A simple truncation criterion in CPCs using constructal theory

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Abstract:

Compound parabolic concentrators (CPCs), whose origin dates back to the mid-1960s, gave rise to anidolic optics. Systems of this type allow the thermodynamic limit of solar concentration to be reached; however, in the case of CPCs, they involve systems with a very large height in relation to the aperture area (svelteness). Thus, arbitrary criteria have been proposed to reduce the height of the systems (truncation) and to be able to give them a real application. These truncation criteria establish the elimination of the upper part of the CPCs at a certain height in order to considerably reduce the height, but this has certainly undesirable consequences, since the geometric concentration (ratio of aperture area to receiver area) decreases with respect to the original design, which can limit the operating temperatures. Alternatively, a geometrical criterion has been proposed to truncate the CPCs without losing geometrical concentration, and still manage to reduce the height of the system by approximately 30%. This criterion consists of avoiding that the parabolic section mirrors do not block the incident rays that enter the aperture area with the maximum acceptance angle, thus defining an optimum truncation angle of 3 times the acceptance angle. However, now, with the help of the constructal law, it is possible to demonstrate that this angle is the optimum from a geometric viewpoint. Additionally, a new dimensionless number is defined for solar concentrating systems, relating the entropy generation distribution ratio, allowing to demonstrate that the Rincón criterion is the optimum for $C_g > 2.28$.

Keywords:

CPC truncation, entropy generation, étendue, constructal law

1. Introduction

The origin of compound parabolic concentrators (CPC) can be traced back to the mid-'60s by the developments of different researchers around the world: Baranov and Melinkov in the URSS [1], Ploke in Germany [2], and Hinterberger and Winston in the US [3], who independently described a novel optical system, unlike traditional systems, were based on the optimal transfer of radiation from the source to the objective, even if this implies forming aberrations and losing the shape of the source at the end of the system. This characteristic led to the coinage of the name non-imaging optics [4], to differentiate it from classical optics, where the image of the source must be conserved [5, 6]. As a result, in nonimaging optics, the light propagation is analyzed in terms of phase-space quantities and energy flow patterns This new optics has some advantages, but the possibility to reach the thermodynamic limit for solar concentrators systems is the principal [7]. Since mid-70s, their potential as collectors of solar energy was pointed out by Winston [8], and the widespread of this technologies has been fruitful.

Today, there is already a wide development of CPCs for solar applications, and several geometries have been generated that take advantage of non-imaging optics to concentrate solar energy on receivers of different shapes (circular, square, triangular, wedge, flat, elliptical...) [4, 9], Fig. 1 shows four typical CPC designs. All these geometries had been used for different purposes, like photovoltaics, solar heating and cooling, solar cooking, solar distillation, among others, that a complete summary of most of the works is out of the scope of the present paper, but several reviews [10–14] summarize some of the many developments, both experimental and commercial, that have been developed to date.

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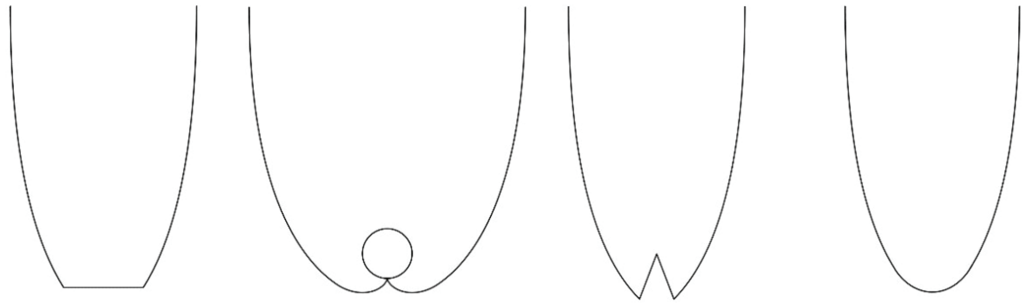

(a) Flat. (b) Circular. (c) Wedge. (d) Elliptical.

