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Development of an optimization-based methodology for subsidy programs of residential buildings

Nico Fuchs[a], Jonas Baumgärtner[b], Dominik Hering[c] and Dirk Müller[d]

[a] RWTH Aachen University, E.ON Energy Research Center (ERC), Institute for Energy Efficient Buildings and Indoor Climate (EBC), Aachen, Germany, nico.fuchs@eonerc.rwth-aachen.de, CA
[b] RWTH Aachen University, ERC, EBC, Aachen, Germany, jonas.baumgaertner@eonerc.rwth-aachen.de
[d] RWTH Aachen University, ERC, EBC, Aachen, Germany, dominik.hering@eonerc.rwth-aachen.de
[e] RWTH Aachen University, ERC, EBC, Aachen, Germany, dmueller@eonerc.rwth-aachen.de

Abstract:

The success of the energy transition in the building sector depends not only on the technical feasibility but also on the economic viability of energy modernization measures. Subsidy programs for building owners and energy prices exert a strong influence on this economic viability and thus on the spread of low-emission building energy systems. Against this background, this study presents a bi-level optimization approach to determine cost-efficient subsidy strategies. At the upper stage, the government aims to reduce subsidies to reach CO_2 targets. At the lower stage, individual building models optimize their total costs by modernizing the heat conversion technology or insulating the building shell. To achieve solvability, the lower model is implemented into the upper model as a discrete set, resulting in a single-stage problem. The model determines the minimal subsidy rates that make the purchase of the technologies and measures worthwhile for the building owner, depending on a CO_2 target aimed by the government. Results show that the introduction of subsidy programs has a considerable steering effect on emission savings. The targeted promotion of low-emission heat supply technologies, such as HPs, with up to 40 % subsidy quota in combination with subsidies for insulation measures contributes significantly to their installation in existing buildings and thus to the achievement of climate protection goals, if the future expansion of renewable energies in the electricity mix is taken into account. With the current costs and emission factors of pellet, the promotion of pellet heating can further contribute to strong emission savings.

Keywords:

Bi-level optimization, energy-efficient buildings, building optimization, subsidy optimization, Subsidy programs for energetic modernization, Residential buildings

1. Introduction

The European Union is committed to decreasing greenhouse gas emissions by a minimum of 55 % by 2030 and achieving climate neutrality by 2050 [1]. These ambitious targets necessitate significant efforts across all energy consumption sectors. As the building sector is accountable for 36 % of European greenhouse emissions [2], it plays a crucial role in achieving emission reductions. The European Commission has reported that 75 % of the European building stock is energy inefficient [3], highlighting the necessity for energy retrofitting. To lower building-related emissions, the European Union intends to double the current retrofit rate, which is currently less than 1 % per year. In practice, the decision of building owners to renovate their buildings is primarily driven by economic considerations. The installation of building envelope insulation or the utilization of low-emission heating technologies entail significant capital expenditures and may not be economically viable in every case. To overcome these financial barriers and promote retrofits, various policy instruments have been developed. According to Vedung [4], there are three general types of policy instruments: regulations, economic means, and information dissemination. Regulations establish energy efficiency standards and partially limit the choice of heating technology in the building sector. Subsidy programs, on the other hand, affect the economic viability of low-emission energy systems. Information flow is critical to educating building owners about sensible renovation measures. Among these policy instruments, Ruparathna et al. [5] identify financial incentives as essential means to encourage the energy retrofitting of bRegulationsuildings. Among these, direct capital grants for retrofit works are the most common financial incentives in Europe [5].

1.1. Subsidy programs in optimization models

In recent years, numerous research papers have incorporated subsidy programs into models for designing and optimizing energy systems for individual residential buildings to quantify their effects. Asadi et al. [6] formulate a multi-objective optimization model that optimizes renovation decisions for discrete insulation measures and

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the installation of solar thermal collectors. The Mixed-Integer Linear Program (MILP) minimizes investment costs, taking into account investment grants, while maximizing energy savings. Ashouri et al. [7] present a MILP for the design optimization of a building energy system. The authors model a large portfolio of heat conversion technologies, plants for renewable energy production and different thermal and electrical storage types. The target function minimizes total costs, including investment grants and feed-in tariffs, and a penalty term punishing thermal comfort violations. The works of Harb et al. [8], Renaldi et al. [9], and Schütz et al. [10] show further examples of the integration of incentives in MILPs for building energy system design optimization. All three models minimize the total costs of the building owner but differ in the portfolio of modernization strategies considered and incentive schemes. Harb et al. [8] focus on CHP systems and feed-in tariffs for CHP electricity. Renaldi et al. [9] consider hybrid systems with HP and implement remunerations for heat generated by HP. Schütz et al. [10] integrate all building-relevant German subsidies in their MILP, including investment grants for single measures, subsidies for reaching efficiency standards, as well as feed-in tariffs for PV and CHP electricity. All models demonstrate that subsidies effect the economic viability of modernization measures and thus the investment decision. While most of the models simplify the conditions of the subsidy programs and omit the constraints under which support is granted, Schütz et al. [10] integrate a significant portion of the funding measures available in Germany along with their restrictions into the model.

