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Thermoeconomic Cost Allocation Approaches in a Simultaneous Heating and Cooling Heat Pump System.

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Abstract:

Thermoeconomics is a branch of engineering that combines concepts from thermodynamics and economics to tackle problems that are hard or cannot be solved by these sciences, separately. The main applications of thermoeconomics are cost allocation, optimization, and malfunction diagnosis of energy systems. Defining the productive structure plays a critical role in thermoeconomic modeling, and exergy is a highly appropriate thermodynamic quantity to correlate with costs. For systems containing dissipative equipment, the use of total exergy in conventional productive diagrams need a subsequent decision, by the analysts, on how to define its fuel and product. In some applications, exergy disaggregation can be elegant options, mostly to deal with dissipative components. Nevertheless, all of them increase the complexity in thermoeconomic modelling. In this work, thermoeconomic cost allocation approaches in a simultaneous heating and cooling heat pump system were performed from the application of some thermoeconomics methodologies. The studied system has one dissipative component (valve) and generates no waste. There are three specific objectives. Firstly, to present the different possibilities to treat and isolate the valve in the thermoeconomics modelling. Secondly, to compare the methodologies presented from the thermoeconomic point of view through the cost allocation in a simultaneous heating and cooling heat pump system. And finally, to show the pros and cons of each methodology applied in this study to support future decision-makings in thermoeconomic modelling. The research demonstrates that the differences between the methodologies used were not significant, and the choice of methodology should depend on factors such as the need for disaggregated equipment. In conclusion, the findings of this study indicate that without certain models that may overestimate the cost of particular components, the thermoeconomic results fall within a narrow range of 4% of the solution line. This suggests that the methodologies used did not yield significantly different results.

Keywords:

Thermoeconomic Approach, Cost Allocation, Physical Exergy Disaggregation, Modelling Complexity, Dissipative Component.

1. Introduction

Thermoeconomics is a branch of engineering that combines concepts from thermodynamics and economics to tackle problems that are hard to solve or cannot be solved by these sciences, separately. The main applications of thermoeconomics are cost allocation, local and global optimization, and malfunction diagnosis of energy systems.

Historically, Keenan [1] was the first researcher to associate the exergy (availability) of the final products of an energy system with their respective costs [2]. Since then, this idea has been applied to several different problems [3], including optimisation of energy systems [4,5]. However, it was only in the late 1980s and early 1990s that modern thermoeconomic methodologies were proposed. In 1994, the CGAM problem was defined to compare results by the application of such methodologies [6]. Since then, thermoeconomics has been widely applied to many energy system problems.

Frangopoulos proposed the Thermoeconomic Functional Analysis (TFA) [7,8], which is originally a methodology for energy systems optimisation. Nevertheless, TFA can be adapted to solve cost allocation problems. Von Spakovsky proposed the Engineering Functional Analysis (EFA) [9], which is an optimisation methodology, but it can be used for cost allocation as well as TFA. Lozano and Valero proposed the Theory

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of Exergy Cost (TEC) [10,11] which was originally developed for cost allocation and malfunction diagnosis, but it can be also applied to formulate optimisation problems. Tsatsaronis and co-workers proposed some exergoeconomic methods [12,13] that became the SPECO method [14], which is a methodology for cost allocation and energy systems optimisation. The methodologies were adapted to deal with environmental impact problems in the same energy systems as well [15–17].

Other thermoeconomic approaches were proposed besides the ones. Erlach et al. proposed the Structural Theory [18], which is a unification approach of the methodologies applied to the CGAM problem. Santos et al. proposed the H&S model [19], which is a modification of the Structural Theory regarding the application of negentropy. Lourenço et al. proposed the UFS model [20], which is an extension of the H&S model. Recently, a method of localized physical disaggregation [21] was proposed to isolate dissipative equipment with less complexity when compared with total disaggregation. However, it can incur a loss in the accuracy of the results. Finally, the latest thermoeconomic methodology presented in the literature is the A&F model [22], a methodology proposed to isolate dissipative components with less modelling complexity.

A key concept used in most thermoeconomic methodologies and approaches is the functional diagram, also known as the productive diagram. The productive diagram is a graphical representation of the inter-relations between the subsystems of the global system and its surroundings. Subsystems are connected according to their respective purposes. Frangopoulos [7] proposed this key concept first in the 1980s, but it has been applied since then until nowadays. The way in which the productive structure is defined is a key point of thermoeconomic modelling. One of the most adequate thermodynamic magnitudes to be associated with the cost is exergy since it contains information from both the first and the second laws of thermodynamics. Most thermoeconomicists agree that exergy is the most appropriate thermodynamic magnitude to associate with cost since it contains information from both the first and the second laws of thermodynamics, qualifies the energy streams, and identifies the irreversibility of the subsystems [23].

The utilization of total exergy in conventional productive diagrams can become a hard task when the systems include dissipative equipment, such as valves. This is because the analyst needs to decide on how to define the fuel and product for the dissipative component, which can be challenging. One solution to this issue is the use of physical exergy disaggregation. Despite the improvement in result accuracy, when the exergy streams are disaggregated, the modelling complexity is increased [14].

