PROCEEDINGS OF ECOS 2023 - THE 36TH INTERNATIONAL CONFERENCE ON
EFFICIENCY, COST, OPTIMIZATION, SIMULATION AND ENVIRONMENTAL IMPACT OF ENERGY SYSTEMS
25-30 JUNE, 2023, LAS PALMAS DE GRAN CANARIA, SPAIN

Efficient solving of time-coupled energy system MILP models using a problem specific LP relaxation

Stefan Kirschbaum[a], Marion Powilleit[b], Merlind Schotte[c] and Furkan Özbeg[d]

[a] Society for the Advancement of Applied Computer Science, Berlin, Germany, kirschbaum@gfai.de, CA
[b] Society for the Advancement of Applied Computer Science, Berlin, Germany, powilleit@gfai.de,
[c] Society for the Advancement of Applied Computer Science, Berlin, Germany, schotte@gfai.de
[d] Society for the Advancement of Applied Computer Science, Berlin, Germany, oezbeg@gfai.de

Abstract:

Energy system optimization models (ESOMs) often contain time coupling constraints, some of which couple short time frames as of daily storages or load changes of components, while other constraints couple longer periods like seasonal storages, peak load prices, or upper bounds to overall yearly CO_2 consumption.
Those ESOMs have binary constraints for minimal loads, efficiency curves, or discontinuous energy prices that are relevant for the short-term operation of the equipment. Calculation times for solving a whole year or longer as a coupled MILP problem are in many cases too high for practical applications that normally should not exceed one hour. Typical decomposition strategies to reduce calculation times are often designed for subclasses of energy system models and are not generally applicable. In order to have a generalized approach to solve these models efficiently, we investigate strategies that are based on a problem specific relaxation of integer constraints and downsampling of the input time series of the models.
A rolling horizon strategy is proposed that relaxes and downsamples the time steps from the end of the rolling horizon to the end of the year to consider the operation during the rest of the year. In order to reduce the error of the relaxation, binary constraints are reformulated to get the best LP approximation of the original MILP model. Using this rolling horizon strategy, models that are almost unsolvable as coupled MILP can be solved efficiently and very robustly and deliver a result that is feasible for the original problem and very close to the optimum of the original problem.

Keywords:

Energy system optimization; Time-coupling constraints; LP-relaxation; MILP; Downsampling.

1. Introduction to solving time-coupled energy system optimization

The optimization of the design and operation of energy supply systems plays an important role in the decarbonization of the industrial, commercial, and communal sector. Due to the vast number of new technologies and the interactions between different kinds of energy (sector coupling), designing and operating energy systems is a complex task which in most cases requires mathematical optimization models. Although some decisions can be made using simulation models, energy system optimization models (ESOM) are much more flexible and versatile to handle different kinds of decisions and target functions. Linear programming (LP) or mixed integer linear programming (MILP) has evolved as the state-of-the-art method to do these kinds of optimizations, although the problems themselves are often nonlinear. The complexity of these kinds of models is high if spatial, temporal, and/or technological dimension is high [1,2].

Many references in scientific literature deal with capacity planning of energy systems of countries [3,4] or even networks of more than one country [5,6]. Because electricity is very hard to store and has to be produced at the same time it is consumed, most of these models focus very much on electricity supply. In this paper, we focus on smaller sized energy systems, for example in industry, commercial buildings, or communal quarters. In contrast to countries or continents, those energy systems have an almost negligible spatial resolution. The technological options and temporal resolution on the other hand are often more versatile than in ESOMs for geographically large networks.

Optimizations are modeled as quasi-stationary states of operation for every piece of equipment and every point in time. Typical resolutions are hourly or quarter-hourly for a whole year or even longer. In cases where the energy suppliers and demands are separated spatially and the energy distribution (grid) is also relevant, a spatial structure of the energy system can be taken into account as well.

In this paper, we focus on models that take hours to solve due to the coupling of time steps by storages or other effects. From a practical standpoint, these models are often considered unsolvable, as for some real-world applications it is not possible to wait for so long. The aim is to solve these models in less time by solving simplified models with results very close to the original model. Other scientific work often focuses on certain kinds of time-coupling variables or equations and propose very good solutions to solve these ESOMs very closely to the original full model. Many of these publications use the target function value of the replacement models as quality measure for the methodology. An overview of different approaches to tackle complexity in ESOMs is given in [1] and [7]. In this work, we want to evaluate the operation of the equipment as well, because for practitioners this is an important criterion to decide whether a solution makes sense.

1.1. Non-scientific requirements in energy system optimization

In this work, we focus on non-scientific use cases of energy system optimization. The GFaI develops a software toolkit called TOP-Energy for the optimization of industrial and communal energy systems [8,9]. The constraints to the methodology discussed here arise from customers and reflect practical considerations like economic constraints or usability constraints.

A very important boundary condition of solving time-coupled ESOMs in a consulting use case is the time to solve the model. Energy consulting projects often have a scope of one or two months and include data acquisition, modeling, scenario optimization, scenario comparison, and presentation of results. It is necessary to solve several models per day, so the time to solve one model should not exceed one hour.

Another important criterion is the comprehensibility of the results. Parameter variations should result in a reasonable change of the solution. This affects parameters like the gap of the MILP solver. Very high gaps can lead to solutions that are close to the optimal solution but are not explainable anymore. The operation of the energy system in the result should be reasonable. This is especially important for design optimization, in which errors in the target function may affect certain time points disproportionately and make them look wrong. This has happened in real world problems in the past and is a problem for the credibility of the ESOM itself.

Some applications exploit the convex nature of some constraints relative to the target function. Electric feed-in for example, usually does not need a constraint that forbids buying and selling electricity at the same time, as prices are usually pointing in the direction of minimizing feed-in. Practical applications nonetheless are very generic and can't exploit the seeming complexity of the constraint. Changing the target function from operating costs to CO_2 emissions for example changes the convexity, but is a very common feature in real-world models.

