PROCEEDINGS OF ECOS 2023 - THE 36TH INTERNATIONAL CONFERENCE ON
EFFICIENCY, **C**OST, **O**PTIMIZATION, **S**IMULATION AND ENVIRONMENTAL IMPACT OF ENERGY SYSTEMS
25-30 JUNE 2023, LAS PALMAS DE GRAN CANARIA, SPAIN

Analysis of first-order model for thermistors in compressible flows temperature measurement

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Abstract:

Thermal variation is assumed to be one of the main source of uncertainty in dimensional measurements and must be controlled in order to analyse and quantify its possible effects. Therefore, as the analysis of temperature control is influenced by different complex non-linear heat transfer effects, the study of this variable is considered a highly application dependent measurement problem. However, temperature measurement devices are often designed primarily to increase their durability, so they remain vulnerable to transient effects. Hence, improvements in mathematical models of general purpose temperature sensors through the knowledge of its response in the application would significantly improve their performance and accuracy.

Keywords:

Thermistor response, Heat transfer, Dynamic response, First-order models.

1. Introduction

One of the coins used to determine the macroscopic state of a system is determined by means of an intensive property known as temperature. This variable has an important dependence on statistical concepts and, for the moment, we shall regard the temperature to be an empirical quantity, measured by a device, such that temperature is proportional to the expansion [1] that occurs whenever energy is added to matter by means of heat transfer. This concept can be understood by imagining a thermometer, where thermal expansion is: liquid metal in a long glass tube, bending of a bimetallic strip or resistivity variation of a semiconductor, among others.

Thermal variation [2] is assumed to be the main source of uncertainty in dimensional measurements and, given these influences, the control of this variable is considered to be a highly application-dependent measurement problem. In fact, it is also known that heat transfer analysis and temperature control is very useful in many engineering systems and that, under stationary conditions, its value does not vary with time at a given point. In contrast, under non-stationary [3] or transient conditions, the temperature varies with time. However, under stationary conditions, effects such as self-heating can occur. For this reason, Darkhaneh [4] proposes a way to quantify the error caused by thermistor self-heating, as it is a problem that generates uncertainty in temperature measurement. In addition, there are areas of thermistor operation that it is preferable to avoid due to certain effects such as blowup, as pointed out in their respective studies by Antontsev and Chipot [5] and Barabanova [6].

On the other hand, nowadays, the precise measurement of the thermal properties of materials is mainly done on the centimetre or millimetre scale [7,8]. It is clear that the miniaturisation of sensors brings undeniable advantages, such as: lower thermal inertia; limited invasiveness of the system to be characterised, which is therefore less disturbed by the sensor; and the possibility of using small amounts of material, which is highly desirable in nanotechnologies, for example. Thus, in this document we refer to miniature sensors as sensor elements whose main dimensions are of the order of a millimetre.

Other problems has been observed that for certain applications involving convection mechanisms, especially in the study of turbulent flows, phenomena are detected during temperature monitoring that introduce uncertainty and lead to confusion in the processes to be analysed. The measurement of transient heat transfer is very important in various areas of scientific importance including the determination of engine temperature,

https://doi.org/10.52202/069564-0034

aerodynamic vehicle temperature in a high velocity flow environment, mentioned by Sanjeev and Kumar [9,10]. From the point of view of analysing the turbulent effects that can occur and generate uncertainty in the temperature measurement, we can refer to the smallest known scale, the Kolmogorov scale. This is the smallest turbulent motion [11] present in a flow and is the scale at which energy is dissipated by molecular viscosity. Taking into account the formation of vorticity at certain points close to the measurement points, near-wall turbulence effects can be considered, as well as the ratio of the velocity and thermal boundary layer established by the Prandtl number.

Hence, improvements in the response of general-purpose temperature sensors through the compensation for response delay would significantly improve their performance and applicability, like mention Tagawa et al. [12]. Thus, this work proposes an experiment to study the turbulence and vorticity generated through the stagnation of a device exposed to an air stream. A device inmersed into the airstream has two built-in thermistors that will measure the stagnation temperature and the static temperature (Fig. 3). In these experiments, it can be seen that an effect is produced that goes against the theory presented and this phenomenon can be associated with a fraction of energy that is dissipated through the leads of the thermistor. This is then corroborated by a series of additional tests in different media with different heat transfer coefficients. Then, this experiment is presented from a mathematical point of view, studying and analysing the effectiveness of first-order models for cases where the sensors are in an airstream. These models provide the necessary tools to approximate real behaviour and by means of a developed algorithm (depending on the selected conditions) it is possible to simulate a thermistor response similar to that of the experiment. However, as will be contrasted with the other tests performed, in certain applications they do not have the necessary precision and induce error in the measurements.