In this study, different thermoeconomic methodologies are applied to a simultaneous heating and cooling heat pump system, which contains a dissipative valve and generates no waste. The study has three specific objectives. Firstly, the research presents the different methodological possibilities for treating and isolating the valve. Secondly, it compares the various thermoeconomic methodologies based on cost allocation in the simultaneous heating and cooling heat pump system. Finally, it demonstrates the advantages and disadvantages of each methodology, which can help guide future decision-makings in thermoeconomic modelling.

By presenting and comparing different cost allocation approaches, this study provides guidance for analysts in their future work and contributes to the development of more effective thermoeconomic modelling.

2. Physical exergy disaggregation

Since 1990, thermoeconomics has used physical exergy disaggregation, which was initially introduced by Kotas [24]. In agreement with [14], disaggregate exergy components enhance the precision of results in thermoeconomics. Physical exergy, disregarding kinetic, potential, and other energy forms, is expressed by Eq. (1), as used by H&S Model approach. It is worth mentioning that all components of exergy presented in this study depend solely on the thermodynamic properties of the flows, previously known.

$$\dot{E}_i^{PH} = \dot{E}_i^{H} - \dot{E}_i^{S} = \dot{m}_i \cdot \left[\left(h_i - h_0 \right) - T_0 \cdot \left(s_i - s_0 \right) \right] \tag{1}$$

Applying the definition of specific enthalpy in Eq. (1), h = u + Pv, Eq. (2) is written. Rearranging Eq. (2), the three terms of UFS Model are obtained and given by Eq. (3).

$$\dot{E}_i^{PH} = \dot{m}_i \cdot \left\{ \left[\left(u_i + P_i \cdot v_i \right) - \left(u_0 + P_0 \cdot v_0 \right) \right] - T_0 \cdot \left(s_i - s_0 \right) \right\} \tag{2}$$

$$\dot{E}_i^{PH} = \dot{E}_i^{U} + \dot{E}_i^{F} - \dot{E}_i^{S} = \dot{m}_i \cdot \left[\left(u_i - u_0 \right) + \left(P_i \cdot v_i - P_0 \cdot v_0 \right) - T_0 \cdot \left(s_i - s_0 \right) \right] \tag{3}$$

Furthermore, Eq. (4) can be obtained by rearranging Eq. (2) in a different way, specifically by combining the first and third terms of Eq. (3).

$$\dot{E}_i^{PH} = \dot{m}_i \cdot \left\{ \left[\left(u_i - T_0 \cdot s_i \right) - \left(u_0 - T_0 \cdot s_0 \right) \right] + \left(P_i \cdot v_i - P_0 \cdot v_0 \right) \right\} \tag{4}$$

The specific Helmholtz energy of a closed system under a heat bath (reservoir at T_0) is given by $a = u - T_0 s$. This can be applied for both i-th and dead states. Eqs. (5) and (6) show the Helmholtz energy term and flow work term, respectively. Thus, the physical exergy could be written as in Eq. (7), according to the A&F Model. It is important to highlight that the principle used in applying the A&F Model to disaggregate physical exergy into its Helmholtz energy terms (Eq. (5)) and flow work term (Eq. (6)) is similar to that used in the H&S and UFS Models.

$$\dot{E}_i^A = \dot{m}_i \cdot \left(a_i - a_0\right) = \dot{m}_i \cdot \left[\left(u_i - T_0 \cdot s_i\right) - \left(u_0 - T_0 \cdot s_0\right)\right] \tag{5}$$

$$\dot{E}_i^F = \dot{m}_i \cdot \left(P_i \cdot v_i - P_0 \cdot v_0\right) \tag{6}$$

$$\dot{E}_i^{PH} = \dot{E}_i^A + \dot{E}_i^F = \dot{m}_i \cdot \left\{\left[\left(u_i - T_0 \cdot s_i\right) - \left(u_0 - T_0 \cdot s_0\right)\right] + \left(P_i \cdot v_i - P_0 \cdot v_0\right)\right\} \tag{7}$$

3. Thermoeconomic modelling

The productive structure is a representation that elucidates the purpose of subsystems by explicitly revealing their input (fuels) and output (products) components in terms of productive flows. In thermoeconomic methodologies, physical or/and productive flows are conventionally utilized to graphically illustrate the productive interconnections among subsystems. This study employs productive and physical flows to construct productive/physical diagrams that facilitate the visualization of the productive structure. Fictitious units (junctions and bifurcations) are used to assist the drawing up of the productive diagrams.

After the definition of the productive structure, each subsystem is represented by means of a cost equation balance relating a thermodynamic magnitude and the unit cost of external resources, and internal flows. The mathematical model lists a set of cost equation balances in each subsystem to calculate the unit costs. A thermoeconomic model should be performed by using Eq. (8).

$$\sum\left(k_{out} \cdot Y_{out}\right) - \sum\left(k_{in} \cdot Y_{in}\right) = k_F \cdot E_F \tag{8}$$

In Eq. (8), E_F is the external fuel exergy consumption (in kW); and Y_{out} and Y_{in} means the generic thermodynamic magnitude of the internal flows at inlet and outlet (including final products) of each subsystem. The solution of the set of equations results in the unit exergy costs of each internal flow and each final product. In this paper, *Y* assumes the thermodynamic magnitudes, such as power (W), total exergy (E), Helmholtz energy term (E^A), flow work term (E^F), internal energy (E^U), and entropic term (E^S). The unknown k_{out} and k_{in} are the unit exergy costs of the internal flows at the outlet and the inlet of each subsystem. The unit exergy cost of a flow is the amount of external exergy unit required to obtain one unit of this flow, meaning that the unit exergy costs of a flow is a measure of the thermodynamic efficiency of the production process when producing this flow [25]. Each subsystem provides a single cost balance equation, thus auxiliary equations are necessary when several products are obtained in a component. Thermoeconomic models which use physical exergy disaggregation, based on the productive diagrams, consider the equality criteria [8,14,26], where productive flows exiting the same productive unit must have the same unit cost. It is worth mentioning that for all the methodologies studied in this research, in the absence of external assessment, the exergy cost of the mass and energy streams entering the plant equals their exergy *(k_w = 1 [kJ/kJ]).*