Figure 1: Different CPC receiver designs.

The main drawback of the CPCs, is that the ratio of height vs the receiver's area (svelteness) is too large, i.e. these concentrators are too high, and this ultimately limits their application even for fixed concentrators [4]. To overcome this problem, different truncation criteria have been proposed to decrease the total height [15–18] and consequently reduce the use of mirrors. These criteria had been developed from a purely geometrical perspective. Therefore, a formal analysis of the geometrical, optical, and thermodynamic aspects is necessary to establish which of the criteria is the most appropriate. This is fulfilled in the present work analyzing the original 2D flat receiver CPC with its geometric related parameters (height-aperture ratio and reflector area-aperture area ratio), optical-energy parameters (average number of reflections and étendue loss) and entropy related parameters (entropy generation number, optimum concentrator temperature and optimum efficiency) with the use of Bejan's constructal law [19, 20]. Although the analysis is done on a flat receiver CPC, the present work can be extended to any CPC design.

2. Brief description of the CPC

Regardless of the type of absorber the CPC may have, rays entering the concentrator with a maximum half acceptance angle θ_{max} (extreme rays), must be reflected by the mirror so that they are incident tangent to the absorber; while all rays entering with an angle θ less than the maximum half acceptance angle (i.e. within the angular full acceptance angle $2\theta_{max}$), are directed to the absorber after passing through the internal optics of the CPC (reflection or refraction).

With this definition, several receivers can be used, but only the flat receiver uses properly parabolas. In the present work, the flat receiver CPCs are considered. Equation 1 are the parametric equations to describe a flat receiver CPC with full height, schematically shown in Fig. 2., where it is assumed that the length of the trough CPC is l. The geometric parameters that define CPCs are the half-acceptance angle θ_0 and receiver size $2a'$. The subscript t stands for the truncated parameter.

$$\begin{cases} x(t) = \dfrac{2a'\,(1+\sin\theta_0)\cos t}{1-\sin(t-\theta_0)} \\ y(t) = \dfrac{2a'\,(1+\sin\theta_0)\sin t}{1-\sin(t-\theta_0)} \end{cases} \quad t \in \left[0, \frac{\pi}{2} - \theta_0\right] \tag{1}$$

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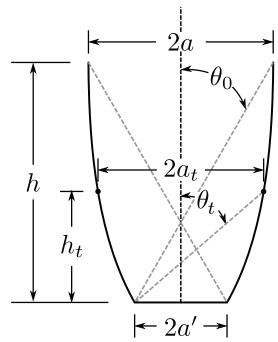

Figure 2: Schematic of the CPC with flat receiver.

From 1, it can easily be demonstrated that the concentration ratio (C_g) for a full CPC is simply:

$$C_g = \frac{A}{A'} = \frac{2al}{2a'l} = \frac{2x\left(\frac{\pi}{2} - \theta_0\right) - 2a'}{2a'} = \frac{1}{\sin\theta_0} \tag{2}$$

With the parametric equations given in 1, the svelteness Sv (height-aperture ratio) and the reflector area-aperture area ratio $r_{m,a}$ can be determined by.

$$Sv = \frac{y(t)}{2x(t) - 2a'} \tag{3}$$

$$r_{m,a} = \frac{\int_0^{t_{max}} \sqrt{x'^2 + y'^2}\mathrm{d}t}{2x(t) - 2a'} \tag{4}$$

2.1. Truncation of the CPC

As mentioned previously, one of the disadvantages of CPCs is the height of the concentrator; however, as seen in Fig. 2, the upper part of the mirrors does not contribute substantially to C_g. For this reason, it is recommended to remove a portion of the mirror to reduce height; this is known as truncation. While truncating the CPCs, material savings are achieved, but on the other hand, C_g will be reduced too since $2a_t < 2a$.