One of the main motivations for the development of this work is that temperature measurement devices are often vulnerable to these transient effects, as they are primarily designed to increase their durability. However, the moderating industry demands smaller and smaller, non-invasive measuring devices for many applications. More accurate measurements of thermal conditions are therefore a real challenge as these companies are demanding more and more complex and precise systems. Accordingly, identifying a suitable mathematical model and the effects that occur in temperature sensors will allow us to evaluate a way to study the effects of turbulence and heat transfer and to be more aware of transient effects.

In the current paper, as a first point of departure, a series of theoretical concepts involved in the test carried out will be described. This is followed by a detailed explanation of the general first-model with the formulation of the governing equations. Next, the description of experiment that has led to the hypotheses put forward in relation to the observed phenomena. Then, the results obtained from such equations will be compared with those of real measurements, thus explaining the phenomena that cause such behaviours. Finally, some perspectives of future are exposed with the conclusions obtained.

1.1. Boundary layer: hidrodynamic and thermal

It is obvious to think that the temperature distribution obtained around a body immersed in a stream will have a similar character to the distribution of velocities in the various layers of a fluid. Therefore, in a flow over a heated surface, both the velocity boundary layer and the thermal boundary layer develop simultaneously (Fig. 1).

This concept of boundary layer was introduced into the science of fluid mechanics by L. Prandtl at the begining of the last century: it has probed to be very fruitful.

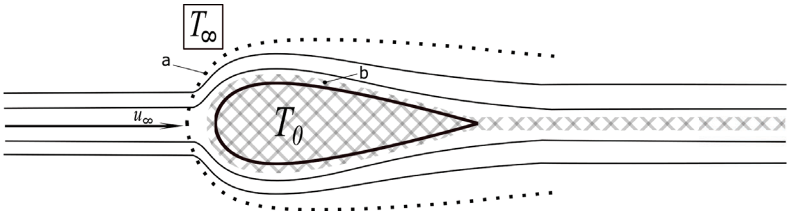

Figure 1: Analogy between temperature and vorticity distribution in the neighbourghood of a body placd in a stream of fluid.

In the Fig. 1, **a** and **b** correspond to the limits of the temperature increase; **a** for low speeds and **b** for high speeds. It should be noted that in the convective transport mechanism two processes take place, the diffusion of heat by the randomly moving molecules in the fluid and the advection of heat due to the overall motion of the fluid. As the molecules of the whole maintain their random motion, the total heat transfer is due to a superposition of the energy transport due to the motion of the molecules and the global motion of the fluid that takes place. Thus, if we imagine a solid body placed in a fluid stream and heated so that its temperature remains above the surrounding environment, then it is clear that the temperature of the stream will increase only in the layer closest to the boda and in a narrow wake behind it, as shown in Fig. 2. Therefore, the greatest diffusion from the hot body to the surroundings occurs in the layers closest to the hot body, which, in accordance with the flow phenomenon, can be called the thermal boundary layer.

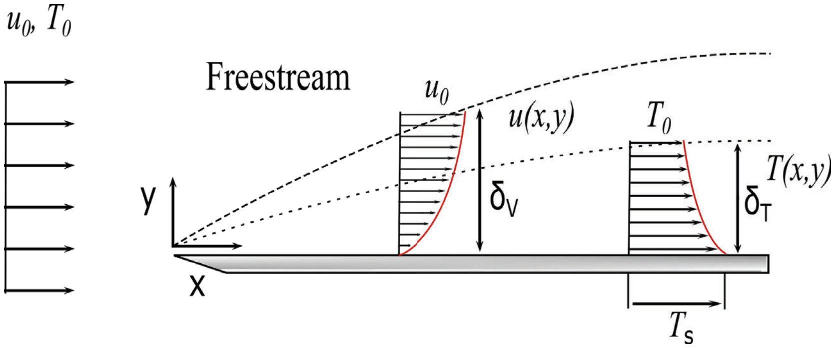

Figure 2: Hidrodynamic and thermal boundary layer.