4. Case study

A simultaneous heating and cooling heat pump system is studied to exemplify the proposal of this paper. This is the same system studied by Nguyen et al. [27]. The flowsheet of the system is shown in Figure 1. The heat pump consists of four components: evaporator, motor-compressor, condenser, and valve. The working fluid of the system is ammonia. The evaporator and the condenser are two water-coupled systems designed for district cooling and heating, respectively. In Figure 1, ‘H’, ‘C’, ‘r’ and ‘s’ correspond to hot, cold, return and supply, respectively, and ‘em’ corresponds to electric motor.

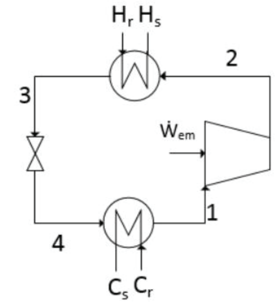

Figure. 1. The physical structure of the heat pump system

Certain assumptions have been made, including the assumption that the processes are in a steady state. Additionally, it is assumed that ammonia is a saturated vapor at state 1 and a saturated liquid at state 3, there is no heat exchange with the environment, there are no changes in kinetic and potential energy, there is no pressure drop for flow through heat exchangers, and chemical exergy is not considered.

Table 1 shows the parameters of the heat pump system.

Table 1: System parameters

Parameter	Symbol	Value	Unit
Condensation temperature	T_{cnd}	70	°C
Evaporation temperature	T_{evp}	7	°C
Evaporator heat transfer rate	$\dot{Q}_{evp}$	250	kW
Compressor isentropic efficiency	η_{cmp}	75	%
Electric motor efficiency	η_{em}	90	%
Water streams pressure	P_{H2O}	300	kPa
Environment pressure	P_0	100	kPa
Environment temperature	T_0	20	°C

Avoiding pinch problems, Eqs. (10)-(13) are used to model the condenser and the evaporator:

$$T_{Hs} = T_{cnd} + 0°C \tag{10}$$

$$T_{Cs} = T_{evp} + 3°C \tag{11}$$

$$T_{Hr} = T_{Hs} - 30°C \tag{12}$$

$$T_{Cr} = T_{Cs} + 6°C \tag{13}$$

Conventional mass, energy and exergy balance equations are applied from the data to each control volume. The simulation is done in Engineering Equation Solver [28]. Table 2 shows the values of electrical power input and mass flow rates of ammonia, hot water, and cold-water streams.

Table 2. Electrical power input and mass flow rates

Variable	Symbol	Value	Unit
Electrical power	$\dot{W}_{cmp}$	110.181	kW
Ammonia mass flow rate	$\dot{m}_{NH3}$	0.271	kg/s
Hot water mass flow rate	$\dot{m}_{H2O}^{H}$	2.783	kg/s
Cold water mass flow rate	$\dot{m}_{H2O}^{C}$	9.958	kg/s
Hot water stream exergy	$\dot{E}_{Qcond}$	37.026	kW
Cold water stream exergy	$\dot{E}_{Qevp}$	6.125	kW

Table 3 shows the thermodynamic properties of ammonia.

Table 3. Thermodynamic properties of the main physical flows of the heat pump system

Flow	P [kPa]	T [°C]	E [kW]	E^A [kW]	E^F [kW]	E^S [kW]	E^U [kW]
1	554	7.00	64.50	68,76	-4,27	-82,50	-13,74
2	3312	179.80	147	129,95	17,04	-65,84	64,11
3	3312	70.00	86.17	122,73	-36,57	-354,08	-231,39
4	554	7.00	76.09	105,58	-29,51	-344,08	-238,47

5. Thermoeconomic methodologies

In this Section, five thermoeconomic methodologies will be studied for the case study presented in this work. One of these methodologies will be presented in a physical diagram (TEC), while the others will be presented using a productive diagram (E Model, UFS Model, A&F Model, and localized physical exergy disaggregation). The aim is to develop the methodologies for the exergy cost allocation of the electrical power to the final products of the system, i.e., heating and cooling. In agreement with [29], this work also concurs that the best productive structure would be one that explains with the greatest depth and simplicity the productive function of the subsystems and flows present in the physical structure of the plant under examination.

5.1. TEC

According to original formulation of TEC [10], one can apply the exergy cost balance equation to each control volume of the system. In addition, it is considered that the final products of the system are the exergy flow increase of both hot and cold-water streams, respectively. In this methodology, the sum of the inlet cost streams is equal to the sum of the exit cost streams for each control volume shown, such as Eq. 8. However, the interpretation of which flows are fuels or products for each subsystem is not a trivial task. Therefore, equations Eqs. (14) - (17) are formulated with the aim of performing the proposed study and Figure 2 shows the physical structure of the heat pump system using TEC.