Winston, Rabl, and O'Gallagher recommend truncating the CPCs about half the fully developed height [7, 8, 17] (named Winston's criterion), while it can be found that some CPC can be truncated up to 2/3 the full height to reduce the loss in C_g. Independently, Rincón et al. [18] have proposed to truncate the CPCs to $3\theta_0$, because at this angle the mirror will not block light in the half-acceptance angle range $\pm\theta_0$ (Rincón criterion).

2.2. Optical energy related parameters

The thermal power that reaches the absorber can be described by a relation $\dot{Q}_u \sim G_b C_g \eta_o$, where G_b is the beam solar radiation and η_o is the optical efficiency, being a function of the average number of reflections n in the mirrors with reflectivity ρ as $\eta_o \sim \rho^n$. As can be seen, the lower the average number of reflections, the higher the optical efficiency and the higher the power that reaches the absorber.

Originally, Rabl [15] studied the average number of reflections , where it was shown that reducing the value of n led to an increase in the useful energy of the receiver. A summary of Rabl's equations for n can be found in [21]. Briefly, the average number of reflections in the CPC is a function of the type:

$$n = \max\left(F\left(\theta_0, x, y, t\right), 1 - \frac{1}{C_g}\right) \tag{5}$$

However, the most important parameter in optics is the étendue $\mathcal{E}$, defined as a geometric quantity that measures the amount of "place" available for light to pass [9], as a measure of the power transmitted along a beam of light. For 2D systems, the étendue is defined as [4, 9, 17, 22]:

$$\mathcal{E} = 2An_{ref}\sin\theta\cos\phi \tag{6}$$

where A is the area where the light passes, n_{ref} is the refractive index, θ is the solid angle where light enters in A and ϕ is the incidence angle. Recalling the concept of steady-state balance equations for any physical property ψ, the imbalance of the property can be determined as $\psi_{imb} = \psi_{in} - \psi_{out}$ [23, 24], where the subscript imb stands for the imbalance (destruction in the case of a negative value, generation for a positive value).

According to the definition of 6, if the rays enter an optical system with imperfections (the étendue is not preserved), then, at the output the rays will tend to scatter over the output area, consequently $\mathcal{E}_{out} > \mathcal{E}_{in}$, thus the imbalance term will be defined as scattered instead of generated. For full CPCs, the étendue is conserved [4, 9, 17, 22]. Any scattering in the étendue will cause the optics to fail to properly redirect the incoming light through the concentrator aperture, so the étendue scattering can be defined as:

$$\mathcal{E}_{sc} = \mathcal{E}_{out} - \mathcal{E}_{in} \tag{7}$$

or as a normalized scattering value as:

$$\mathcal{E}_{sc}^{*} = \frac{\mathcal{E}_{out} - \mathcal{E}_{in}}{\mathcal{E}_{in}} \tag{8}$$

which will have a value of 0, if there is no light scattering (perfect system, with conserved étendue); or 1 if the light is completely scattered (imperfect system, with non-conserved étendue).

2.3. Entropy related parameters

As described by Bejan [25], the minimization of the entropy generation rate implies the maximization of the useful power. In general, the entropy generation in a concentrating solar system, as in the thermal devices, will be due to the heat transfer process ($\dot{S}_{gen}^{th}$) and the fluid friction ($\dot{S}_{gen}^{fl}$) [25], but the étendue loss also contributes to an entropy generation $\dot{S}_{gen}^{\mathcal{E}_{sc}}$, so the total entropy generation has the basic form $\dot{S}_{gen}^{total} = \dot{S}_{gen}^{fl} + \dot{S}_{gen}^{th} + \dot{S}_{gen}^{\mathcal{E}_{sc}}$.