The thickness of the thermal boundary [13] layer (ΔT) at any location along the surface is defined as the distance from the surface at which the temperature difference ($T - T_s$) is equal to $0.99 \cdot (T_0 - T_s)$, which establishes some relationship to the velocity boundary layer. So, it can be said that the rate of convective heat transfer anywhere along the surface is directly related to the temperature gradient at that location. Hence, the shape of the temperature profile of the thermal boundary layer imposes convective heat transfer between the solid surface and the fluid flowing over it. Since the fluid velocity will have a strong influence on the temperature profile, the development of the velocity boundary layer relative to the thermal boundary layer will have a strong effect on the convective heat transfer. So, it is evident that flow phenomena and thermal phenomena interact to a high degree.

1.2. Temperature increases through adiabatic compression; stagnation temperature.

Temperature changes caused by dynamic pressure variation in a compressible fluid are important from the point of view of heat transfer. In particular, it seems useful to compare the difference in temperatures resulting from heat generation by friction with those caused by compression. For this reason, if we look at Fig. 1, it is logical to think that if the velocity varies along the contour, the temperature will also vary along the shape. If we assume an adiabatic and reversible process due to the low value of conductivity and the high right rate of change in the thermodynamics propierties of state will, in general, prevent any apreciable exchange of heat with the sorroundings. Then, the temperature increase (ΔT) which occurs at the stagnation point of a body in a fluid stream and that is due to the compression that is generated when the fluid makes contact with the solid, as can be deduced from Fig. 1.

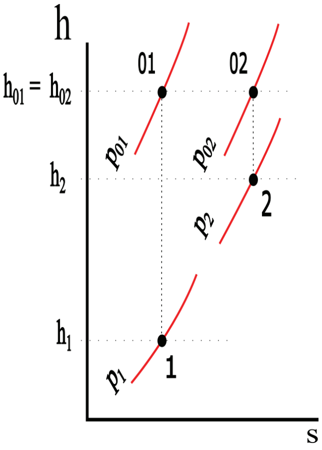

Figure 3: h-s diagram.

The h-s diagram indicates that the stagnation enthalpies - total energy level - are constant, but the area of interest focuses on the static variables and the kinetic energy terms. Thus, it is possible to know the energy state of the fluid and to determine the conditions that are introducing uncertainty into the temperature measurements.

2. First-order model

As mentioned in the introduction, the first model corresponds to the classical approach proposed by the manufacturer to obtain the thermistor properties. It is therefore a generic model that encompasses any situation to which the sensor is subjected. In other words, this type of proposal does not focus on the heat flow through the wires, as well as other thermophysical parameters.

Accordingly, a scenario showing a thermistor immersed in airstream and the heat transfer due to convection process between sensor and surrounding is shown in Fig. 4. In this schematic, it can be seen that the thermistor is in an environment whose fluid temperature is (T_∞) and surrounding temperature (T_{sur}). Also, the boundary layer around of thermistor (control volume) has an associated a convective resistance (R_h).

On the other hand, at this point, it is important to point out that the influences of radiation heat transfer ($\dot{q}_r$) , as well as the self heating (S_h) producing in the thermistor are of minimal relevance in this study, as will be discussed in the conclusions section. Therefore, for now we will focus on the determination of the convection ($\dot{q}_h$) phenomena.

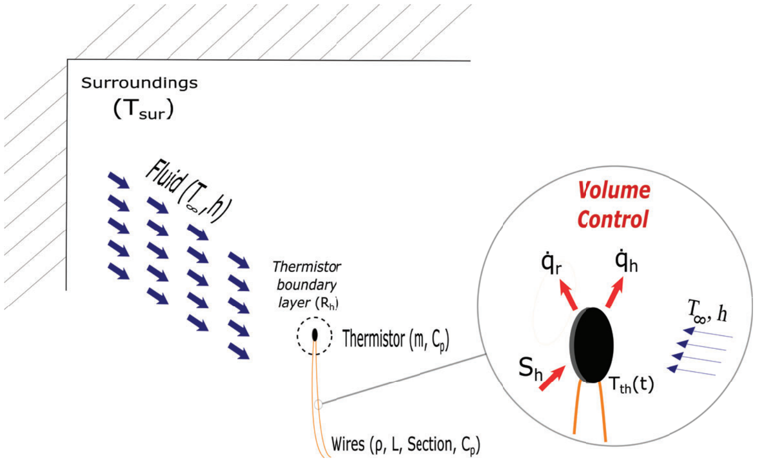

Figure 4: First-order model diagram.