1.2. Idea of the paper

The idea of the paper is to propose a method that can handle all kinds of relevant time-coupling constraints in models of typical industrial or communal energy systems without exploitation of certain convex parts of the model or behavior of technical components. Some decomposition approaches only work for a subset of ESOMs. We are trying to use a very general approach that is based on a relaxation of binary variables and a downsampling of the time series rather than a decomposition. We do not want to use Benders or Dantzig-Wolfe decomposition because they rely on a very specific mathematical structure of the problem that cannot be guaranteed in every case. In order to get a reasonable mode of operation for every device in the final solution, the previously relaxed binary variables have to be set to 0 or 1 in a later stage of the optimization. By doing this, the solution presented to the user is feasible for the original problem.

We propose a multi-stage optimization approach, in which the first couple of stages determine the design variables and the peak power prices, and the last stage uses a rolling horizon with a relaxed look-ahead in order to calculate results that fulfill all constraints of the original problem. The complexity of the original problem is thus reduced by relaxation of binaries and downsampling of the input time series in a way, that the difference between the original and the downsampled time series is as small as possible.

In order to improve the quality of the relaxed solution, problem specific substitute formulations are used for specific binary variables. They are used to replace piecewise-linear functions and SOS1 formulations for feed-in prices in a way that the difference between the binary formulation and the LP formulation is as small as possible.

2. Structure of optimization problem

2.1. Mathematical structure of the design and operation optimization problem

The mathematical formulation of the optimization problem was stated similarly in many cited publications. We look for a generic formulation, which works for many energy conversion units (electricity, heat, cooling, steam, compressed air), many operational side conditions (minimum part load, minimum runtime, ramp-up behavior, maintenance), and many operating costs (costs per operating hour, costs per full load hour, costs per year).The mathematical formulation will only be given very briefly. In most cases, the total annualized costs (TAC) are

https://doi.org/10.52202/069564-0249

minimized, although in some cases minimizing CO_2 emissions is also relevant. So, inspired by [10] the target function is formulated like this:

$$\min_{\dot{V}_{n,t},\dot{V}_{e,t},\dot{V}'_{e,t},\delta_{n,t}} TAC = OPEX + CAPEX$$

$$= \sum_{t\in T}\left(\Delta t_t\left(\sum_{n\in C} f_{n,t}(\dot{V}_{n,t}) + \sum_{e\in E} c_{e,t}\dot{V}_{e,t} - r_{e,t}\dot{V}'_{e,t}\right)\right) + \sum_{e\in E}\hat{c}_{e,t}\dot{V}_{e,t}^{max} + a_{inv}\sum_{n\in C} f_n^{inv}(\dot{V}_n^N)$$

$$\sum_{n\in C^e}\dot{V}_{n,e,t} - \sum_{n\in C^e}\dot{V}'_{n,e,t} + \dot{V}_{e,t} - \dot{V}'_{e,t} - \dot{D}_{e,t} = 0 \quad \forall\, e\in E\ \wedge \forall\, t\in T \quad (1) \quad \text{(energy balance)}$$

$$\dot{V}_{n,t} = f_n^{eff}(\dot{V}'_{n,t}) \quad \forall\, n\in C\ \wedge \forall\, t\in T \quad (2) \quad \text{(energy conversion efficiency)}$$

$$\dot{V}_n^N \geq \dot{V}_{n,t} \quad \forall\, n\in C\ \wedge \forall\, t\in T \quad (3) \quad \text{(nominal power per device)}$$

$$\dot{V}_{e,t}^{max} \geq \dot{V}_{e,t} \quad \forall\, e\in E\ \wedge \forall\, t\in T \quad (4) \quad \text{(peak power grid connection)}$$

$$0 \leq \dot{V}_{n,t} \leq \delta_{n,t}\dot{V}_n^{max} \quad \forall\, n\in C\ \wedge \forall\, t\in T \quad (5) \quad \text{(minimum part load)}$$

$$\Delta t_t\dot{V}_{n,e,t} \leq V_{n,e,t}\ \wedge \Delta t_t\dot{V}'_{n,e,t} \leq V_{n,e,t}^n - V_{n,e,t} \quad \forall\, n\in C_{st}\ \wedge \forall\, t\in T \quad (6) \quad \text{(charging and discharging)}$$

$$\Delta t_t(\dot{V}_{n,e,t} - \dot{V}'_{n,e,t}) = V_{n,e,t} \quad \forall\, n\in C_{st}\ \wedge \forall\, t\in T \quad (7) \quad \text{(filling level storage)}$$

$$\delta_{n,t}\in[0,1];\ \dot{V}_{n,e,t}\in\mathbb{R}^+;\ \dot{V}'_{n,e,t}\in\mathbb{R}^+; V_{n,e,t}\in\mathbb{R}^+;\ (\dot{V}_{e,t},\dot{V}'_{e,t})\in SOS1$$

The operating expenditures consist of operating costs for running a certain piece of equipment $n \in C$ at a certain point of time $t \in T$. The operating costs are a function of the amount of energy $\dot{V}$ of a certain energy form $e \in E$ at that point in time. Additional costs for purchasing energy from the grid $c_{e,t}\dot{V}_{e,t}$ and revenues from selling energy to the grid $r_{e,t}\dot{V}'_{e,t}$ as well as peak power costs $\hat{c}_{e,t}\dot{V}_{e,t}^{max}$ are included in the OPEX. The capital expenditures (CAPEX) consist of the investment costs for a device n multiplied by the annuity factor a_{inv}. The investment costs are a function of the nominal power $\dot{V}_n^N$. The target function has to be minimized subject to other constraints, most importantly the energy balance. This implies that the demand of each energy form at a certain point in time $\dot{D}_{e,t}$ has to be met by the sum devices producing that energy form $\sum_{n\in C^e}\dot{V}_{n,e,t}$ plus the supply from the grid minus feed-in and the consumption of that energy form by other devices. The feed-in and grid supply are part of a special ordered set I (SOS1), so both cannot be non-zero at the same time. The energy supplied by a device is a function of other energy forms consumed. This function can be a constant efficiency of a piecewise-linear function. The minimum part load is modeled using a binary variable $\delta_{n,t}$ and a Big-M formulation using $\dot{V}_n^{max}$ as Big-M. Storage components (C_{st}) can only supply the amount of energy represented by their actual filling level $V_{n,e,t}$ and can only be charged up to the maximum filling level $V_{n,e,t}^n$.