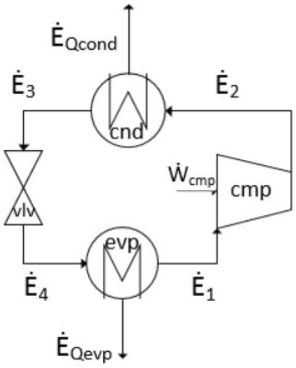

Figure. 2. Physical structure of the heat pump system using TEC

$$k_w \cdot \dot{W}_{cmp} + k_1 \cdot \dot{E}_1 = k_2 \cdot \dot{E}_2 \tag{14}$$

$$k_2 \cdot \dot{E}_2 = k_3 \cdot \dot{E}_3 + k_{cnd} \cdot \dot{E}_{Qcond} \tag{15}$$

$$k_3 \cdot \dot{E}_3 = k_4 \cdot \dot{E}_4 \tag{16}$$

$$k_4 \cdot \dot{E}_4 = k_1 \cdot \dot{E}_1 + k_{evp} \cdot \dot{E}_{Qevp} \tag{17}$$

Whether there is removal of exergy from a mass stream within the control volume being considered then the unit exergy cost of the output stream is equal to the unit exergy cost of the correspondent input stream. Therefore, Eqs. (18)-(19) are written.

$$k_3 = k_2 \tag{18}$$

$$k_1 = k_4 \tag{19}$$

According to the authors [29], a limitation of the theory of exergetic cost, as it was originally formulated, consisted of defining the productive structure in relation to the same flows and components present in the physical structure. One of the resulting difficulties lies mainly in the adequate treatment of the dissipative units and of the residues of the plant [29].

5.2. E Model

The E Model utilizes total exergy to define fuels and products of the subsystems of the plant. However, when a valve is present in the system as a dissipative equipment, the analyst must decide how to treat the valve. It is not possible to assign a productive purpose to the valve using only total exergy. Usually, dissipative equipment is combined with other subsystems to aid in the production of the final product. Moreover, this plant has the characteristic of having two products, which makes the analyst's decision more difficult. Therefore, this study presents three options to treat the valve using total exergy, as illustrated in Figure 3. The first option (Figure 3a) is to combine the valve with the condenser (where the valve's irreversibility is entirely attributed to heat), the second (Figure 3b) is to combine it with the evaporator (where the valve's irreversibility is entirely attributed to cold), and the third option (Figure 3c) is to separate it until temperature T_0 (ambient temperature), where the valve's irreversibility up to that point will be attributed to the condenser, and below temperature T_0, it will be attributed to the evaporator. This third option appears to be more reasonable since it is consistent to consider (from a thermodynamic point of view) that until temperature T_0, the valve contributes to the heat production and below the ambient temperature, it contributes to the cooling production.

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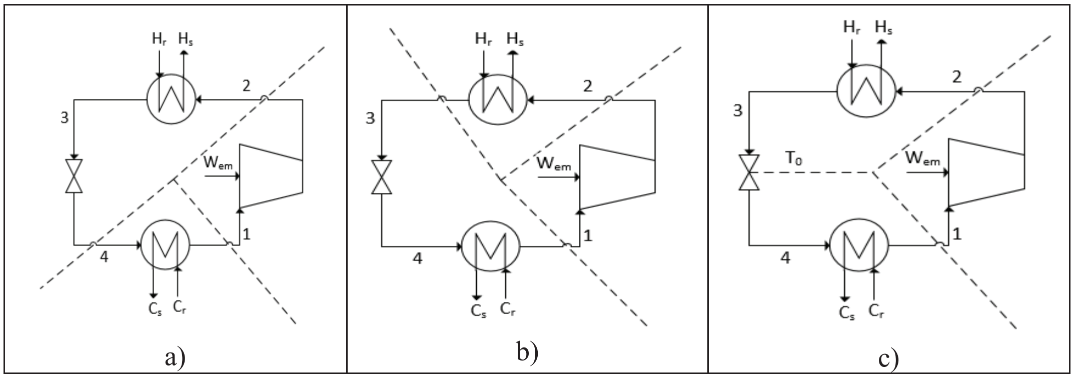

Figure. 3. Three physical structure layouts for the treatment of the valve: a) *valve with the condenser; b) valve with the evaporator; c) valve until T_0 with condenser, below T_0 with evaporator.*

Figures 4 to 6 depict the production diagram for the three possible treatments of the valve using Model E. It is worth noting that the main difference between the three diagrams is the input of the subsystem where the valve is merged. As previously explained, however now represented in the productive diagram, in Figure 4 the valve is merged with the Condenser (*E Model - CV*), in Figure 5 it is merged with the evaporator (*E Model – VE)*, and in Figure 6 the irreversibility of the valve is divided between the condenser and the evaporator.

In all the productive diagrams analysed in this research, the rectangles represent the actual units or subsystems that correspond to the physical equipment of the system. The rhombus and circles are fictitious units utilized to connect and/or divide the productive streams. Each subsystem includes input arrows to indicate its fuel or resources, and output arrows to indicate its products. The determination of productive streams is based on the specific exergy term variation between the inlet and outlet, with positive variation classified as a product and negative variation as fuel [19]. In this research, the auxiliary equation for each bifurcation is formulated using the multiproduct method, which assumes that the same unit costs apply to all productive streams that leave the same subsystem, owing to shared resources and irreversibilities inherent within the subsystem [30].