Then, from the energy balance in the control volume (CV) of the last diagram, is deduced:

$$-\dot{q}_h = m \cdot c_p \cdot \frac{dT_{th}}{dt} \tag{1}$$

Keep in mind the minus sign of the equation means that the heat transfer is gave up to surrounding area.
By the other hand, the Newton´s cooling law is:

$$\dot{q}_h = h \cdot A \cdot (T_{th}(t) - T_\infty) \tag{2}$$

So, substituting Eq. (2) into Eq. (1) and developing, it is obtained:

$$-h \cdot A \cdot (T_{th}(t) - T_\infty) = m \cdot c_p \cdot \frac{dT_{th}}{dt} \tag{3}$$

$$(T_{th}(t) - T_\infty) + \frac{m \cdot c_p}{h \cdot A} \cdot \frac{dT_{th}}{dt} = 0 \tag{4}$$

$$(T_{th}(t) - T_\infty) + \tau \cdot \frac{dT_{th}}{dt} = 0 \tag{5}$$

At this point, a linear time-invariant system (LTI) is founded. By this:

$$\frac{1}{\Delta T_{th}} \cdot dT = -\frac{1}{\tau} \cdot dt \tag{6}$$

$$\Delta T_{th_1} = \Delta T_{th_2} \cdot e^{-t/\tau} \tag{7}$$

$$T_{th_1} = (T_{th_2} - T_\infty) \cdot e^{-t/\tau} + T_\infty \tag{8}$$

If $t = \tau$, the equation is:

$$\frac{(T_{th_1} - T_\infty)}{(T_{th_2} - T_\infty)} = 0.632 \tag{9}$$

That is, the time period during which temperature effectiveness ($T_{th_1} - T_\infty$) becomes 63.2% of the temperature width ($T_{th_2} - T_\infty$) is taken as τ.

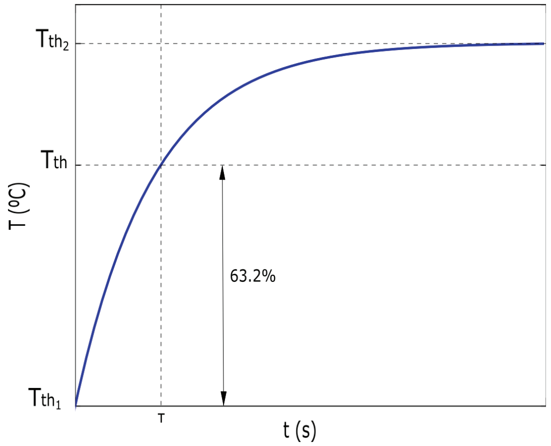

Figure 5: Thermal time constant.

2.1. Thermal-electrical analogy for first-order model

Generally, the manufacturer proposes a first order system as the characteristic response of a temperature sensor. This kind of representation makes it possible to show mathematically and in a simple manner how the thermistor (T_A) behaves over time when an external heat source (E_s) is applied to it. However, this model omits some effects that may be of special interest for the phenomena described in the previous paragraphs. Nevertheless, as a first analysis, to which certain terms will later be introduced and compared, this model is presented in which a series of conditions will be taken into account. Thus, as proposed by Sanjeev and Kumar in [7], the one-dimensional (1D) heat transfer equation for a semi-infinite solid body can be a starting point to obtain the value of the transient temperature at the sensor surface. So, from the schematic shown in Fig. 4, does not involve heat flow through the wires ($\dot{q}_c$), radiation heat transfer ($\dot{q}_r$) and the self heating (S_h). Thus, using the electrical analogy for the study of the heat transfer of a system, this first-order model results as follows:

For this case, initially only convective resistance of the CV (R_h) and heat capacity of thermistor (C_1) will be taken into account.
Therefore, from the energy balance at point A it is obtained:

$$E_s - \dot{q}_h = C_1 \cdot \frac{dT_A}{dt} \tag{10}$$

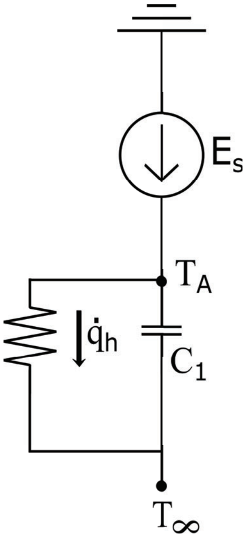

Figure 6: Thermal-electrical diagram for first-order model.