2.2. Time-coupling constraints

Typical ESOMs for optimizing the design and operation of energy systems can have different kinds of time-coupling constraints, which can be divided into two different categories: On the one hand, there are couplings that introduce a variable as an upper limit for another variable for every single time step. This is the case for design variables, like the nominal power of devices that limit the power of a device for every single time step, and grid peak power variables that limit the power that can be taken from an energy supply grid for every single step in time. These coupling variables introduce complicating variables into the MILP. On the other hand, there are cumulating variables. Examples for these constraints are storage variables and upper limits to CO_2 emissions of an energy system. These variables introduce complicating constraints into the MILP.

One way to deal with these complicating constraints and variables in an MILP that otherwise has a block diagonal structure is Bender or Dantzig-Wolfe decomposition. Because we deal with all kinds of energy systems with very heterogeneous mathematical formulations and mixtures of complicating constraints and variables, these kinds of mathematical decompositions are not used in this work. There are some approaches to use Bender and Dantzig-Wolfe decomposition in literature ([11–13]), but they are tailored to a subset of use cases that are investigated in this work. Instead, the approach of this paper is a generic use of relaxations and downsampling to simplify the overall MILP model.

2.2.1. Upper limit (power price, capacity)

Complicating variables that represent an upper limit to another variable in every single time step occur, for example, in peak power pricing where a certain price has to be paid for the highest energy amount consumed from the electric grid per 15 minutes ((4) in Section 2.1). Another common example of variables representing an upper limit for every time step is the nominal power of a technical component in a design problem. The nominal power is an upper limit to the energy produced by that component per time step ((3) in Section 2.1).

2.2.2. Daily storage

Storages introduce complicating constraints into the ESOM. The state of charge of the storage couples two consecutive time steps and depends on the charged and discharged energy ((7) in Section 2.1).

Storages can be distinguished on the basis of their main application. Storages are economically most feasible, when they have many charging cycles throughout a year. Therefore, daily storages, which store electricity from photovoltaic, for example, are very common today. ESOMs containing these storages can be decomposed using typical days for calculating the storage operation ([14–19]). This kind of decomposition typically uses a cyclic constraint that couples the filling level of the first hour with the last hour of a typical day thus separating the solution of different typical days from each other.

2.2.3. Seasonal storage

Seasonal storages work exactly as daily storages, and the mathematical formulation of the overall problem looks exactly the same. However, the use case is different. Seasonal storages are designed to store energy for very long periods. They often have only one charging cycle throughout the year. A typical example is an ice storage that freezes water during the winter using the cold side of a heat pump. This ice can be used during the summer to cool buildings and thus be thawed again. Because we focus on models with a time frame of only one year, a cyclic constraint for seasonal storages has to be added to the mathematical formulation above, which couples the state of charge of the first time step with the last time step:

$$V_{n,e,0} = V_{n,e,max} \quad \forall\, n \in C_{st}$$

Due to the long-term storing of energy, decomposed typical days do not work for the design of these kinds of applications. There are approaches to couple typical days by a superimposed state of charge variable for every day of the year ([17,20,21]). These approaches do not perform very well for the design of seasonal storages because the order of the typical days has a very big influence on the design of the storage. Calculations with simplified energy system models resulted in deviations in the size of the storage between the full model and the decomposed model of over 85 %. Typical days are often determined using a clustering algorithm. Grouping of typical days by “charge-days” and “discharge-days” improves the design of the storage but still does not give reliable results.

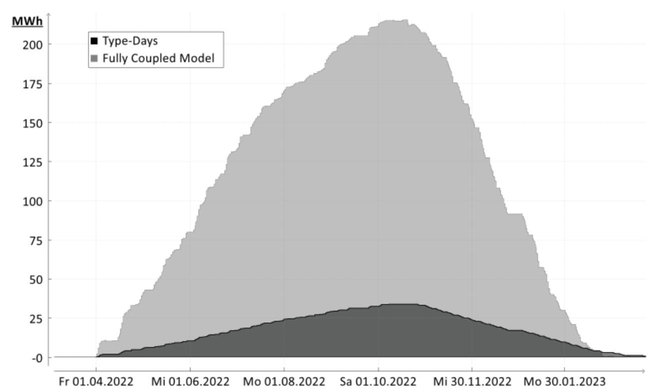

Figure. 1. *State of charge of a seasonal storage designed using typical days compared to the fully coupled original model.*

Figure. 1 shows the resulting state of charge using coupled typical days compared to the result of the original model. The underlying model consists of an oversized photovoltaic plant that produces electricity for hydrogen during the summer in order to produce electricity in a fuel cell during the winter season. The hydrogen is stored in a hydrogen storage of the size which is shown in the figure above. The fully coupled model results in a storage size of 215 MWh, whereas the decomposed model calculates a size of 33 MWh. The design of seasonal storages is one of the main reasons a method is developed that maintains the order of time steps in the reduced model in this paper.

2.2.4. Integral values

Another reason to have complicating constraints from cumulating variables in an ESOM are integral values. Typical examples are upper limits for CO_2 emissions of an energy system or performance indicators that should be met throughout the year. These constraints often come from corporate ecological goals or government regulations [22]. Equations of integral values are not included in Section 2.1. A typical formulation might look like this:

https://doi.org/10.52202/069564-0249

$$\sum_{e\in E}\sum_{t\in T}\theta_{e,t}\left(\dot{V}_{e,t}-\dot{V}'_{e,t}\right)\leq\theta_e^{max}\qquad\forall\, e\in E\qquad(8)\quad\text{(CO}_2\text{ emission limit)}$$

$$\frac{\sum_{e\in n}\sum_{t\in T}\dot{V}_{e,t}}{\sum_{e\in n}\sum_{t\in T}\dot{V}'_{e,t}}\geq\hat{\eta}_c\qquad\forall\, n\in C\qquad(9)\quad\text{(Overall efficiency constraint)}$$

In formulation (8), θ_e^{max} is an upper limit of an integral value: in this case, the CO_2 footprint of all energy forms bought from the grid minus the ones fed into the grid. In (9) the quotient of produced energy and consumed energy throughout the year of one component (overall efficiency) should be higher than a target efficiency $\hat{\eta}_c$. These formulations exist in cogeneration subsidies in Germany, for example.