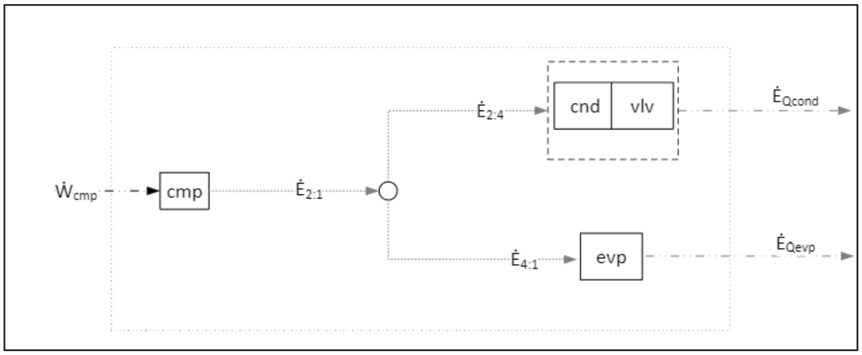

Figure. 4. Productive diagram for the heat pump system using E Model - CV.

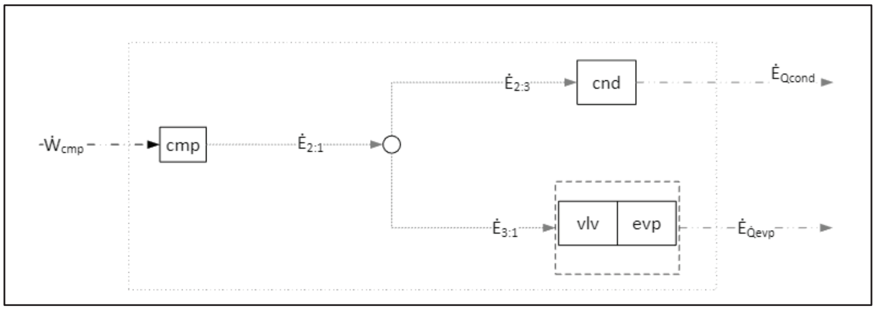

Figure. 5. Productive diagram for the heat pump system using E Model - VE.

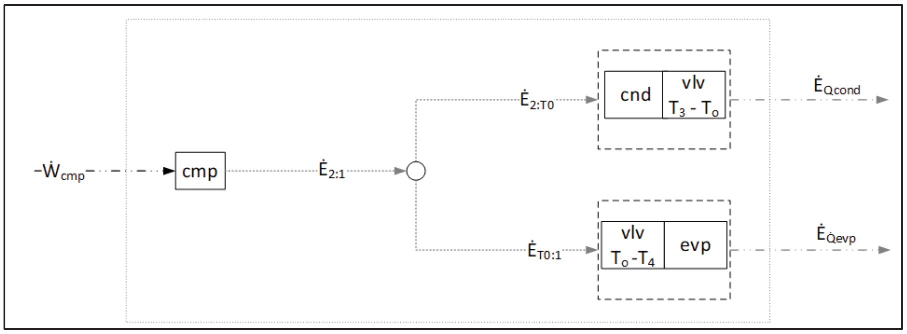

Figure. 6. Productive diagram for the heat pump system using E Model - CVE.

5.3. UFS Model

Despite the increased computational effort required and modelling complexity compared with the E Model, the application of the UFS Model [20] is justified because there is a valve in the system. To be more specific, there are two additional exergy terms for every equipment in the productive diagram to define fuel and product, compared to the E Model. The valve is now isolated, and there is no longer a need to merge this component with another to achieve its intended production purpose. Figure 7 represent the productive diagram of the heat pump system using the UFS Model. As already explained, this model utilizes physical exergy disaggregated into internal energy (E^U), flow work (E^F), and entropic term (E^S).

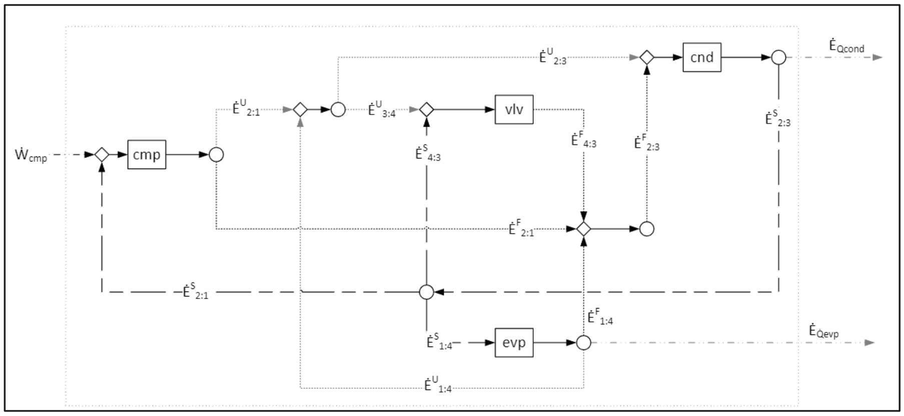

Figure. 7. Productive diagram for the heat pump system using UFS Model.

5.4. A&F Model

The diagram depicted in Figure 8 showcases the use of the A&F Model [22] in the heat pump system for the isolation of valves and the determination of their products and fuels via the utilization of Helmholtz energy (E^A) and flow work (E^F) terms.