1.2. Optimization models for the improvement of subsidy programs

While building owners aim to minimize total costs through modernization, funding agencies aim to enhance the energy performance of buildings while minimizing the costs of subsidies. The goal of the funding agency is to allocate subsidies effectively to individual buildings, providing incentives to undertake energy-efficient retrofitting and achieve emission savings. Different research papers attempt to formulate optimization models to model the relationship between the funding agency and energy systems with the goal to improve subsidy programs from the perspective of a funding agency. Liu et al. [11] present an optimization model for multi-energy systems on an urban scale. The model is formulated as a Stackelberg competition, commonly used in game theory. In this case, the state as funding agency aims to minimize total subsidies while meeting an emission reduction target or achieving a share of renewable energy. The upper level perspective is modeled as a mixed-integer linear program (MILP) with binary variables determining the minimum size of plants required for subsidies. Subsidies include investment grants and remuneration for renewable energy production. At the lower level, a city determines how to meet its energy demand at the lowest possible total annual cost (TAC). The lower problem is modeled as a linear program (LP) with plant size and operation as continuous decision variables. Both perspectives result in a bi-level optimization problem (BiOP). In order to solve the BiOP, Liu et al. reformulate the original problem. Since the lower problem is modeled with continuous variables and linear inequalities, it is continuously differentiable, and the Karush-Kuhn-Tucker (KKT) conditions can be applied to include the lower problem in the upper problem. By reformulating the resulting non-linear terms through discretization of variables and the use of the Big-M method, Liu et al. are able to reformulate the BiOP into a single-stage MILP. Martelli et al. [12] introduce a model to determine optimal incentives and CO_2 prices to achieve emission reduction targets for a multi-energy systems on a district scale. The upper problem of the BiOP minimizes incentives for produced renewable heat and electricity while maximizing the income from CO_2 prices. The overall budget and emissions are constrained, and the costs for the lower problem must not exceed a maximum value. The lower level problem minimizes the TAC to cover the energy demand of the district. Technology choices on the lower level are implemented with binary variables resulting in a MILP. As the problem is not continuously differentiable, a problem reformulation by means of the KKT conditions is not possible, and the problem is solved with a gradient-free direct search algorithm (Particle-Generating Set-Complex Algorithm) which was developed by the authors and is not open source available. Prada et al. [13] determine the influences of investment grants in Italy by formulating a BiOP. Both levels are solved separately for different investment grants, and the solution of both problems is compared. At the upper level, the funding agency aims to minimize the primary energy demand of the building stock and its own investment, resulting in a multi-objective target function. The lower level model minimizes the primary energy demand and the TAC. By comparing variations, the originally formulated problem is not necessarily solved to optimality. Another example of solving a BiOP to determine optimized subsidy programs in energy science is the work of Zhou et al. [14] . The model minimizes subsidies for renewable electricity production on the upper level while achieving a minimum share of renewables in a electricity production system. The lower level minimizes the total cost of the portfolio and is modeled as a graphical flow problem with nodes for producers as well as consumers and with discrete technology choices resulting in a MILP. The authors solve the BiOP by a combination of a modified CPA on the lower level and a heuristic approach on the upper level.

1.3. Rationale of the work

The aforementioned approaches demonstrate the potential of subsidy programs to influence investment decisions and present various models for determining optimized subsidies. The studies utilize BiOPs to optimize investment grants and remunerations for renewable energy production. To ensure solvability, the BiOPs are ei-

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ther reformulated into a single-level problem if possible or solved using heuristics at the upper level. The BiOPs presented in the studies focus on connected urban and multi-energy systems. Prada et al.'s BiOP [13] takes the single building perspective into account but is not solved to optimality. Therefore, the following aspects have been identified as research gaps based on the findings of the literature review:

- How can subsidies be optimally allocated from the perspective of a funding agency to reduce building stock emissions, while considering individual investment decisions at the level of single residential buildings?

The formulation of a BiOP based on the literature presented shows promise. However, compared to previous works on multi-energy systems, the consideration of building stocks requires the inclusion of multiple entities in the lower-level problem. It is not sufficient to assume, as in Liu et al. [11], that every technology is chosen at the lower level resulting in a linear programming problem. As a result, the main contribution of this work is to address this challenge by:

1. Formulating a BiOP to determine the minimum subsidies required for individual modernization measures to achieve an emission reduction target.
2. Integrating the perspectives of multiple building owners into the BiOP for optimized subsidy allocation.
3. Reformulating the BiOP as a single-level optimization problem and presenting an approach to solve it using deterministic optimization methods.

The paper is organized as follows: The second section describes the methodology and model formulation and illustrates the use of the German single-family-house building stock as a case study as well as the input data used. The third section analyzes the results, and the fourth section constitute the conclusion.

2. Methodology

To determine optimized subsidy schemes, a BiOP is utilized in this study. At the lower level, individual building entities aim to minimize their costs, while on the upper level, a funding agency aims to encourage energy-efficient modernizations by disbursing subsidies to achieve an emission reduction target. The method employed in this paper is designed for this two-stage approach. Subsection 2.1. outlines the model used on the lower level, while Subsection 2.2. formulates the BiOP. In Subsection 2.3., the application of the method is presented through a case study of the German residential single-family house building stock, including the necessary input data. Figure 1 gives an overview about the methodology in this paper.