2.3. Binary variables and their relaxations

The main idea of this paper is to use LP relaxations combined with the downsampling of time steps in order to reduce the complexity of the ESOM, while maintaining the chronological order of the time steps. Therefore, different typical applications of binary variables in ESOMs will be discussed in the following Section.

2.3.1. Part load characteristics and cost functions

Some variable relations in ESOMs are non-linear and have to be linearized in order to use them in MILP models. Typical examples are part load characteristics of technical equipment, which represent energy conversion efficiencies in part load ((2) in Section 2.1). Another common example are cost functions of technical components that usually have an economy of scale effect, which means the specific price of a technology is lower, when the plant size is bigger (f_n^{inv} in Section 2.1). The relation between variables is mostly described using piecewise linear functions (PWL) that are characterized by supporting points and interpolation in between those points. These PWLs can be modeled using special ordered sets (SOS) using the following formulation:

$$\sum_{i=1}^{n}\lambda_i x_i = x;\ \sum_{i=1}^{n}\lambda_i y_i = y\,;\sum_{i=1}^{n}\lambda_i = 1$$

$$\lambda_i\geq 0;\ \lambda_i\in\text{SOS2};\ i=1,\dots,n$$

in which the PWL is defined by the supporting points $(x_1,y_1),(x_2,y_2),\dots,(x_n,y_n)$. In a SOS constraint of type 2 (SOS2), not more than two variables are allowed to take a non-zero value, and these non-zero variables must be consecutive in the list. SOS constraints are often reformulated using binary variables.

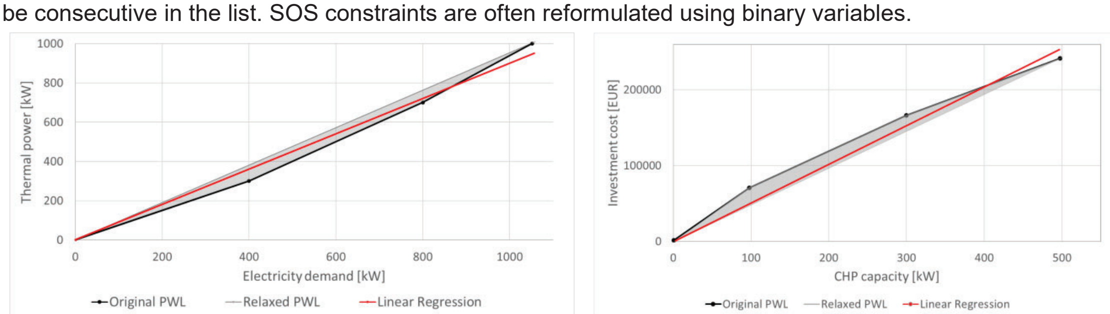

Figure. 2. *Typical structure of part load curves and investment cost function.*

The figure above shows two piecewise linear functions (black) and the space of possible solutions with relaxed binaries (grey). In this paper, we use a linear regression of the PWL function instead of an LP relaxation. This way, the deviation between the LP formulation and the PWL is smaller. The linear regression is also shown in Figure. 2 (red).

Typical part load curves have a convex structure due to inefficiencies in part load. Cost functions are usually concave due to economy of scale effects. However, in relation to the objective function, the PWLs are typically concave. In case of the part load curve, the objective function favors higher output power; in case of the cost function, the objective function favors lower investment costs. Therefore, binaries are needed for the formulation of the PWLs, and regression functions are used as replacement formulation in this work.

2.3.2. Semi-continuous variables for minimum part load and start/stop restrictions

Many energy conversion devices cannot be operated continuously between 0 % and 100 % of power output. In most cases, the part load behavior is continuous down to a point called minimum part load below which the device has to be turned off ((5) in Section 2.1). Therefore, the use of semi-continuous variables is very common in order to model this kind of behavior. Semi-continuous variables can either be 0 or in a range between a lower (ℓ) and an upper bound (u). They are reformulated using a binary variable (δ) by MILP solvers:

$$\ell \cdot \delta \leq x \leq u \cdot \delta$$
$$\delta \in \{0,1\}$$

Relaxing these kinds of variables leads to a continuous solution space between 0 and u for the variable x. So, in the LP relaxation, the minimum load of the component is not taken into account.

Because the binary variable δ is also the indicator for whether a component is on or off, this also cannot be distinguished in the LP relaxation leading to other constraints not being considered. Start and Stop constraints, for example, are often modeled in a way that a component cannot be switched on and off very frequently. If the on/off state cannot be determined anymore, start/stop constraints can also not be included.

2.3.3. Binary variables for buying and selling energy

Another typical application for special ordered sets represented by binaries are prices for buying electricity from the grid and the respective revenues for selling electricity to the grid. This can be represented by an SOS1 constraint, which means electricity can either be bought or sold, but not both at the same time. The amount of electricity bought and sold is represented by a variable that is part of an SOS1 in each case. So only one of them can be different from 0 at the same time step. These variables are then multiplied with the purchase or feed-in price and added to the target function.

In the LP relaxation, the variables in the SOS1 can both be different from 0 at the same time, which means buying and selling electricity at the same time step is allowed. As prices for buying electricity are usually much higher than the revenues for selling electricity to the grid, this is not a problem in most cases. Because the use cases discussed here are very heterogeneous, there might be situations where the price for buying electricity is very low. This can happen, when the selling price is determined by a subsidy (e.g., PV) and the buying price by a spot market (e.g., EPEX intraday). In these cases, the LP relaxation will buy cheap electricity from the grid and feed it back for a higher price, leading to an unbound MILP. This issue can be resolved by setting the same price for buying and selling electricity in the LP relaxation in these rare cases.

In cases in which the target function is not operating costs or annualized costs but rather CO_2 emissions or primary energy, simultaneous supply from the grid and feeding into the grid is possible. In these cases, the MILP will not become unbound, but the cost results of the model will be wrong.