Considering an isothermal concentrator with no flow of mass on the concentrator (i.e., heating a plate), it is possible to eliminate the fluid friction entropy generation. The thermal entropy generation, as described by Bejan [25], includes the heat transfer coefficient U_r, the net solar transfer rate $\dot{Q}_s$ captured by the concentrator with an aperture A and receiver A' areas, the ambient temperature T_0, and the apparent sun temperature as an exergy source $T^* = \frac{3}{4}T_s$, as suggested by Petela [26], by:

$$\dot{S}_{gen}^{th} = \frac{U_rA'\left(T_r - T_0\right)}{T_0} - \frac{\dot{Q}_s}{T^*} + \frac{\dot{Q}_s - U_rA'\left(T_r - T_0\right)}{T_r} \tag{9}$$

The entropy generation for the etendue loss can be related trough the concepts of statistical thermodynamics, resembling that in general $S = k\log\Omega\left(E, V, N\right) + \text{const.}$ [27], and since the étendue is a measure of the transmitted power along a beam of light, a proper relation between this two variables can be developed. Winston et al. [28] have stated an entropy-étendue per photon relation as:

$$S^{\mathcal{E}} = k\log\mathcal{E} + \text{const.} \tag{10}$$

where the constant is related to a thermal quantity that can be set aside, since it applies only in the case of a wavelength shift [28, 29] or computed independently as done in 9, so the relation between entropy and étendue is firmly established. The entropy change for an irreversible process is equal to the entropy generation, so the entropy generation in terms of the étendue:

$$\dot{S}_{gen}^{\mathcal{E}_{sc}} = \dot{S}_{out}^{\mathcal{E}_{out}} - \dot{S}_{in}^{\mathcal{E}_{in}} = \dot{N}k\ln\left(1 + \mathcal{E}_{sc}^{*}\right) \tag{11}$$

where $\dot{N}$ is the number of photons per unit time that cross the aperture area of the concentrator. Note that under the numerical conditions of 8, the value of the entropy generation due to étendue scattering is always positive. The number of photons can be determined with $\dot{Q}_s$ and the average photon energy $E_{ph} \sim 10^{-19}$J as $\dot{N} = \dot{Q}_s/E_{ph}$.

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Therefore, the total entropy generation rate in the concentrator is

$$\dot{S}_{gen}^{total} = \frac{U_r\,(2a')\,(T_r - T_0)}{T_0} - \frac{\dot{Q}_s}{T^*} + \frac{\dot{Q}_s - U_r\,(2a')\,(T_r - T_0)}{T_r} + \frac{\dot{Q}_s}{E_{ph}} k \ln\left(1 + \mathcal{E}_{sc}^*\right) \tag{12}$$

In view of this two discussed entropy generation terms, a dimensionless parameter relating the entropy (irreversibility) distribution ratio can be defined as $\dot{S}_{gen}^{\mathcal{E}_{sc}}/\dot{S}_{gen}^{total}$, with extreme values of 1 when the étendue scattering irreversibility dominates, 0 is the opposite limit at which irreversibility is dominated by heat transfer effects, and 0.5 is the case in which the heat transfer and the étendue scattering entropy generation rates are equal. In addition, when the entropy distribution ratio is zero, the étendue conservation condition is fulfilled. Thus, it is justified to define this parameter from the étendue concept and not from the heat transfer perspective.

Caution must be taken, since this relation can be confused with the Bejan number (Be) defined by Paoletti et al. [30] for the generation of entropy through heat and flow. This new relationship involves an optical parameter (the étendue) as an element that also generates entropy. This dimensionless parameter was originally proposed for entropy analysis in CPCs by González-Mora [31] but can be further generalized to any concentration geometry, since étendue conservation is a major concern when designing optical systems [4, 9, 22]. Consequently, it is suggested to call the entropy distribution ratio as Mo. The Mo number can then be expressed with the help of 12 as:

$$\mathrm{Mo} = \frac{\gamma\,(\theta_{max} - 1)\ln\left(1 + \mathcal{E}_{sc}^*\right)}{\theta^*\left[\theta_r^2 - 2\theta_r + \theta_{max}\right] + \theta_r\,(\theta_{max} - 1)\left[\gamma\theta^*\ln\left(1 + \mathcal{E}_{sc}^*\right) - 1\right]} \tag{13}$$