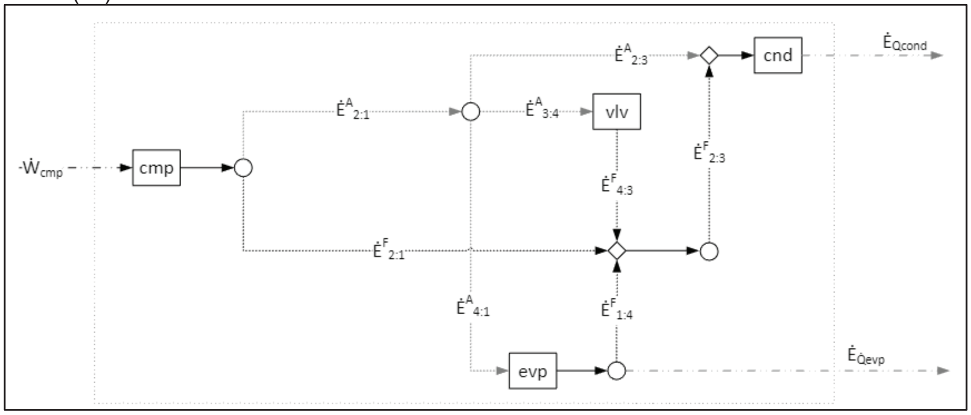

Figure. 8. Productive diagram for the heat pump system using A&F Model.

By employing these terms, the model allows for the appropriate handling of this dissipative component. The pro of the A&F Model over the E Model, in this study, is attributed to its ability to isolate valves. Regarding the UFS model, both models can isolate dissipative components, but the A&F Model achieves this via a simpler exergy disaggregation using only two terms, as opposed to the three terms used by UFS. Consequently, the A&F Model's simplicity and universality are its main advantages. Furthermore, compared to the UFS Model, the A&F Model reduces the number of flows, junctions-bifurcations, and cost equations, resulting in a significantly lower degree of complexity and computational requirements.

5.5. Localized physical exergy disaggregation (E Model – LD).

The localized physical exergy disaggregation [21] is used to disaggregate physical exergy only in dissipative equipment where the total exergy is not enough to define a productive purpose for the equipment. In this case study, the valve is a dissipative equipment, and therefore, the disaggregation of physical exergy into its Helmholtz energy (E^A) and flow work (E^F) terms was only used in the definition of fuel and product for this equipment. It should be noted that other methodologies could be used to isolate the valve as long as fuel and products could be defined. However, in this work, the A&F model was chosen, which is the thermoeconomic model with the fewest exergy terms capable of defining input and output for the valve. In other words, where total exergy can be used, it is used; where it is not possible, the disaggregation of physical exergy is a viable solution. This model has an advantage over other models that use physical exergy disaggregation, as it can isolate the valve with fewer flows. The Figure 9 shows the productive diagram for the heat pump system using localized physical exergy disaggregation.

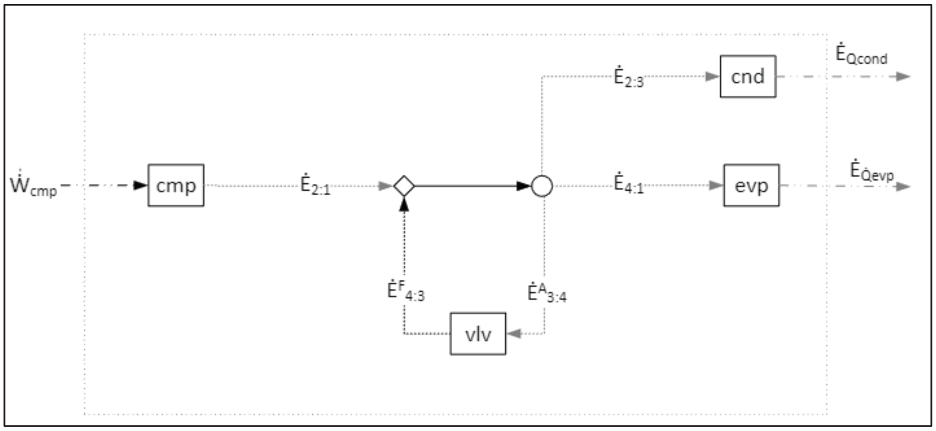

Figure. 9. Productive diagram for the heat pump system using localized physical exergy disaggregation.

6. Results and discussions

The cost allocation models had to determine the unit exergy cost for both heating and cooling, which was a challenge. Regardless of the method used for allocation, the result was a pair of unit exergy costs for both final products that lie along a defined straight line, which can be mathematically represented by Eq. (20).

$$\dot{W}_{cmp} = k_{E_{Qcond}} \cdot \dot{E}_{Q_{cond}} + k_{E_{Qevp}} \cdot \dot{E}_{Q_{evp}} \rightarrow k_{E_{Qcond}} = -\frac{\dot{E}_{Q_{evp}}}{\dot{E}_{Q_{cond}}} \cdot k_{E_{Qevp}} + \frac{\dot{W}_{cmp}}{\dot{E}_{Q_{cond}}} \tag{20}$$

Thus, for each model, as the unit exergy cost of heating increases (k_{EQcond}), the unit exergy cost of cooling (k_{EQevp}) decreases. The six points in Figure 10 represent pairs of unit exergy costs for heating and cooling obtained from the different methods analysed in the study, and they are consistent from a thermoeconomic point of view since they belong to the same straight-line solution.