The Newton´s cooling law is:

$$\dot{q}_h = h \cdot A \cdot (T_A - T_\infty) \tag{11}$$

Then, substituting the Eq. (11) into Eq. (10) and developing the result according ours considerations it is obtained:

$$\frac{C_1}{h \cdot A} \cdot \frac{dT_A}{dt} + (1 + \frac{1}{h \cdot A \cdot R_1}) \cdot T_A - \frac{1}{h \cdot A} \cdot H_s = 0 \tag{12}$$

Where the term ($\frac{C_1}{h \cdot A}$) is the known thermal time constant (τ). The form of which is as follows:

$$a_2 \cdot y' + a_1 \cdot y + a_0 = 0 \tag{13}$$

Each coefficient accompanying the variable corresponds to those in Eq. (12), so they must be taken into account in the solution. Also, the b term in the following equation is related to the amplitude of the external heat source (E_s). This turns out to be a first order linear ordinary differential equation, the solution of which is:

$$T_A = b \cdot e^{(-a_1/a_2) \cdot t} - \frac{a_0}{a_1} \tag{14}$$

As mentioned in the section on the thermal time constant, this Eq. (14) can be divided into a heating and a cooling stage, thus obtaining two responses with corresponding time constants. Obviously, this depends on the external source, among other factors, so we will focus on the generic equation as such.

3. Overview of experimental procedure.

The way to relate the study of turbulence and heat transfer is for the thermistor to be excited by an external source that causes the same disturbances and uncertainties as the vortices in an air current. Thus, in order to identify the limitations to which these temperature sensors are subject, a device and an experiment are designed to study their behaviour. This test consists of placing two thermistors in a small tube so that each sensor measures a different temperature: static temperature (T_1) and stagnation temperature (T_{01}). This refers to the diagram presented in Fig. 3.

Finally, this device is placed inside an isolated tube through which an airstream circulates.

The morphology of the described device is schematically represented in the following image.

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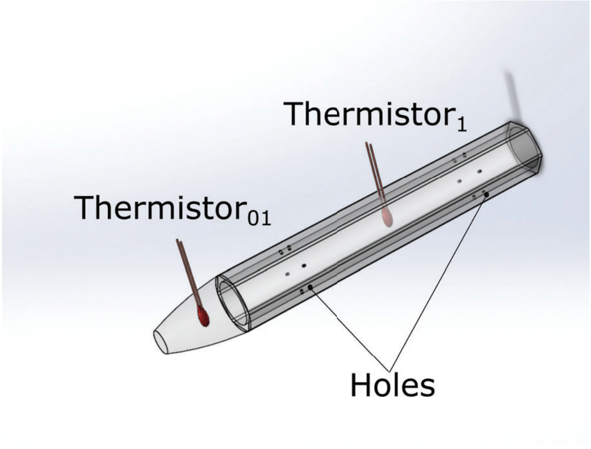

Figure 7: Sensor.

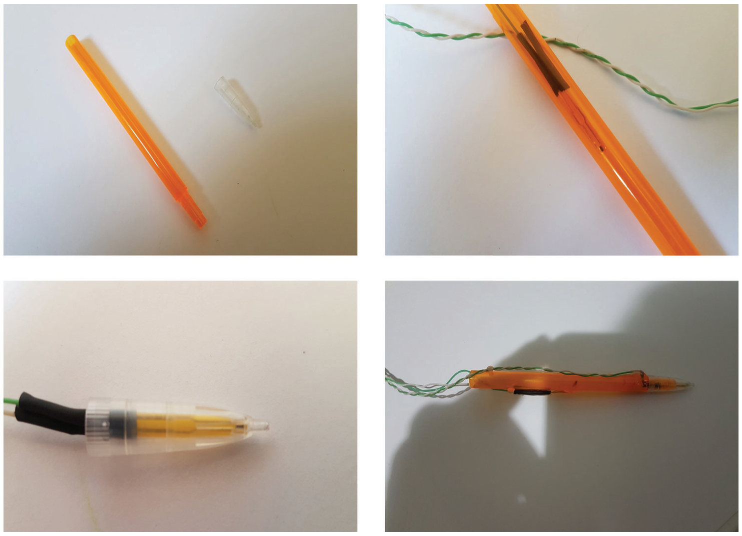

Figure 8: Sensor.