where $\theta_r = T_r/T_0$, $\theta^* = T^*/T_0$, $\theta_{max} = T_{r,max}/T_0 = 1 + \dot{Q}_s/U_r A' T_0$ and $\gamma = T_0 k/E_{ph}$. Furthermore, from 12 the optimal receiver temperature can be defined as the one that minimizes the total entropy generation rate, i.e. $\mathrm{d}\dot{S}_{gen}^{total}/\mathrm{d}\theta_r = 0$, resulting:

$$\theta_{r,opt} = \theta_{max}^{1/2} \quad \Leftrightarrow \quad T_{r,opt} = \sqrt{T_0 T_{r,max}} \tag{14}$$

2.4. Constructal law

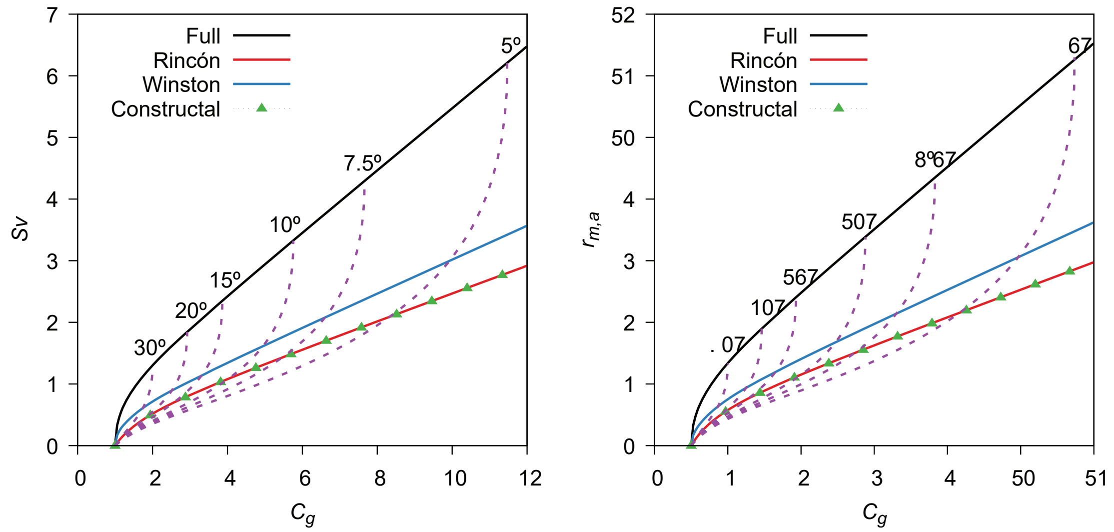
The aforementioned truncation criteria were established only through a purely geometrical approach. However, Rincón's criterion was recently demonstrated on a simplified parametric energy basis with a first approach of the constructal law [32]; while González-Mora [31] gave the first approach for entropy related parameters suggesting the evaluation of the so-called Mo number, as a response to several inquiries to rename the Rincón criterion. Therefore, a complete discussion of the geometric, optical and entropy parameters of CPCs is required for an objective comparison of the truncation criteria. To compare these criteria, constructal theory is used to define a constructal truncation criterion.

In 1997, Bejan [19] stated "For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed currents that flow through it", which is known today as constructal law, with a vast theory under it [20, 33]. The constructal law has been applied to various engineering systems [34], and recently in renewable energy systems analyzing a solar chimney and an oscillating water column [35], in addition to the first approaches in CPCs [31, 32], so the path for the constructal law in sustainable energy systems is clear and ongoing.