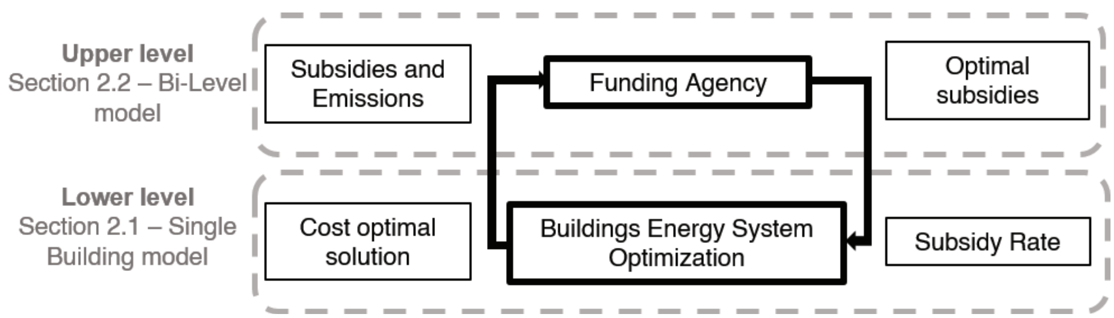

Figure 1: Overview over the Bi-Level approach with the funding agency on upper level and individual buildings on lower level

2.1. Single Building model

At the lower level, we use a mixed-integer linear programming (MILP) design optimization model developed by Schütz et al [10]. The model's detailed features can be found in Schütz et al. [10]. We present only the main features and adaptations of the model for this work. The model selects the optimal energy system in terms of total annualized costs considering available system technologies and insulation measures. The paper considers gas boilers (GAS), electrical air-source heat pumps (HP) with electrical heaters (EH) as backup heaters, and pellet boilers (PEL) as heat generators. Photovoltaic collectors (PV) and solar thermal collectors (STC) are available as solar energy harvesting options. Additionally, batteries (BAT) and thermal energy storages (TES) are included as possible energy storage options. In addition, we consider the insulation of the building's outer wall (WALL), roof (ROOF), ground floor (FLORR) and window replacement (WIN) as possible measures.

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The optimization program offers two retrofit scenarios, retrofit and advanced retrofit, to reduce the building's energy demand. As the model's input were updated compared to the original version of Schütz et al., section 2.3.2. describes the updated input data. To reduce computing time and to consider extreme days, we implement a k-MILP clustering [15] algorithm to represent the annual profiles of weather data, domestic hot water (DHW), and electricity demand through eight weighted type days complemented by two extreme days. The target function minimizes the total annualized costs (TAC) for the building owner, which are calculated based on the VDI 2067 [16] standard. The cost function includes annualized investments (c^{inv}), demand costs (c^{dem}), and revenues from the sale of surplus electricity ($r^{el,sell}$) or subsidies (r^{sub}).

$$\min(c^{inv} + c^{dem} - r^{el,sell} - r^{sub}) \tag{1}$$

If the observation period exceeds the service life of a system, the residual value is calculated based on a replacement investment. The investment is comprised of acquisition costs for the system technology and renovation costs for building envelope measures. Demand costs and profits from surplus electricity fed into the grid are annualised using a dynamic factor and annuity factor. Binary variables x are utilised to map system selection, and to bound the nominal heat output Q^{nom}_{dev} of a heat generator *dev* with its minimum and maximum size.

$$\dot{Q}^{nom}_{dev} \leq \dot{Q}^{nom}_{max,dev} \cdot x_{dev} \quad \text{and} \quad \dot{Q}^{nom}_{dev} \geq \dot{Q}^{nom}_{min,dev} \cdot x_{dev} \tag{2}$$

For the PV and STC systems, the integer variable z specifies how many modules are installed on the roof. The area of all installed solar modules must not exceed the maximum roof area A^{roof}_{max}.

$$\sum_{sol \in \{STC,PV\}} z_{sol} \cdot A_{sol} \leq A^{roof}_{max} \tag{3}$$

For the building envelope components, the decision variable $x^{retrofit}$ represents whether the component is retrofitted and to which retrofit level. The retrofit levels considered are standard (strd), retrofit (retrofit) and advanced retrofit (adv). The *strd* level is associated with no costs and each component of the building envelope has to select one level.

$$\sum_{i \in \{strd,retrofit,adv\}} x^{retrofit}_{shell,i} = 1, \forall \text{ shell in Building Shell} \tag{4}$$

The heat demand is calculated according to DIN EN 12831 [17] and includes the heat required for room heating and DHW production. The DHW demand is considered by time series as a fixed input parameter, while the space heating demand is computed based on the building envelope renovation and the difference between indoor and outdoor temperatures. The resulting heat demand must be provided at each time step. Further details on the modeling of heat generators and technical devices can be found in Schütz et al [10] . This study focuses on the impact of investment subsidies for individual retrofit measures. Under the German subsidy scheme for individual measures, a percentage of the investment costs is subsidized for the installation of HP, PEL, PV, STC, BAT, or for retrofitting components of the building shell to a maximum heat transfer coefficient U^{max} (WALL: 0.2 $\frac{W}{mK}$, WIN: 0.95 $\frac{W}{mK}$, ROOF: 0.14 $\frac{W}{mK}$, FLOOR: 0.25 $\frac{W}{mK}$).

2.2. Bi-Level model

To determine optimized subsidy schemes for modernization measures, a model is described which includes the objectives and goals of the funding agency as well as the funding recipients. Between the funding agency and the funding recipient exists a dynamic behaviour and their decision making process is intertwined. Within the studies of game theory in economics, this market situation is known as the Stackelberg Competition. The game consists of a leader and a group of follower. The leader makes an action which can be observed by all followers. After that, the followers react with an action best suited for their needs. With regard to the optimization of federal subsidy funding, the state takes the position as leader who decides about subsidy shares for different renovation measurements and building owners are the followers who use the provided subsidy share to minimize their total annualized costs.