2.3.4. Indicator constraints for if-then relations in control statements

Sometimes in ESOMs a constraint is only active if another condition is met. This can be modeled using indicator constraints, which themselves are represented using a binary variable with a Big-M formulation. An example is a heat pump that can only produce a certain amount of heat, when the heat source has a temperature above a certain value. The temperature of the heat source may be determined by another component (e.g., geothermal) that determines whether the source temperature can be reached. In this case, the heat pump can produce only a certain amount of heat, when the geothermal system is running. The on/off variable of the geothermal indicates the maximum power of the heat pump in this case. Using an LP relaxation of the original model, these indicator constraints cannot be modeled anymore, and the relation between the binary and the indicator constraint is lost.

3. Methodology

The methodology of this paper is based on a multi-stage approach using an LP relaxation and a downsampling in the first couple of stages to calculate the complicating variables and a rolling horizon approach in the last stage to calculate the complicating constraints and all other results. Using 5 different energy system models, times for solving LP relaxations were determined [23]. The measure to compare solving times are gurobi [24] work units, because they are independent of other processes running on the same machine and thus reproducible. A gurobi work unit is almost the same as a second on a single core processor. The measurements show a significant reduction in the LP relaxation. LP relaxations of MILP Models that do not solve in hours can be calculated in less than an hour.

The LP relaxations used in this paper are not only relaxations of the original model but in fact reformulations of characteristic curves and some energy prices in order to generate a formulation that is closer to the original model than the LP relaxation without using any binaries. These reformulations are described in Section 2.

An additional downsampling of the LP leads to another significant reduction of the solving time typically to less than a minute. We experimented with other time series reduction algorithms as described in [17]. Especially feature-based segmentation methods [25] that do not produce typical days but keep the order of the time steps intact were tested without significant improvements in the solution quality. Nevertheless, further investigations of these methods should be done in the future.

3.1. Substitute formulation of binary variables

In order to get as close as possible to the original unrelaxed formulation with the new LP formulation, some constraints can be reformulated in a way that does not require any binary variables. This has been done for piecewise linear functions using a linear regression with an axial intercept at 0/0. The advantages of this formulation were tested in [23]. It could be shown that a regression is faster. It is also more accurate in most

https://doi.org/10.52202/069564-0249

cases, but depending on the convexity of the PWL, it might also be worse than a relaxation. Nevertheless, we favor the regression because we do not want to exploit the convexity of PWL functions.

In the case of grid connections that have different purchase and feed-in prices, we use a different approach. In [23] we investigated different reformulations of the SOS1 constraint for feeding and purchasing electricity. Relaxing the SOS1 constraints led to high shifts in the target function value in time steps where the feed-in price is higher than the purchase price. Setting a medium price for buying and selling electricity would be a solution but also leads to high errors. So in this work, we propose for time steps in which electricity is bought and sold at the same time to add binary steps in later stages.

3.2. Fixing complicating variables (first stages)

Results of the LP relaxation are used to determine and fix complicating variables in the first stage of the optimization process. Because the LP relaxation violates a lot of binary constraints, the results of the complicating variables can be improved by re-adding some binary constraints that are violated in stage one back into the model. A solution of this partially relaxed model is stage two.

Further studies on two different ESOMs (see **Figure. 3** and **Figure. 5**) show the work units of models with different numbers of unrelaxed time steps (**Figure. 4** and **Figure. 6**). The figures show a significant increase in work units above 100 unrelaxed time steps. Another significant increase happens above 1000 unrelaxed time steps.

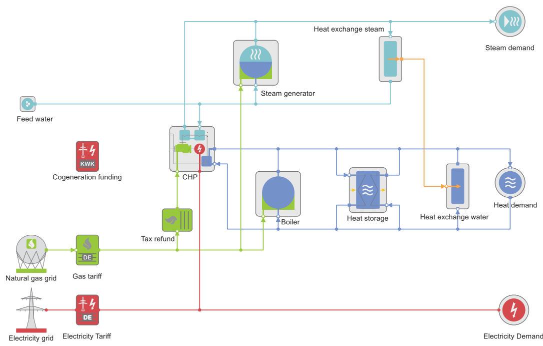

Figure. 3. *Heat and steam supply with efficiency constraint (Model A).*

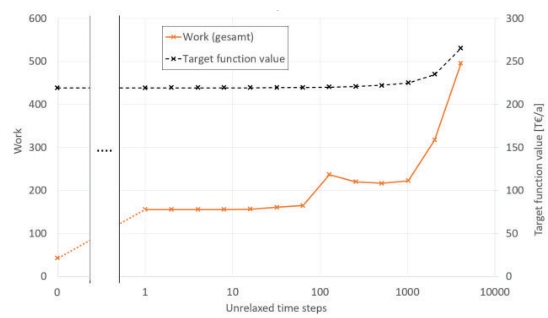

Figure. 4. *Work units and target function value over the number of unrelaxed steps (Model A).*

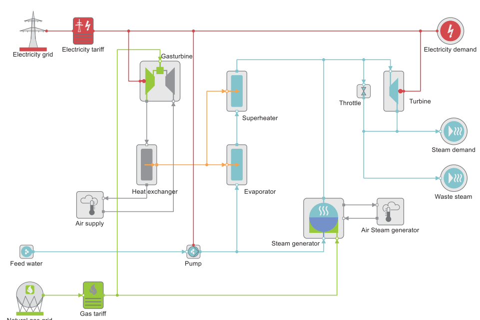

Figure. 5. *Steam supply with gas turbine and peak power price (Model B).*

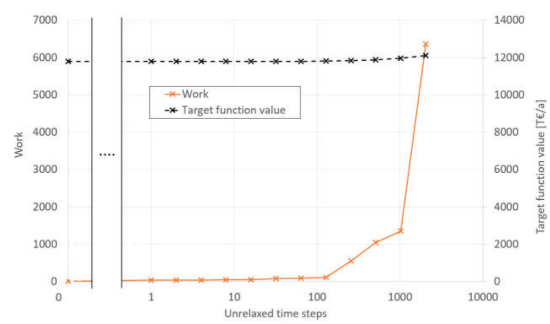

Figure. 6. *Work units and target function value over the number of unrelaxed steps (Model B).*

Model A consists of 259 continuous and 31 binary variables per time step of which presolve of gurobi removes about 90 %. Model B has 566 continuous and 70 binary variables per time step of which 96 % are removed during presolve.