Fig. 8 below shows the actual assembly of the device:

Some parameters of the thermistor used in the experiment are shown in the Table 1 but, the most important one is the thermal time constant that will be compared with the numerical solution proposed.

The airstream is produced by fan controlled by a microcontroller. Inside of the tube there is the device with two thermistors NTC that read the different temperature changes (T_{01}) and (T_1). Also, there is an additional thermistor outside that reads atmosphere temperature but is not integrated in the system as its value is merely informative. These values, are reading using an integrated development environment. With this, it is possible to collect and show data comming from microcontroller.
Regarding to data acquisition, we have these devices:

- An adaptative stage of signal (Voltage divider): consists of $10k\Omega$ resistors and the power supply ensures a constant voltage of 5V.
- An analogical to digital converter of 16 bits (ADS1115 Texas Instruments): will allow a higher accuracy of

Table 1: Parameters of NTC thermistor used.

Parameter	Value
Resistance at 25 ºC	10.000$\Omega \pm 1\%$
Temperature rating	-30º to 90 ºC
β (25º to 85ºC)	3.435 K nominal
Dissipation constant	0,7 mW/ºC nominal (Still air)
Thermal time constant	5 seconds nominal (air)

the temperature values read. It is also powered by the 5V power supply.

- A microcontroller ATmega328 (Atmel) implemented in an embebed system (Arduino): to prevent the microcontroller that records the data from crashing, another one is available to control the shutter. In turn, both are managed by an external application developed that allows real-time reading of the data.

Although, other devices are used in this experiment and keeping in mind all system described, Fig. 9 shows a schema of this:

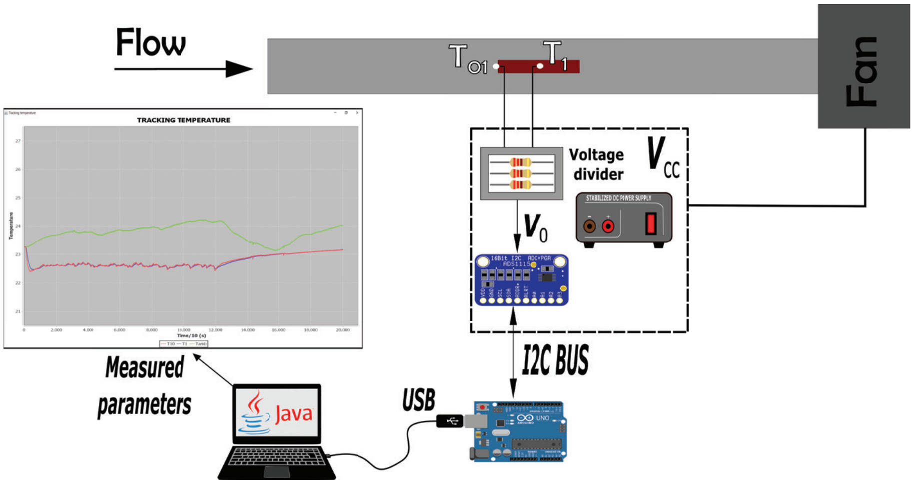

Figure 9: Acquisition and data capture for the experiment.

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In Fig. 10, the response obtained during an experiment is shown. As can be seen, the temperature gradient value is proposed on the vertical axis and the test time on the horizontal axis. When the measurement is started, the fan is started and the evolution of the data is analysed in real time. This information is also stored for later study.

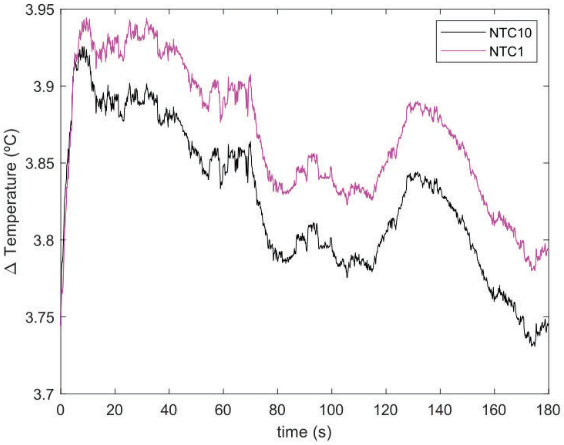

Figure 10: Response with tube experiment and two thermistors.

4. Results and discussion.

4.1. Response of thermistor in airstream

In this section, the results carried out in the tube with the device to measure temperatures will be presented. It should be noted that each test has a sample frequency of 10 data per second. The duration of the tests can be variable. This depends on the amount of data to be collected. Generally, however, tests last between 60 and 300 seconds.