The fundamental concept of the constructal law is to establish the configuration that facilitates the flows in the system, by defining different parametric relations that establish the system degrees' of freedom and evaluating the behavior of the system with its proper restrictions. In the present analysis, the degrees of freedom are C_g and θ_r for the parametric relations that are described in the previous sections. Thus, accoring to the different truncation criteria, different configurations can be easily compared , where the main objective function is the Mo number described in 13.

3. Results

These results are shown graphically below, including the constructal law results applied with the PIKAIA genetic algorithm [36]. Although the objective function is the Mo number, all parameters were evaluated and discussed below. In all the plots, the same color palette is used: black for the full CPC, purple for the truncated CPCs, red for the truncated CPC under the Rincón criteria, blue for the truncated CPC under the Winston criteria, and green for the CPC results using the constructal law.



(a) Svelteness for different CPC designs.

(b) Reflector area-aperture area for different CPC designs.

Figure 3: Geometric results.

3.1. Geometric results

In Fig. 3, the svelteness Sv and the reflector area-aperture area $r_{m,a}$ are plotted as a function of the concentration ratio C_g. As can be seen, Winston's criterion (blue line) reduces the height of CPCs, but Rincón's criterion (red line) reduces the height even further compared to the full CPC (black line). Here, a remark must be done for this, and all the following plots, since Winston's criterion reduces the concentration ratio one must find the intersection of the blue curve with the green curves to read the desired parameter, while the Rincón criterion attains no reduction in concentration ratio since a new acceptance angle must be determined to keep the original C_g value, and parameters can be read directly.

When applying the constructal law to the geometric parameters (green line), the same result is obtained as the Rincón criterion. Therefore, from a purely geometric perspective, the Rincón criterion is a geometric result justified by the constructal law.

3.2. Optical-energy results

The average number of reflections and the étendue scattering are shown in Fig. 4 and can be read like the previous plots. As can be seen, the Winston criterion (blue line) considerably reduces the average number of reflections, but the Rincón criterion (red line) reduces even more the average number of reflections, being the same for C_g between 2.04 and 5.01; however, Rincón's criterion maintains an almost average value n close to the minimum average number of reflections. An interesting behavior occurs for the étendue loss, since the Rincón criterion results in beneficial results only for $C_g > 2.28$, otherwise the light will start to spread over the receiver. In view of these results, the optical efficiency will be maximum for the CPC that is truncated with the Rincón criterion due to a greater reduction of n and, in general a minimum loss of the étendue.

Similarly, the application of the constructal law (green line) to the optical energy parameters yields a curious behavior. For the average number of reflections, the results are in agreement with the Rincón criterion, but not so for the étendue scattering. The behavior of the truncation criterion according to the constructal law shows that the Winston criterion is beneficial up to $C_g = 2.28$, and subsequently, the Rincón criterion is the one that minimizes the dispersion of the étendue in the optical system for $C_g > 2.28$.

3.3. Entropy results

In this case, in addition to C_g, θ_r is another degree of freedom. Although a 3D surface could be generated, its interpretation would be complicated. The parameter θ_r is the dimensionless temperature of the receiver, so it is greater than 1, and necessarily less than θ_{max}, with the possibility of being $\theta_{r,opt}$. Under these constraints, in the present analysis θ_r is proposed as an average of these values.

The entropy parameters under the described conditions are shown in Fig. 5. As expected, for the full CPC (black line), Mo=0 regardless of C_g. In the case of the Winston criterion (blue line), Mo results lower up to