The idea of the algorithm is to add binary variables back into the relaxed model and solve it until the violation of binary constraints is acceptable or calculation time are getting unacceptable. When this algorithm terminates, the complicating variables are fixed. These are nominal powers of technical components and peak power of the electric grid.

3.3. Calculating unrelaxed results using a rolling horizon

With the complicating variables fixed, the other results are calculated using a rolling horizon approach. The rolling horizon is designed in a way that results are calculated using unrelaxed time steps that are not downsampled. The rest of the year is taken into account using a downsampled a relaxed time frame. This way, the development of complicating constraints by cumulating variables can be considered in the decision for the unrelaxed time frame. The advantage of this approach over a relaxed calculation is that the results do not violate any binary constraints and are not downsampled. The result is a solution of the original model, which is a requirement by most users. Analog to [26] the solution could be used to warm-start the original model. This has not been implemented in this work, but is subject to further investigation.

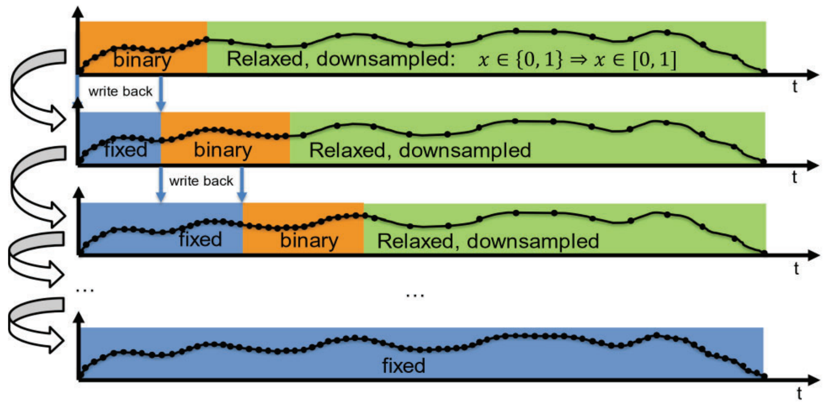

Figure. 7. *Illustration of binary rolling horizon, writing back, fixed past, and relaxed future.*

4. Runtime experiments

Whereas preliminary studies used many different models (Section 3.1) final experiments used a model with an oversized photovoltaic system and a power-to-gas unit to produce hydrogen out of a surplus of electricity. The hydrogen can later be used to produce electricity in a fuel cell. Both fuel cell and power-to-gas produce low temperature heat as a by-product. This heat can either be dumped in an emergency cooler or upgraded in a heat pump to high temperature heat. The high temperature heat from the heat pump is used to supply the high temperature heat demand. Heat that cannot be produced by the heat pump has to be generated using a common boiler. The ESOM was modeled using the modeling framework TOP-Energy. Most of the runtime measurements have been done using a python reimplementation of the same model. The model was chosen, because the full unrelaxed solution to a of gap 0.1 percent with 8760 time steps can be obtained in a reasonable time (about 30 minutes). Other example models did not solve in hours, which made it hard to do lots of evaluations with them. The prove of optimality for this model would take more than 24 h and has therefore not been carried out. A scheme of the ESOM is shown in Figure. 8.

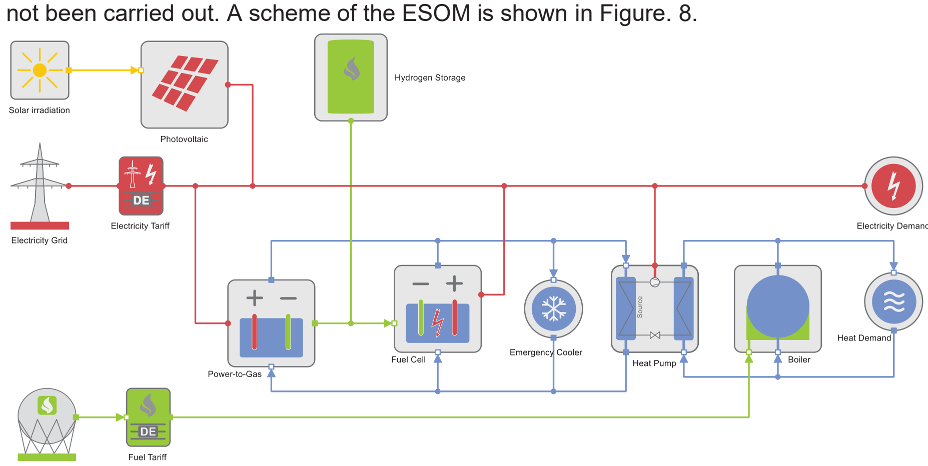

Figure. 8. *ESOM with photovoltaic and hydrogen storage for electricity and heat supply (Model C).*

The model contains two complicating constraints. One is the size of the hydrogen storage, and the other one is the peak power taken from the electric grid. The fuel cell has a minimum part load of 20 % and a nominal power of 600 kW. The heat pump has a characteristic curve that describes the part load behavior. The hydrogen storage has a characteristic curve that describes the investment cost function with an economy of

https://doi.org/10.52202/069564-0249

scale effect. Both characteristic curves have three supporting points which result in three variables in the SOS2 formulation.

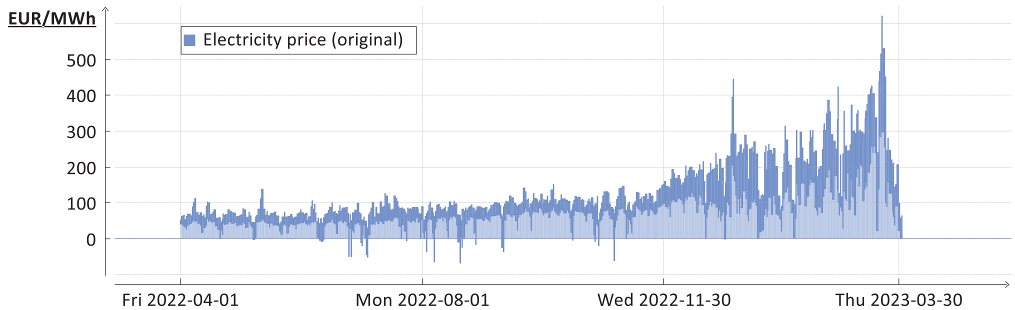

Figure. 9. *Electricity prices based on EPEX spot prices from 2021.*

The electricity price was taken from the EPEX day-ahead auction in Germany in 2021 [27] and moved 8 months back to cover the time frame of the model which is from April 2020 until the end of march 2021 (to get a better charging and discharging regime of the seasonal storage).