In Figure 10, it can be observed in the upper corner that when assigning the entire cost of the fuel of the system to a single final product, the unit exergy costs will be (0;2.98) and (17.99;0) for the separately produced of heating and cooling, respectively. In this same region of the Figure 10, it is possible to identify a green square that delimits an area where the costs allocation corresponding to all thermoeconomic methodologies presented in this work are contained, highlighting that the results obtained for the unit exergy costs do not present significant differences. More specifically, when considering all possible thermoeconomic results for the unit exergetic costs of heating and cooling, it is possible to notice that the employed methodologies are concentrated within a specific range, with a variation of around 17.5% for heating and cooling. This observation can be interpreted as an additional indication that these methodologies demonstrate yield similar results with respect to the values found.

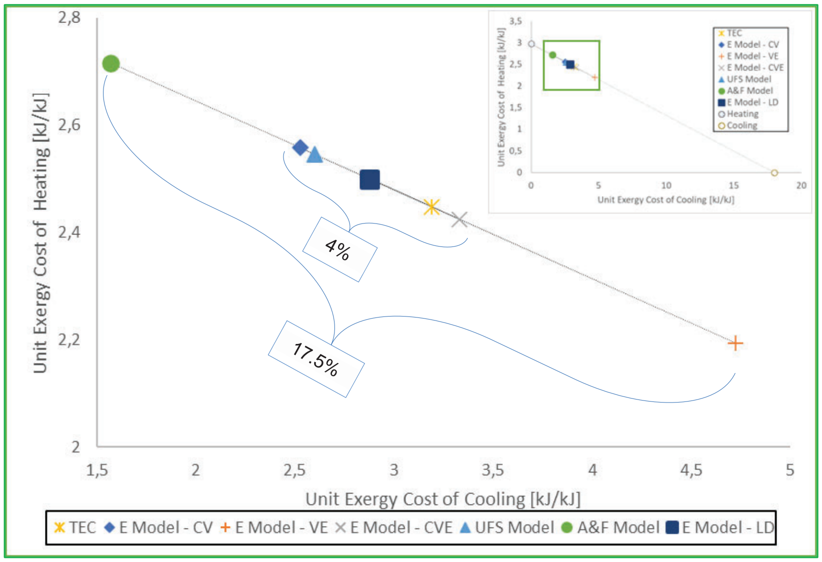

Figure. 10. Results of final unit exergy costs of products

It is not possible to determine which methodology presents the best/right cost through cost allocation, as all the methodologies presented in this study produce coherent results. However, some comparative analysis can be made such as among the three possibilities for using Model E presented in the study. It is observed that associating the valve with the condenser results in the highest cost of heating among the three possibilities, which was expected because all the irreversibility related to the valve is associated with heat, which increases his cost. Associating the valve with the evaporator leads to an increase in the cost of cooling, as all the valve irreversibility is associated with the evaporator. When there is a rational criterion for dividing the irreversibility of the valve (Model E-CVE), an intermediate value is obtained for the costs. It can be observed that the A&F model exhibits the highest unit exergy cost of heating. This can be attributed to the fact that all the irreversibility of the valve is associated as fuel to the condenser, along with a portion of the irreversibility of the evaporator that is also linked as fuel to the condenser (see Figure 8). This is similar to the UFS model. However, in the UFS model diagram, a part of the irreversibility of the condenser returns to the cycle as fuel to the compressor, which slightly reduces the cost of the condenser product.

When analysing the TEC, E Model - CVE, and E Model - LD, it is noted that all of them have an implicit or explicit rule for dividing the irreversibility of the valve for both final products, which results in intermediate costs for these products. It is worth noting that it is more thermodynamically reasonable for the irreversibility of the valve to be divided between both final products. However, not all methodologies presented in this study follow this procedure, possibly due to the subjectivity of the productive diagram, regarding the productive unit interconnection, adopted by the analyst during the implementation of these methodologies. By conducting a more detailed analysis of cost allocation, excluding methodologies that overload the cost of heating (A&F Model and E - CV Model) or cooling (E Model - VE), it can be observed that the methodologies that apply a more rational criterion for dividing valve irreversibility among final products (TEC, E Model - VCE, E MODEL - LD, and UFS Model) are concentrated within a specific range, with a variation of about 4% for heating and cooling. This conclusion is the result of a more rigorous and precise analysis, which allowed for a clearer identification of the methodologies that may be more effective in allocating these costs.

The information presented in Tables 4 – 5 displays the values of fuel (Fu), product (Pr) and irreversibility (Ir) for each individual component of the plant, for each used methodology. It should be noted that although different productive structures (fuel and product) are defined, no matter the methodology, the irreversibility (fuel-product difference) of each component remains the same. Irreversibility is one of the responsible for generating costs in thermoeconomics. However, the methodologies obtain different results since different fuels and products are used for the productive units. It is important to highlight that the costs generated by each productive units depend on the product-fuel ratio (efficiency). Thus, different cost values are obtained for each methodology.