For example, the two experiments shown below (Fig. 11 and Fig. 12) have a duration of 80 seconds. As soon as the temperature reading begins, the fan starts running and the data is recorded. In both tests, the magenta line corresponds to the static temperature (thermistor inside the device) and the black line to the stagnation temperature (thermistor at the tip of the device).

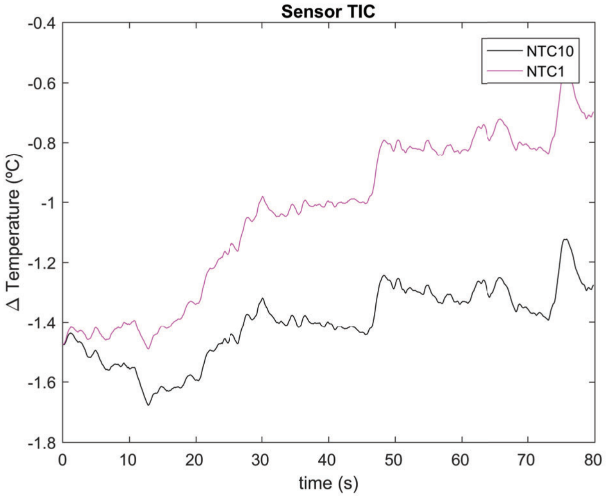

Figure 11: Response with tube experiment and two thermistors.

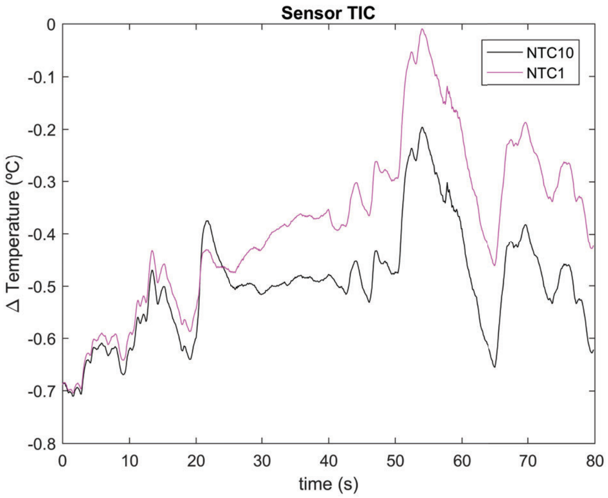

Figure 12: Response with tube experiment and two thermistors.

As can be seen in the previous graphs, the difference in temperature of both variables reaches maximums of 0.2 ºC. However, the most curious thing is the value that the stagnation temperature takes, when it is supposed to increase due to friction effects. This was later corroborated by a simple experiment that simulated the behaviour of the fluid inside the tube. This test consisted of exciting the isolated thermistor with an external heat source in a chamber in which two completely different fluids were introduced separately. This heat source allowed different excitation powers to be applied to the thermistor, thus contrasting the different media. Then, the typical response of the thermistor used to an external heat source that is applied for a certain time and then disappears is shown. It should be noted that these experiments are carried out in still fluids and under controlled conditions. Also, what is demonstrated is that the response of the sensor itself is simply reproducible from Eq. (14), as can be seen in the graph below.

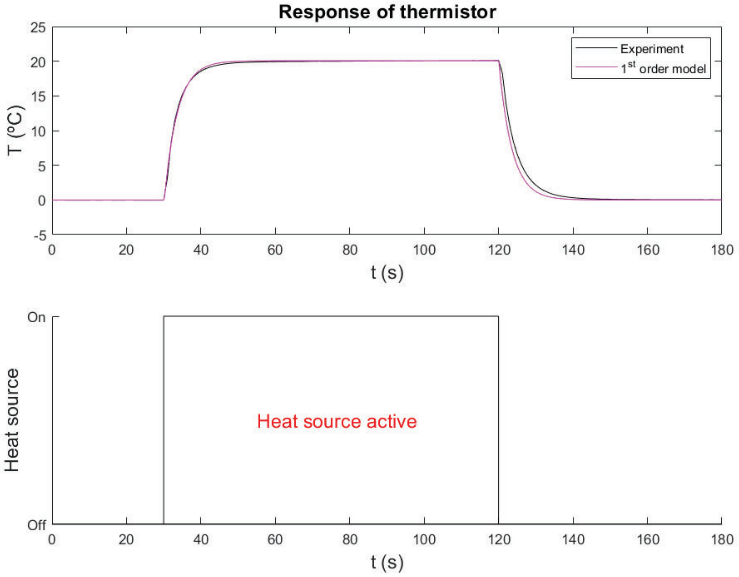

Figure 13: Response of thermistor during experiment using external heat source.