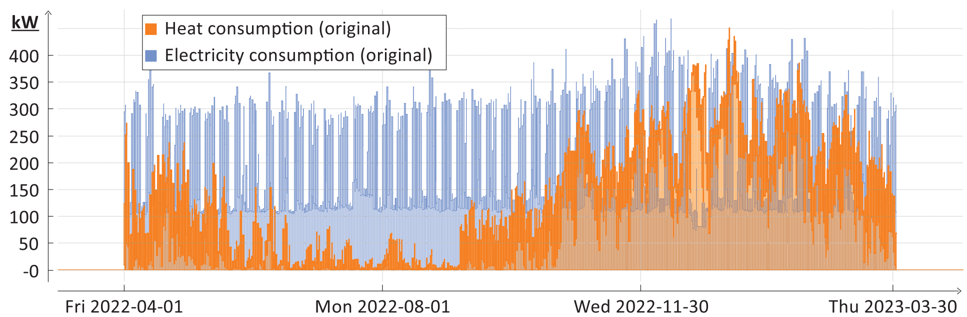

Figure. 10. *Heat and electricity demand.*

The electricity demand was measured at an office building and scaled up by a factor of 10. The heat demand was generated from a temperature profile of Berlin. The electricity production of the photovoltaic system is calculated using solar irradiation data of Berlin for a typical year. These data are provided by the German Meteorological Service DWD. A study from 2020 [28] states a price of 0.6 €/kWh for a very big cavern storage of 26 GWh. To make the storage economical for the sake of the study, these prices were further reduced. The specific costs of the hydrogen storage were set to 0.3 €/kWh for small storages (up to 300 MWh) and 0.255 €/kWh for bigger storages.

4.1. Runtime for stage one

The best available result for the complicating variables in stage 1 of model C was achieved by running the whole time frame (8760 steps) with binary variables and a gap of 0.1 %. This calculation took 1661 work units. Lower gap values took too long to calculate. This calculation results in 288 kW peak power for the grid connection and 569 MWh capacity of the hydrogen storage.

Table. 1. *Work units and errors of downsampled solutions in stage 1.*

Timesteps	Work units	Peak power	Error	Hydrogen storage	Error
1095	0.608	318 kW	10.4 %	554.92 MWh	2.5 %
1460	0.844	261 kW	9.4 %	475.36 MWh	16.5 %
2190	1.932	267 kW	7.3 %	492.49 MWh	13.5 %
4380	7.664	272 kW	5.6 %	494.53 MWh	13.1 %
8760	30.942	273 kW	5.2 %	490.47 MWh	13.8 %

The table above shows the work units of certain levels of downsampling and the respective results for hydrogen storage size and peak power. The downsampled models do not contain any binary variables. All binaries are replaced by substitute formulations or relaxed. While the peak power is getting better with a higher number of time steps, the storage size gets worse. This is not reproducible for different setups, so we assume it is a random behavior. The target function value of the overall problem is 211,126 Euro, so the error of the peak power as well as the error of the hydrogen storage size led to errors in the peak power price and the hydrogen storage price close to 0.1 % of the target function value in a solution with gap 0.1 %.

In this study, a downsampling rate of 4 was chosen for this example. Because the results from **Table. 1** are not known beforehand, we start the investigation with a downsampling to 4-hour intervals. This seems to be a reasonable resolution to account for fluctuating renewable energies, and it led to acceptable calculation times in the other models as well.

4.2. Adding binary steps in stage 2-5

In the next stages, the violation of binary constraints is fixed by adding binary steps to the relaxed model. The results of the stages are shown in the table below. Although the solution time other than the work units may depend on other tasks on the same processor, solution times are given next to work units for reference.

Table. 2. *Complicating variable results of the first 7 stages of optimization.*

Stage	Binary Steps	Work	Time	Peak Power	Error	Hydrogen Storage	Error
1	0	1.93	1.51 s	267 kW	7.3 %	492.49 MWh	13.5 %
2	137	5.12	4.37 s	265 kW	8.0 %	497.97 MWh	12.5 %
3	204	5.86	4.82 s	261 kW	9.4 %	511.89 MWh	10.0 %
4	220	4.13	3.90 s	259 kW	10.1 %	513.32 MWh	9.8 %
5	239	4.54	4.99 s	267 kW	7.3 %	518.92 MWh	8.8 %
6	250	3.66	4.22 s	267 kW	7.3 %	521.42 MWh	8.4 %
7	250			Stage 7 not carried out (no change)			
	Result 2-6:	25.26	23.80 s	267 kW	7.3 %	521.42 MWh	8.4 %

A peak power of 267 kW and a hydrogen storage size of 521,427 kWh is used for the last stage.

4.3. Rolling horizon stage

In the last stage, a rolling horizon of 84 time steps is used. Each frame is calculated using 96 unrelaxed time steps that are not downsampled and the rest of the year with a relaxed downsampling of 4. The first 84 time steps of each calculation are stored as results and fixed for the next frames. 103 calculations are needed to calculate the whole year. These 103 calculations take another 70 work units and 105 seconds. The sum of all stages took 95 work units and 129 seconds compared to 1661 work units and 1956 seconds for the full model.