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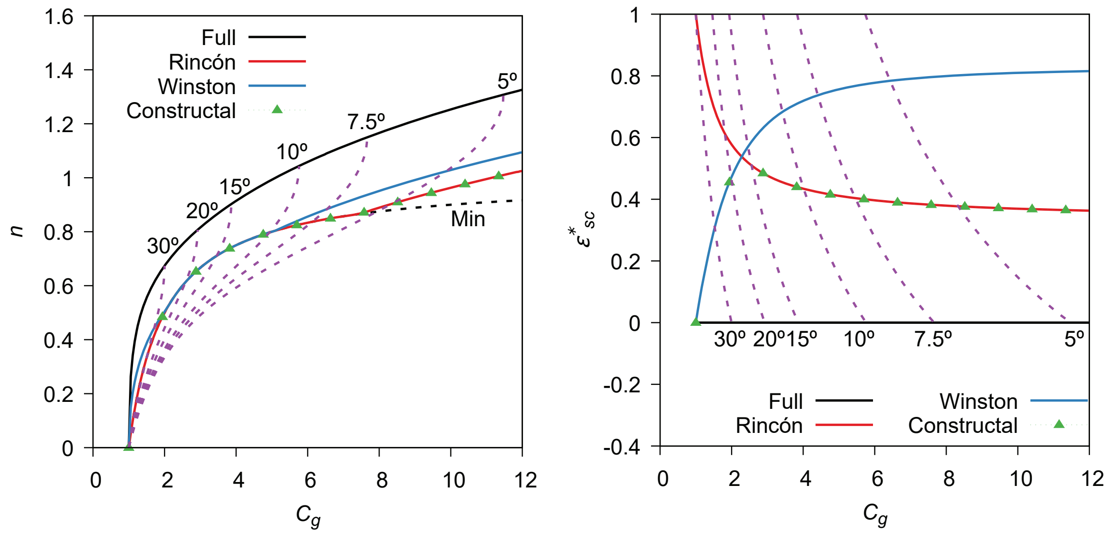


(a) Average number of reflections for different CPC designs.

(b) Étendue scattering for different CPC designs.

Figure 4: Optical-energy results.

$Cg < 2.28$, subsequently, the Rincón criterion (red line) presents lower values. Applying the constructal law (green line) gives a result very similar to the dispersion of the étendue, since the truncation of the CPC is optimized as a function of C_g. As can be seen in the graph of the optimum temperature, the Rincón criterion allows the optimum temperature of the complete CPC to be reached, just as if the constructal law were applied.

4. Conclusions

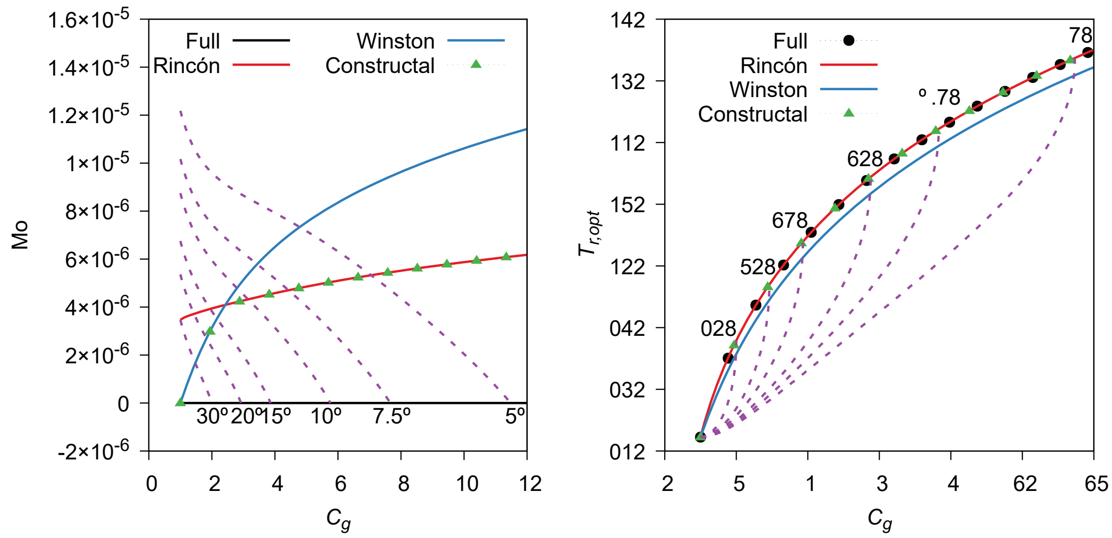
The CPC has several advantages for concentration systems; principally the possibility to get mid temperature ranges with a fixed concentrator, however, its height is the major drawback for its application. As a result, several truncation criteria had been proposed to increase its use, but, up to date, all had been justified by its height reduction, despite a first approximation with a simplified energy analysis.