The following table shows the temperature gradient reached by the thermistor for each of the four powers during the tests with different fluids. Also, a quality estimator (QE) is shown to assess the accuracy with which the first-order model is adapted to the response obtained. Fluid 1 has a lower heat transfer coefficient and after the tests a time constant of 3.57 s was obtained. For fluid 2, which has a higher heat transfer coefficient, the time constant was found to be 1.29 s.

Table 2: Temperature gradient results in different fluids and excitation powers.

	Fluid 1			Fluid 2		
Power	ΔT_1 ($^\circ C$)	STD_1	QE_1	ΔT_2 ($^\circ C$)	QE_2	STD_2
P_1	25,41	0,1	230,81	11,77	0,03	87,77
P_2	20,19	0,01	85,67	10,07	0,002	17,54
P_3	10,47	0,01	40,57	4,43	0,001	13,18
P_4	9,80	0,04	50,89	4,38	0,05	18,99

With all this, it is understood that there is an important fraction of heat flow that is lost through the cables and that is not analysed accordingly. Thus, from the first test in which the stagnation temperature and the static temperature were measured, results were obtained that suggested that certain phenomena related to the turbulent behaviour of the fluid were taking place and that heat flow through the cables was being generated.

4.2. Discussion.

Due to minimal surface of thermistors in a gaseous medium, there is a behaviour with a high dynamic responses. This high dynamic response of sensors proves to be a great advantage in several applications, in which temperature evolution measurements are sought with great precision in phenomena that occur in a short period of time, in which the temperature undergoes very abrupt variations. Also, due to the high transient response to which both thermistors are subjected in the temperature measurement during the experiment, it is unfeasible to obtain such behaviour by means of a first-order mathematical model. However, for applications where the measurements are more stable and are not constantly changing abruptly, such as Fig. 13, it is possible to resort to first-order mathematical models.

On the other hand, even if it is possible to find a model that fits the response obtained in the conditions of the tube experiments (Fig. 11 and Fig. 12), it must be stressed that there are other phenomena that are not considered. As mentioned above, the stagnation temperature will tend to increase its value due to the friction generated in the layers closest to the thermistor. But as can be seen, the opposite is true. This suggests that there is a fraction of energy that is flowing through the conductors and is inducing this contradictory behaviour. Therefore, it is necessary to have tools that allow us to discern the applications, models and sensors to be used in order to obtain more precise measurements.

5. Conclusions.

This article presents experiments carried out under specific conditions that have allowed conclusions to be drawn about first-order models and their accuracy in real applications.

Therefore, the existence of conduction phenomena through the cables is evident, which in certain applications introduce uncertainty in the measurements. Thus, all this leads us to consider that in order to use this type of sensors in temperature control systems, it is important to take into account the conditions of the fluid to be monitored. Thus, as mentioned, due to the high dynamic responses offered by these sensors, it is necessary to know the uncertainties introduced by boundary layer effects and vorticity. These phenomena can lead to errors in high-precision processes in terms of temperature knowledge, as we have seen.

However, not everything should be seen from a negative point of view, as this type of device could be used to determine the turbulence near the wall or to detect the boundary layer detachment zone in order to improve aerodynamics. Thermistors, knowing their influence, have an attractive application in this type of field due to their small size and the reduced instrumentation required. Therefore, the development of measuring devices that implement this type of sensors to study the conditions of a fluid is not ruled out. Likewise, the quantification of the energy lost through the cables could be a future motivation, as the introduction of tools would allow more detailed processes to be obtained.

Nomenclature

Letter symbols

A	area, m^2
c_p	specific heat at constant pressure, $J/(kgK)$
h	heat transfer coefficient, W/m^2K

$\dot{m}$	mass flow rate, kg/s
p	pressure, N/m^2
$\dot{q}$	heat flow rate, J/s
T	temperature, K

Greek symbols

Δ	gradient
τ	time constant

Subscripts and superscripts

h	convection
∞	fluid
r	radiation
s	surface
sur	surrounding
th	thermistor

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