	2nd story	18.4	6.13	19.1	6.37	3.54
	l st story	62.5	20.8	66.7	22.2	6.69
	5th story	21.6	7.20	22.1	7.37	2.22
	4th story	22.3	7.43	23.0	7.67	3.07
5 stories	3rd story	22.5	7.50	23.4	7.80	3.92
	2nd story	21.3	7.10	22.2	7.40	4.27
	1st story	64.9	21.6	70.8	23.6	8.97



Figure 27: The results of capacity spectrum method in case of 3-story without gravity model



Figure 28: The results of capacity spectrum method in case of 3-story with gravity model



Figure 29: The results of capacity spectrum method in case of 4-story without gravity model



Figure 30: The results of capacity spectrum method in case of 4-story with gravity model



Figure 31: The results of capacity spectrum method in case of 5-story without gravity model



Figure 32: The results of capacity spectrum method in case of 5-story with gravity model

7 TIME HISTORY RESPONSE ANALYSIS

7.1 STANDARD MODEL ANALYSIS RESULTS

Table 11 shows the average values of maximum story displacement and story deformation angle by time history analysis using five artificial seismic waves. The rate of increase in interlayer displacement due to the P- Δ effect of the standard model was -34.4 % to 22.2 %. Figs. 33 to 38 show the analysis results of the standard models.

 Table 11: Comparison time history response analysis in standard models

		Without	Gravity	With C	Gravity	Increase rate due to $P-\Delta$ effect
		Relative story displacement	story deformation angle	Relative story displacement	story deformation angle	Relative story displacement
		$_1\delta[mm]$	$\theta[\times 10^{-3} rad]$	₂ δ[mm]	$\theta[\times 10^{-3} rad]$	$(_{2}\delta{1}\delta)/_{1}\delta \times 100[\%]$
	3rd story	88.8	29.6	58.2	19.4	-34.4
3 stories	2nd story	77.3	25.8	94.1	31.4	21.7
	lst story	41.5	13.8	45.7	15.2	10.0
	4th story	66.8	22.3	49.4	16.5	-26.1
4	3rd story	75.0	25.0	78.9	26.3	5.24
stories	2nd story	47.3	15.8	52.8	17.6	11.8
	lst story	42.3	14.1	40.7	13.6	-3.66
	5th story	69.3	23.1	55.9	18.6	-19.4
	4th story	58.1	19.4	66.8	22.3	15.1
5 stories	3rd story	41.1	13.7	50.2	16.7	22.2
	2nd story	41.8	13.9	43.5	14.5	4.09
	l st story	39.2	13.1	40.2	13.4	2.64



Figure 33: The results of time history response analysis in case of 3-story without gravity model



Figure 34: The results of time history response analysis in case of 3-story with gravity model



Figure 35: The results of time history response analysis in case of 4-story without gravity model



Figure 36: The results of time history response analysis in case of 4-story with gravity model



Figure 37: The results of time history response analysis in case of 5-story without gravity model



Figure 38: The results of time history response analysis in case of 5-story with gravity model

7.2 SOFT FIRST STORY MODEL ANALYSIS RESULTS

Table 12 shows the average values of maximum story displacement and story deformation angle by time history analysis using five artificial seismic waves. The rate of increase in interlayer displacement due to the P- Δ effect of the soft first story model was 0.494 % to 11.9 %. Figs. 39 to 44 show the analysis results of the soft first story models.

Table 12:	Comparison	time	history	response	analysis	in soft
first story	models					

		Without	Gravity	With C	iravity	Increase rate due to P-∆ effect
		Relative story displacement	story deformation angle	Relative story displacement	story deformation angle	Relative story displacement
		₁ δ[mm]	$\theta[\times 10^{-3} rad]$	₂ δ[mm]	$\theta[\times 10^{-3} rad]$	$(_{2}\delta{1}\delta)/_{1}\delta \times 100[\%]$
	3rd story	10.9	3.64	11.4	3.79	4.30
3 stories	2nd story	10.7	3.58	10.9	3.65	1.73
	1st story	29.3	9.76	29.7	9.90	1.46
	4th story	12.2	4.05	12.9	4.31	6.41
4	3rd story	12.6	4.20	12.8	4.26	1.61
stories	2nd story	12.6	4.19	12.6	4.21	0.494
	1st story	29.8	9.92	33.3	11.1	11.9
	5th story	14.1	4.70	15.1	5.05	7.43
	4th story	15.3	5.11	15.6	5.21	1.88
5 stories	3rd story	15.1	5.02	15.8	5.27	5.01
	2nd story	14.5	4.83	15.4	5.14	6.35
	1st story	36.0	12.0	40.1	13.4	11.5
180 150 150 120 90 60 30						
-1st i	0 floor a	40 nalysis results	80 — 1st floor — 3rd floor	120 R assumed wall	160 clative story —2nd fle	200 240 displacement [mm] oor analysis results

Figure 39: The results of time history response analysis in case of 3-story without gravity model

–Artificial seismic waves 1 -- Artificial seismic waves 2 -- Artificial seismic waves 3

- Artificial seismic waves 4 - Artificial seismic waves 5 - Average



Figure 40: The results of time history response analysis in case of 3-story with gravity model



Figure 41: The results of time history response analysis in case of 4-story without gravity model



Figure 42: The results of time history response analysis in case of 4-story with gravity model



Figure 43: The results of time history response analysis in case of 5-story without gravity model



Figure 44: The results of time history response analysis in case of 5-story with gravity model

8 CONCLUSION

We aim to clarify the influence of the P- Δ effect by pushover analysis, capacity spectrum method, and time history response analysis for wooden buildings of 3 to 5 stories, and to obtain basic data for seismic design method. In the pushover analysis, it was confirmed that the analysis using wallstat was appropriate by comparing the load deformation relationship of the wall assumed with the analysis result. The increase rate of relative story displacement due to the P- Δ effect was up to 8.97% for the capacity spectrum method and up to 22.2% on average for 5 artificial seismic waves in the time history response analysis. Looking at the results of the capacity spectrum method, it can be said that the effect of the P- Δ effect on the seismic performance of medium-rise wooden buildings is small. However, the rate of increase may reach 22.2% in the time history response analysis, so future examination is required.

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