Now, the analysis has been extended to include the rate of entropy generation, identifying that this production occurs by two factors: dispersion of the étendue and heat transfer process; leaving aside the entropy generation by fluid friction when considering only the heating of a flat plate for CPC with flat receivers. However, this analysis can be extended to any receiver geometry.

The present analysis has focused on comparing the behavior of the concentration ratio (C_g) for full CPCs with respect to truncated CPCs according to Winston's and Rincón's criteria and by applying Bejan's constructal law. This comparison is made by means of six parameters: two geometrical (svelteness and reflector area-aperture area ratio), two optical-energy (average number of reflections and étendue dispersion) and entropy (a new dimensionless group proposed Mo that relates the entropy generation distribution and the optimum receiver temperature).

When the study is carried out for different values C_g for the geometric parameters, the results of the constructal law and the Rincón criterion are optimal as Sv and $r_{m,a}$ decrease significantly to a value close to $1/3$ of the full CPC. With respect to the optical-energy parameters, the Rincon criterion and the construct law can be identified as the conditions for minimizing n; however, with respect to the dispersion of the étendue, the Rincon criterion is only beneficial for $C_g > 2.28$, for lower concentrations, the Winston criterion is a better choice, as shown by the constructal law. For the entropy results, again the Rincon criterion is only beneficial for $C_g > 2.28$, as shown by the constructal law; while for the optimal temperature, the Rincón criterion allows reaching the full CPC temperature.

In view of the results obtained, Rincón's criterion, although proposed from a purely geometric perspective, is practically a result of the construction law, provided that $C_g > 2.28$. This allows, without any doubt, to establish that, for the great majority of low- and medium-temperature applications, using the Rincon criterion allows obtaining the best performance in CPCs, as had been stated as an assumption (without a formal demonstration) in other previous works.



(a) Mo number for different CPC designs.

(b) Optimal temperature for different CPC designs.

Figure 5: Entropy results.

Acknowledgments

EGM would like to acknowledge PhD Diogo Canavarro (Universidade de Évora) for his insight and explanation of étendue. Although he may not agree with all the interpretation of this paper, any error is by the authors, and should not tarnish the reputation of this esteemed colleague.

Nomenclature

Letter symbols

A aperture area, m^2

a aperture width, m

A' receiver area, m^2

a' receiver width, m

Cg concentration ratio, -

F function, -

l CPC length, m

E energy, J

G irradiation, W/m^2

h heat transfer coefficient, W/m^{2}K; specific enthalpy, kJ/kg

k Boltzmann constant, J/K

L lenght, m

Mo Mora number, -

$\dot{N}$ number of photons per unit time, photons/m$^2 \cdot$ s

n average number of reflections, -

$\dot{Q}$ heat transfer rate, W

https://doi.org/10.52202/069564-0152

r reflector area-aperture area ratio, -

$\dot{S}$ entropy rate, W/K

Sv svelteness, -

T temperature, K

t parameter (angle), rad

U heat transfer coefficient, $W/m^2 \cdot K$

x parametric equation, m

y parametric equation, m

Greek symbols

γ dimensionless photon energy

$\mathcal{E}$ étendue, m^2

θ half acceptance angle, dimensionless temperature, -

ϕ incidence angle, -

Subscripts and superscripts

$*$ sun

0 ambient

ap aperture

bn beam normal

fl flow

gen generated

imb imbalance

in inlet

out outlet

ph photon

r receiver

s solar

sc scattering

t truncated

th thermal

total total

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