As an assumption, the funding agency aims to minimize its use of paid subsidy funds and thus its costs. At the same time, a CO_2 goal is defined for the considered building stock and the funding agency has to provide at least as much subsidy funds as needed to achieve the CO_2 goal.
The decision variables of the funding agency are the funding shares provided for a device or a building shell modernization measure and are denoted with ϕ^{inv}. The individual buildings decide for their best investment strategy with the variables c^{inv}(y) which derives from the solution of the lower problem (LowP). In addition, the variable $em(y)$ obtained from the LowP is the amount of CO_2 emitted, which must not exceed the parameter em^{lim}.

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This results in the following form of the upper problem (UppP):

$$
\begin{aligned}
UppP: \quad \min \quad & \phi^{inv} \cdot c^{inv}(y) \\
\text{s.t.} \quad & \\
& em(y) \leq em^{lim} \\
& y \in \text{argmin}(LowP(\phi))
\end{aligned}
\tag{5}
$$

In the above formulation, the decision variables of the UppP and LowP are multiplied and thus constitute a non-linearity. To avoid this non-linear terms, the linearization following the approach of Liu et al. [11] is applied. For this purpose, the variable ϕ^{inv} is mapped as a discrete interval KX with a step length KM and the binary variable δ_k and the continuous auxiliary variable CX_k are introduced. The equations describing the linearization are displayed in equations 6a - 6e.

$$\sum_{k \in KX} \delta_k \leq 1 \tag{6a}$$

$$CX_k \leq \delta_k \cdot M \quad , \forall k \in KX \tag{6b}$$

$$c^{inv} - CX_k \leq (1 - \delta_k) \cdot M \quad , \forall k \in KX \tag{6c}$$

$$CX_k \leq c^{inv} \quad , \forall k \in KX \tag{6d}$$

$$\phi^{inv}_{min} + (\phi^{inv}_{max} - \phi^{inv}_{min})/KM \sum_{k \in KX} k \cdot \delta_k = \phi^{inv} \tag{6e}$$

Using equation 6e, the auxiliary variable δ_k can be translated back to the original variable ϕ^{inv}. The UppP of the BiOP can be represented using only linear terms and can be stated as follows:

$$
\begin{aligned}
UppP: \quad \min \quad & \phi^{inv}_{min} \cdot c^{inv} + (\phi^{inv}_{max} - \phi^{inv}_{min})/KM \sum_{k \in KX} CX_k \\
\text{s.t.} \quad & \\
& em(y) \leq em^{lim} \\
& \text{Equations (6a)} - \text{(6e)} \\
& y \in \text{argmin}(LowP(\phi))
\end{aligned}
\tag{7}
$$

As there are still binary variables present in this model, a reformulation of the BiOP to a single-stage optimization program using the KKT condition is not possible. Instead, the solution space of the LowP is discretized and included in the UppP to formulate the optimization problem as a single-level model.

The result of a single-building optimization can be roughly divided into energetic renovation measures of the building envelope, which improve the heating load of the building, and technology for heat generation, which covers the heating load. Three levels are considered as renovation measures for the building components WALL, WIN, ROOF and FLOOR. The heating load can be covered either by GAS, HP with back up EH or PEL. Furthermore, the roof can be used for PV or STC. As a simplification, the roof area is divided into the discrete steps no usage, half usage, or full usage. Within the cases of full usage PV, a further distinction is made between the possibility to buy BAT or not. For the discretization of the solution space, all combinations between the components are calculated and stored. The single building model is then reformulated making use of the discrete combinations. A binary variable λ_i for each possible combination i indicates whether the combination i is chosen as the optimal solution. Using the TAC_i and the investment c_i^{inv} of the combination i, the optimization program of the single building model can thus be represented as follows.

$$
\begin{aligned}
LowP: \quad \min \quad & \sum_{i \in \mathcal{K}} \lambda_i (TAC_i - c_i^{inv} \cdot \phi^{inv}) \\
\text{s.t.} \quad & \\
& \sum_{i \in \mathcal{K}} \lambda_i = 1 \\
& \phi^{inv} \in \text{argmin}(UppP(\lambda_i))
\end{aligned}
\tag{8}
$$

We use the following approach to resolve the bi-level structure of the BiOP: If only the constraints of the LowP are included in the UppP, the optimizer selects a package of measures, thereby minimizing the funding agency's objective function. However, the selected package of measures is not necessarily the preferred solution for the single building. To reflect this, equations are added using the Big M method to ensure that the selected measures are also optimal for the single building with the current solution of the UppP. Assuming that

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a combination i_1 from the set of combinations $\mathcal{K}$ can be considered as the optimal set of measures. Then the total cost minus the subsidies of combination i_1 must be less than or equal to the total cost minus the subsidies of any other possible combination i_2 in $\mathcal{K}$. Generalizing this constraint for any i_1, i_2 in $\mathcal{K}$ yields:

$$TAC_{i_1} - c_{i_1}^{inv} \cdot \phi^{inv} - (1 - \lambda_{i_1})M \quad \leq \quad TAC_{i_2} - c_{i_2}^{inv} \cdot \phi^{inv} \quad , \forall i_1, i_2 \in \mathcal{K} \tag{9}$$

From equation 9 follows that λ_{i_1} can only take the value 1 if no other combination $i_2 \in \mathcal{K}$ exists for which there is a lower cost under the current solution ϕ^{inv}. Although equation 9 ensures that the optimal package of measures is selected for the individual building, this formulation of constraints is not efficient. Since the set $\mathcal{K}$ is iterated over twice, the number of constraints is quadratic to the possible combinations in $\mathcal{K}$ leading to high computation time to generate the constraints of the optimization model. To counteract this, we use a Cutting Plane Algorithm (CPA) and systematically add only the constraints that restrict the solution space. The optimization problem is first initialized without equation 9 and a branch-and-bound algorithm (B&B) is started. If an admissible solution is found, the combination i for which λ_i takes the value 1 is determined. If another combination $j \in \mathcal{K}$ exists for which the total cost is lower than with i, the specific constraint is added, by which the current solution i is no longer admissible. The flow of the CPA is outlined in algorithm 1.