Table 5 presents a comparison between the subsystems merged in the E model. It is observed that, in the E-Model-CV, the irreversibility of the Condenser-Valve subsystem is exactly equal to the sum of the irreversibility of the equipment when they are separated in the other thermoeconomic methodologies. Similarly, the same result can be observed in the E-Model-VE. For the E-Model-CVE, two inputs, two outputs, and two irreversibilities were presented, as the valve is partly merged with the condenser and partly with the valve. Nevertheless, it is still possible to compare the irreversibility, as the sum of the two irreversibilities presented in the E-Model-CVE is the same as the sum of the irreversibility of these subsystems when separated in the other thermoeconomic methodologies.

Table 4. Exergy balances of each productive units of the heat pump system.

Model	Condenser			Valve			Evaporator			Compressor		
	Fu [kW]	Pr [kW]	Ir [kW]	Fu [kW]	Pr [kW]	Ir [kW]	Fu [kW]	Pr [kW]	Ir [kW]	Fu [kW]	Pr [kW]	Ir [kW]
TEC	60.8	37.0	23.8	86.2	76.1	10.1	11.6	6.1	5.5	110.2	82.5	27.7
UFS	349.1	325.3	23.8	17.1	7.1	10.1	261.6	256.1	5.5	126.8	99.2	27.7
A&F	60.8	37.0	23.8	17.1	7.1	10.1	36.8	31.4	5.5	110.2	82.5	27.7
E - LD	60.8	37.0	23.8	17.1	7.1	10.1	11.6	6.1	5.5	110.2	82.5	27.7

Table 5. Exergy balances of the E Models with heat pump subsystem division.

Model	Condenser Valve			Valve Evaporator			Condenser Valve Evaporator			Compressor		
	Fu [kW]	Pr [kW]	Ir [kW]	Fu [kW]	Pr [kW]	Ir [kW]	Fu [kW]	Pr [kW]	Ir [kW]	Fu [kW]	Pr [kW]	Ir [kW]
E - CV	70.9	37.0	33.9	-	-	-	-	-	-	110.2	82.5	27.7
E - VE	-	-	-	21.7	6.1	15.6	-	-	-	110.2	82.5	27.7
E - CVE	-	-	-	-	-	-	67.2 15.2	37 6.1	30.2 9.1	110.2	82.5	27.7

7. Conclusions

This study aimed to present and compare different thermoeconomic methodologies for unit exergy cost allocation in a heat pump system with a dissipative valve. The study focused on presenting different options for treating the valve in thermoeconomic modelling and comparing the advantages and disadvantages of each methodology. The methodologies presented in this study were TEC, E Model, UFS Model, A&F Model, and localized physical exergy disaggregation.

Different models can be used to define fuels and products of subsystems in the presence of a dissipative equipment like a valve. The E Model uses total exergy, but it can be challenging to assign a productive purpose to the valve. The UFS Model is more complex but allows for the isolation of the valve. The A&F Model isolates valves using Helmholtz energy and flow work terms, reducing complexity and computational requirements when compared with UFS Model. Finally, localized physical exergy disaggregation can disaggregate physical exergy only in dissipative equipment, like valves, where total exergy is not enough to define a productive purpose. It is important to observe that while the models define different productive structures, with different fuels and products utilized in each production unit, the irreversibility of each component remains constant. Therefore, it can be concluded that the models are consistent with thermodynamic principles.

The results demonstrate that although each methodology presents different results, the differences are not significant. Therefore, the methodology to be used will depend on various factors, such as the need to have disaggregated equipment or not. Regarding the E Model - CV and E Model - VE, it was observed that they tend to overload the cost of heating and cooling, respectively. Conversely, the UFS and A&F Models disaggregate the valve but were presented in this study using the productive diagram, which may have some arbitrariness in relation to connections, due to the use of productive flow only. Thus, for these methodologies, the use of the comprehensive diagram, which combines both productive and physical flows, may be a more appropriate option, since the TEC method, which uses physical flows, presents an intermediate cost.

When considering all possible thermoeconomic results, i.e., all the solution straight line, the obtained results are concentrated within approximately 17.5% of the line. However, excluding methodologies that overload the cost of heating or cooling, this value would drop to 4%, which indicates that these methodologies did not produce significantly distinct results.

In conclusion, this study provides valuable insights for future decisions in thermoeconomic modelling and highlights the importance of a suitable productive structure to explain the productive function of the subsystems and flows present in the physical structure of the plant.

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Nomenclature

Latin

A	Helmholtz energy [kJ]
a	Specific Helmholtz energy [kJ/kg]
A&F	Helmholtz energy and flow work
cmp	compressor
cnd	condenser
E	Physical exergy
evp	evaporator
evp	evaporator
F	Flow work [kJ]
H	Enthalpic term [kJ]
h	Specific enthalpy [kJ/kg]
Ir	Irreversibility
k	Exergetic unit cost [kW/kW]
LD	Localized physical exergy disaggregation
P	Pressure [kPa]
Q	heat exergy
S	Entropy [kJ/K]
s	Specific entropic [kJ/kgK]
T	Temperature [°C or K]
TEC	Theory of the exergetic cost
trb	turbine
U	Internal energy [kJ]
UFS	Internal energy, flow work, entropic term
v	Specific volume [m^3/ kg]
vlv	valve
Y	thermodynamic magnitudes
$\dot{m}$	Mass flow [m^3 /s]

Greek

η	efficiency
x	quality

Subscript

0	Environmental conditions
cnd	condenser
evp	evaporator
F	Fuel
i	Internal flow
in	Inlet
out	Outlet
ph	Physical
trb	turbine
vlv	valve

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