Algorithm 1 CPA for iterative restricting the solution space

```
Initialize B&B
Solve LP-relaxation
if Solution i ∈ ℤ then
    if TAC_i − c_i^inv · φ^inv ≤ TAC_j − c_j^inv · φ^inv, ∀j ∈ K then
        Solution accepted
    else
        for j ∈ K : TAC_j − c_j^inv · φ^inv ≤ TAC_i − c_i^inv · φ^inv do
            Add constraint:
            TAC_i − c_i^inv · φ^inv − (1 − λ_i)M ≤ TAC_j − c_j^inv · φ^inv
        end for
    end if
end if
```

Since the funding agency wants to achieve a CO_2 target for a stock consisting of different buildings, the set $\mathcal{B}$ is defined which includes the different building types. Using the discretization of the single building optimization presented in equations 10, the CPA presented in algorithm 1 and considering a building stock defined by the set $\mathcal{B}$, the BiOP can be reformulated as a single stage MILP by the following equations 10.

$$\begin{aligned}
\min \quad & \sum_{b\in\mathcal{B}} \Big(\sum_{i\in\mathcal{K}_b} (\phi_{min}^{inv} \cdot \lambda_{i,b} \cdot c_{i,b}^{inv}) + (\phi_{max}^{inv} - \phi_{min}^{inv})/KM \sum_{k\in KX} CX_{k,b} \Big) \\
\text{s.t.} \quad & \\
& \sum_{i\in\mathcal{K}_b} \lambda_{i,b} = 1 && , \forall b \in \mathcal{B} \\
& \sum_{b\in\mathcal{B}} \Big(\sum_{i\in\mathcal{K}_b} \lambda_{i,b} \cdot em_{i,b} \Big) \leq em^{lim} \\
& \sum_{k\in KX} \delta_{k,b} \leq 1 && , \forall b \in \mathcal{B} \\
& CX_{k,b} - \delta_{k,b} \cdot M \leq 0 && , \forall k \in KX, b \in \mathcal{B} \\
& \sum_{i\in\mathcal{K}_b} \lambda_{i,b} \cdot c_{i,b}^{inv} - CX_{k,b} - (1 - \delta_{k,b}) \cdot M \leq 0 && , \forall k \in KX, b \in \mathcal{B} \\
& CX_{k,b} - \sum_{i\in\mathcal{K}_b} \lambda_{i,b} \cdot c_{i,b}^{inv} \leq 0 && , \forall k \in KX, b \in \mathcal{B} \\
& \lambda_{i,b}, \delta_{k,b} \in \{0,1\} && , \forall i \in \mathcal{K}_b, k \in KX, b \in \mathcal{B}
\end{aligned} \tag{10}$$

2.3. Application at case study

2.3.1. Building stock data

The focus of this study is on the residential single-family house stock in Germany. The European TABULA [18] project provides 12 archetypes that classify the German residential building stock based on the building's energetic quality, with each archetype representing a specific age class of buildings. Since buildings constructed after 2001 are assumed to already have good energy performance, they are not included in this study. To achieve comparability, all buildings are assumed to have a heated net floor area of 150 m^2 and three occupants per household. The building geometry is parameterized based on the TABULA database, using relation factors between net floor area and buildings shell components similar to Lauster et al [19]. Potsdam is chosen as the location to represent a typical moderate German climate.

2.3.2. Inputs on building level

Different technical and economical inputs as well as time series data are necessary inputs on single building level: Typical modernization measures and heat transfer coefficients for the different archetypes originate from the TABULA typology [18]. Technical device efficiencies are obtained from manufacturer data, while the HP's behavior is modeled according to DIN V 18599 [20]. Emission factors of the energy sources originate from [21] and [22]. General economic parameters are based on the past 10-year development and are presented in Table A.2. Energy prices are scenario dependend and explained in section 1. Investment costs for building shell retrofitting are obtained from IWU [23] and include fixed and area-related costs. Installation costs for heating technologies are sourced from BDEW [24], while costs for PV and STC are from the online construction database Sirados [25]. Device investment costs are derived from market research and include fixed and power-related (for heating technologies) or area-related (for PV and STC) costs. Operation and maintenance costs are presented as percentages of component costs. Hourly time series data for outdoor temperature and solar radiation are obtained from the German Meteorological Service (DWD) [26] to account for the interaction with the outdoor climate. Load profiles for domestic hot water and electricity demands are generated using the RichardsonPy [27] tool and serve as hourly input data. These inputs are summarized in Appendix A.

2.3.3. Scenarios

Two scenarios are considered to estimate the impact of future changes in energy prices and emission factors. The transition to renewable energy sources is expected to change the electricity mix, leading to fluctuations in electricity prices and emission factors. Additionally, the pellet market is affected by demand changes and the use of wood in other sectors. Therefore, scenario A represents the current market situation in Germany, while scenario B considers prognosed energy prices and emission factors for the year 2030 [28]. For scenario A, the emission factors for 2021 are based on current data, while for scenario B, the emission factor for electricity is expected to decrease according to the German climate targets for the energy sector. A linear regression model is used to extrapolate an emission factor for electricity from the grid in 2030. The emission factor for pellets in scenario A is based on sustainable forestry practices in Germany [21]. However, Röder et al. [29] found that the production conditions highly influence the emission factor of pellets. To account for possible increased demand effects and non-sustainable forestry practices, scenario B assumes an emission factor of 100 $\frac{g_{CO_2}}{kWh}$ for pellets. Table 1 summarizes the assumptions for both scenarios.
Pehnt et al. [30] propose a system of emission classes to rate buildings based on their maximum CO_2 emissions relative to their living space. These classes range from A+++ for buildings emitting less than -10 $\frac{kg_{CO_2}}{m^2}$ to H for those emitting more than 60 $\frac{kg_{CO_2}}{m^2}$. To incentivize PV electricity feed-in, the system offers an emission credit that can result in negative emissions. In this study, we adopt Pehnt et al.'s emission classes and investigate the minimum required subsidies to attain classes A to C in average over all buildings. Therefore, the area-related climate targets are multiplied by the living space and the numbers of buildings in the considered building stock. Table 1 shows the resulting emissions targets A,B and C, which are the same for both scenarios.

Table 1: Energy prices Emissions factors in different scenarios

		Scenario A	**Scenario B**
Energy prices in $\frac{EUR}{kWh}$	Electricity	0.400	0.327
	El. HP	0.350	0.246
	Pellet	0.104	0.110
	Gas	0.120	0.125
Emission factors in $\frac{g_{CO_2}}{kWh}$	Electricity	420	164
	Pellet	36	100
	Gas	202	202
Emission targets in Mil. t_{CO_2}	A	6.9	6.9
	B	16.6	16.6
	C	27.6	27.6

3. Results

The optimized subsidy strategy yields different outcomes based on the desired climate target of the funding agency. Figure 2 (a) summarizes the optimized subsidy shares for individual measures for the emission targets A, B, and C for both scenarios A and B. In addition to the subsidy rates, an analysis of the investments indicates

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the effects of the subsidy schemes. Therefore, Fig. 2 (b) shows the annualized investment costs per emission target and scenario divided into a bar for plant technology and a bar for shell measures.

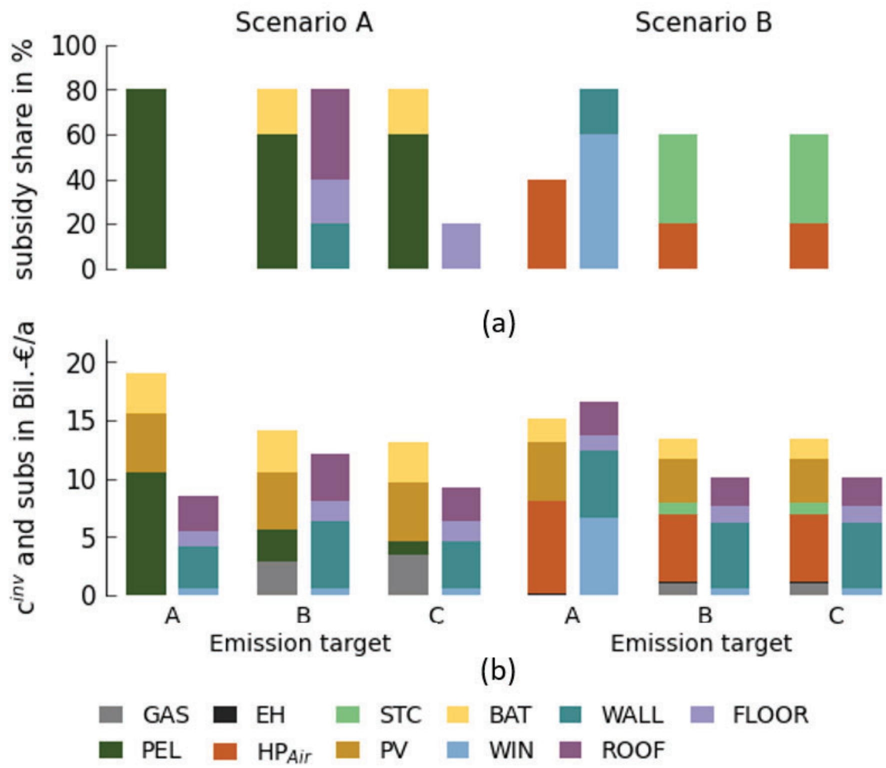

Figure 2: Investment costs for plant technology (left bar) and insulation measures (right bar) for different emission targets and scenarios

In the case of emission target A and parameters of scenario A, the unique subsidized technology is PEL with an investment grant of 80 %. Under this funding rate, all building entities install PELs as their heat conversion technology, making the investment in PELs the highest cost position for emission target A in scenario A. Pellets are nearly climate neutral and given the emission factors of scenario A, lead to a significant CO_2 reduction. Compared to the heat generation by HP, pellet heat supply is cheaper, given the price assumptions of scenario A. Moreover, due to the higher emissions factor of the current German electricity mix, the heat supply by pellet leads to lower emissions per kWh than an alternative supply by HP. Besides the investment in PEL, Fig. 2 shows high investments in PV and BAT systems. Installing PV and BAT is advantageous in all buildings due to the high electricity prices, resulting in cost savings in electricity supply by the means of PV and BAT. Therefore, no subsidies for PVs and BAT are required. The investment in PELs leads to a decrease in emissions even without any subsidies for shell modernization measures due to the low pellet emission factor. Even without incentives, measures on the roof and ground floor are chosen in old buildings as they come with comparatively low investment costs. In old buildings with poor insulation standards, the insulation of the outer wall is favorable under the high energy prices of scenario A even without subsidy. Newer buildings refrain from modernizing the building shell. However, the results of high penetration of PELs should be considered with regard to demand effects and the associated increases in the pellet price and emission factor, as shown in scenario B (see section 1). In scenario B, the optimal solution is to promote HPs with a 40 % investment grant, while also providing subsidies for window replacement (60 % share) and wall insulation measures (20 % share). The differences to scenario A can be explained by the lower electricity prices and emission factors and the increased pellet emission factor making the HP the best option in terms of CO_2 avoidance costs. Figure 2 shows high investments in HPs and PVs in this scenario. Electrification of the energy system of each building achieves the emission target for emission target A. Investments in PV's and BAT reduce costs from the building owner's point of view. To reach the ambitious emission target, investments in the insulation of the building shell are necessary as Fig. 2 demonstrates. As wall insulation is highly effective in heating demand reduction, it should be promoted to retrofit old building ages with poor insulation standards. As window replacement is expensive in relation to energy and cost savings, its implementation must be promoted with subsidies of 60 %. The results show that the subsidies for the building shell lead to modernization in older buildings with poor insulation standards, while buildings with higher insulation standards refrain from renovation measures.

Optimization results for emission target B show similarities with regard to the subsidised technologies but differ in subsidy shares and promoted renovation measures. In scenario A, the investment in PELs should be promoted with a 60 % investment grant. Compared to emission target A, Fig. 2 demonstrates that the total investment in PELs for emission target B is only a quarter of the investment in PELs for emission target A. The PEL subsidy share of 60 % leads to PELs only being used in modern buildings, where the outer wall is not retrofitted after the optimization. These modern buildings initially have a better insulation standard than older buildings. In buildings with retrofitted outer wall, the investment in a GAS is preferred to the choice of a PEL due to the low investment costs in combination with only slightly higher demand related costs. PVs and BAT are again, like in all scenarios, chosen even without subsidies and lead to high emissions savings. Instead of stimulating the investment in low-emission heat conversion technologies on a large scale, the energetic modernization of the outer wall, the ground floor and the roof is promoted for emission target B in scenario A. The increased subsidies lead to higher investments in insulation measures, especially for the insulation of outer walls and the roof. Scenario B shows a different subsidy strategy for emission target B compared to emission target A: HPs are promoted with a 20 % investment grant and STC with a 40 % investment grant. This subsidy scheme leads to HPs being installed in all buildings except for old buildings with a low insulation standard, where they operate with lower efficiency than in better insulated buildings. The reduction of the investment in HPs amounts to 25 %. In older buildings with low insulation standards, GAS are installed. STC supports the heat supply in buildings with GAS, where they can achieve relevant emission savings.

For emission target C in scenario A, the results show an optimized subsidy share of 60 % for PEL and 20 % for FLOOR. In this scenario, the PEL only supplies older buildings with poor insulation standard, that do not modernize the outer wall. The investments in PELs, therefore, decrease by more than half compared to the results for emission target B. The lower share of PELs is replaced by GAS, resulting in lower investment costs but higher emissions. The investments in PVs and BAT are the same as for emission target B. A subsidy share of 20 % for floor measures ensures that all buildings undertake a floor modernization. The subsidy strategy and investments for emission target C in scenario B are exactly the same as for emission target B. No feasible solution seems to lie between emission target C and B.

Examining the amount of subsidies disbursed and the consequent emission avoidance cost can provide insights into the efficient utilization of subsidy funds. Figure 3 illustrates the total annualized subsidies and the CO_2 avoidance costs for emission targets A, B, and C under both scenarios A and B. It is apparent that the

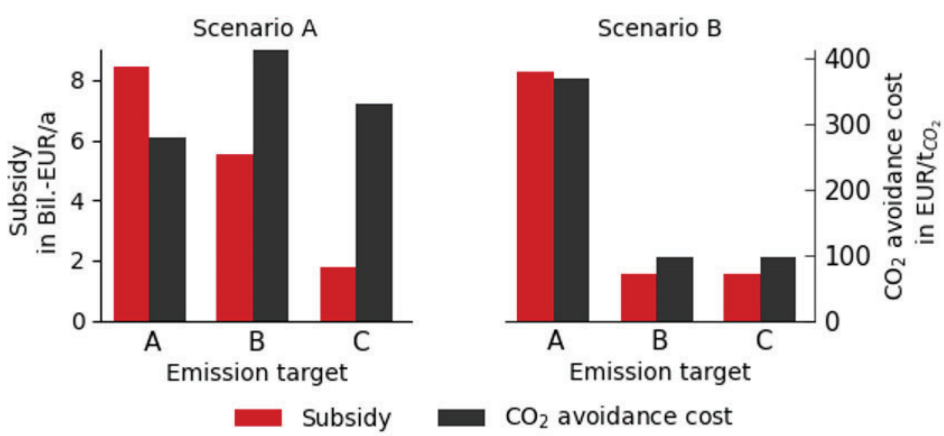

Figure 3: Total amount of annualized subsidies and emissions for different emission targets and scenarios

required subsidies increase significantly with more ambitious climate objectives. In scenario A, complete decarbonization of the building stock is achieved for emission target A, even if the climate target permits higher emissions. The low emission factor of pellets in combination with an emission credit for electricity fed into the grid, results in a balance of approximately zero emissions for the building stock. Compared to emission targets B and C, the CO_2 avoidance cost for target A are comparatively low due to the high emission savings. For emission target B, the building stock's emissions rise to 16.3 million tons of CO_2 per year, while subsidies

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decrease by 30 %. When aiming for emission target C, the emissions increase by one-third compared to class B, while the costs decrease by two-thirds, leading to lower CO_2 avoidance costs. Lower climate targets are found to be more cost-effective in terms of emission savings. In scenario B, the emission targets B and C are achieved with significantly lower CO_2 avoidance costs than in scenario A. The use of heat pumps in combination with lower emission factors of electricity leads to cost-efficient emission reductions. In the region between the emission targets C and B, only one optimal solution prevails over all others and leads to the same subsidy strategy. The inference that ambitious objectives necessitate significant subsidies can also be drawn from the results of scenario B.

4. Conclusion

This study introduces a BiOP for the optimal allocation of subsidies to achieve emission targets in a building stock. The method is applied to the residential single-family house building stock of Germany. Results show that an optimized strategy depends on aimed emission targets, energy prices, and emission factors of energy sources. If sustainable forestry can supply pellets with low emission factors, the best strategy is to promote PELs with high subsidy shares of up to 80 %. In combination with PV systems PELs can contribute to the decarbonization of the building stock. As the investment in PELs and the associated investment grant is comparatively expensive, the optimized subsidy strategy for less ambitious climate targets provides lower subsidy shares for PEL and additional subsidies for insulation measures for outer wall and floor insulation amounting to 20 %. If the electricity mix improves as aimed by the government and predicted by recent forecasts, the subsidy of HPs will be optimal in terms of minimized subsidy cost for emission reduction. High emission reductions must be promoted with a subsidy share of 40 % for HPs and accompanied by the promotion of window replacement with 60 % and outer wall insulation with 20 %. For lower emission targets, the promotion of insulation measures is not necessary. A subsidy rate of 20 % for HPs and 40 % for STCs which are combined with GAS achieves significant emission savings. With the progressive expansion of renewable energies in the electricity mix, funding agencies are advised to promote the electrification of heat supply through HP. These incentives should be accompanied by incentives for building insulation in poorly insulated old buildings.

The method presented allows a detailed technical examination of the investment decision of different building entities and proposes an approach for reformulating a bi-level problem with a lower-level MILP. The consideration of MILP at both levels of the bi-level problem and the application to a stock of individual buildings, each with its own decisions, is a novelty in the literature. The optimization program presented supports the decision-making process for subsidizing various energy-related measures and can facilitate the decision-making process of a funding strategy. When reformulating the problem into a one-stage problem, the solution space had to be discretized. This discretization could lead to inaccuracies in the problem solution. In addition, a very simple approach to mapping the building stock is used in this work. For the demonstration of the methodology, this seems justified. Future work should attempt to achieve a more accurate spatially resolved mapping and map the current state of modernization in buildings. A temporal resolution of the refurbishment process as well as the inclusion of limited resources such as investment budgets or craftsmen's capacities can be achieved by the method. Demand effects, which could lead to rising energy prices, can only be taken into account by the method through parameter variations.

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Appendix A Economical and technical Inputs

Table A.2: General Economical parameters and energy prices

Variable	Value	Unit	Variable	Value	Unit
Observation period	15	a	Yearly inflation	1.014	-
Interest rate	0.02	-	Energy tax gas	0.0055	EUR/kWh
Yearly el. price change	1.0388	-	Gas grid connection costs	118.7	EUR/a
Yearly gas price change	1.001	-	El. grid connection costs	96.0	EUR/a
Yearly pel. price change	1.0158	-			

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Table A.1: Economical Parameters Devices

Device	Installation Cost	Fix Investment Costs	Variable Investment costs	OM Cost
GAS	3,500 EUR	2,781.20 EUR	94.86 EUR/kW	2.5 %
HP Air	6,020 EUR	2,072.20 EUR	677.5 EUR/kW	2.5 %
Pellet	7,700 EUR	8,308.20 EUR	146.42 EUR/kW	5 %
STC	4,300 EUR	0.00 EUR	245.22 EUR/m^2	1.5 %
PV	8,000 EUR	0.00 EUR	95.42 EUR/m^2	1%
TES	0 EUR	460.00 EUR	608.6 EUR/m^3	2%
BAT	0 EUR	2,500.00 EUR	709.4 EUR/kW	1%
EH	100 EUR	111.00 EUR	8.00 EUR/kW	3%
WALL	Included in Fix.	3.3 EUR/(m)	112.2 EUR/(m^2m)	0 %
WIN	Included in Fix.	-2.7 EUR/(m)	876.1 EUR/(m^2)	0 %
ROOF	Included in Fix.	2.8 EUR/(m)	39.1 EUR/(m^2m)	0 %
ROOF	Included in Fix.	1.8 EUR/(m)	63.0 EUR/(m^2m)	0 %

Nomenclature

Abbreviations

BAT Battery

BiOP Bi-Level Optimization Program

DHW District Hot Water

FLOOR Ground floor insulation

HP Heat pump

GAS Gas boiler

KKT Karush-Kuhn-Tucker

MILP Mixed-Integer Linear Program

PEL Pellet boiler

PV Photovoltaic collector

ROOF Roof insulation

STC Solar thermal collector

TAC Total Annual Cost

WALL Outer Wall insulation

WIN Window replacement

BES Building energy system

Variables

A_{sol} solar module area

c^{dem} demand cost

c^{inv} invest cost

$r^{el,sell}$ revenue from feed-in electricity

r^{sub} revenue from subsidy

$\dot{Q}_{dev}^{nom}$ nominal heat output

x purchase decision variable

$x_{shell}^{retrofit}$ decision variable for renovation measure

ϕ^{inv} subsidy share

λ decision variable for measure pattern

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