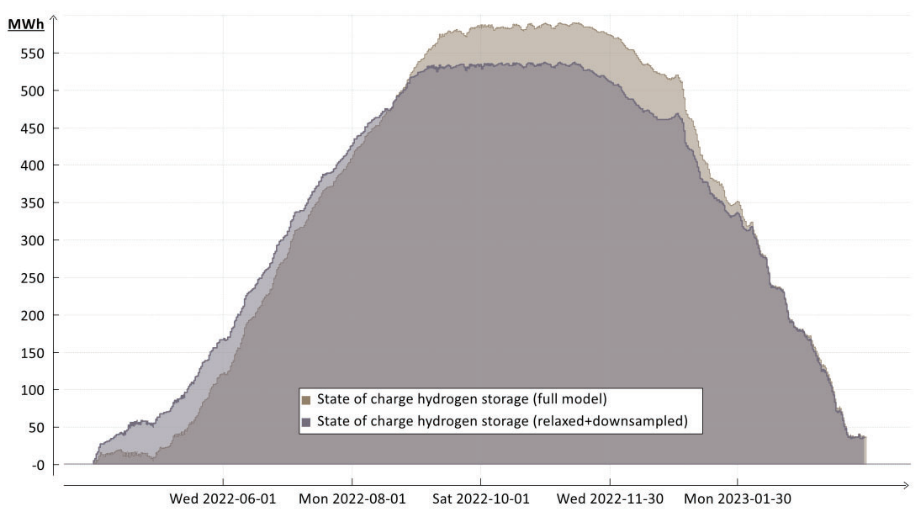

Figure. 11. *State of charge of the full model compared to the result acquired by the methodology.*

The patterns of charging and discharging the hydrogen storage are similar between the unrelaxed solution based on 8760 time steps and the relaxed and downsampled solution. The grid consumption of electricity (which is not shown here) is also very similar. For this example, the methodology delivers a reasonable result, which fulfils the requirements.

https://doi.org/10.52202/069564-0249

4.4 Sensitivity analysis

One requirement for the methodology was the traceability of results. Therefore, we carry out a sensitivity analysis for the original solution and the relaxed and downsampled solution to see whether the dependency on peak power pricing and hydrogen storage price are similar.

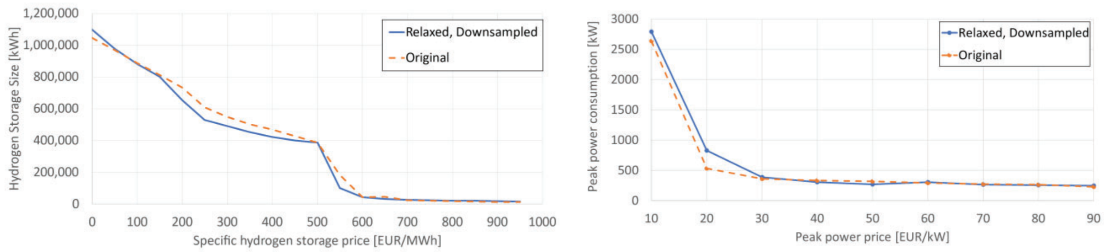

Figure. 12. *Dependency of prices and peak power (r) and storage size (l).*

The figure above shows the dependency of the consumed peak power on the peak power price and the dependency of the hydrogen storage size on the specific price of the hydrogen storage. The figure shows that the general behavior of the correlation is the same, and the methodology represents the values quite good.

5 Conclusions and future work

The examples above show that a relaxation of binary variables combined with a reformulation of some constraints together with a downsampling of time steps is capable of replacing the original more complex formulation and producing good results for design variables and peak power prices. In a multi-stage approach adding binary steps back into the problem in later stages and then calculating a feasible solution using a relaxed rolling horizon approach, reasonable results for problems with all kinds of time-coupling could be calculated.

The statements made here are tested against a small set of energy system models and should be investigated further. The fact that for some of the tested ESOMs the original solution is not known makes this evaluation a hard task.

In future work it should be tested whether the solution can be improved even further when the final solution of the rolling horizon approach (which is feasible for the original problem) is used to warm start the solution of the original problem. Having this lower bound of the original problem has proven to have big advantages [26].

Another way to improve the solution is to use a multivariate feature based segmentation [29] to reduce the number of time steps. This means removing steps from the input time series that change the time series as little as possible. An implementation based on the bottom-up approach from [25] was tried by the authors, but did not have a significant effect yet.

References

[1] Kotzur L, Nolting L, Hoffmann M, Groß T, Smolenko A, Priesmann J et al. A modeler's guide to handle complexity in energy systems optimization. Advances in Applied Energy 2021;4:100063.

[2] Scholz Y, Fuchs B, Borggrefe F, Cao K-K, Wetzel M, Krbek K von et al. Speeding up Energy System Models - a Best Practice Guide; 2020.

[3] Mehleri ED, Sarimveis H, Markatos NC, Papageorgiou LG. Optimal design and operation of distributed energy systems: Application to Greek residential sector. Renewable Energy 2013;51:331–42.

[4] Kydes AS. The Brookhaven Energy System Optimization Model: Its Variants and Uses. In: Ziemba WT, Schwartz SL, editors. Energy Policy Modeling: United States and Canadian Experiences: Volume II Integrative Energy Policy Models. Dordrecht: Springer Netherlands; 1980, p. 110–136.

[5] Leuthold FU, Weigt H, Hirschhausen C von. A Large-Scale Spatial Optimization Model of the European Electricity Market. Netw Spat Econ 2012;12(1):75–107.

[6] van der Voort E. The EFOM 12C energy supply model within the EC modelling system. Omega 1982;10(5):507–23.

[7] Cao K-K, Krbek K von, Wetzel M, Cebulla F, Schreck S. Classification and Evaluation of Concepts for Improving the Performance of Applied Energy System Optimization Models. Energies 2019;12(24):4656.

[8] Augenstein E, Herbergs S, Wrobel G. TOP-Energy - Ein Framework für Softwarelösungen in der Energietechnik. In: Oberweis A, Weinhardt C, Gimpel H, Koschmider A, Pankratius V, editors. eOrganisation Service-, Prozess-, Market-Engineering 8. Internationale Tagung Wirtschaftsinformatik: Universität Karlsruhe; 2007, p. 947–954.

[9] Kirschbaum S. Entwicklung eines Softwarepakets zur Simulation industrieller Produktionsprozesse unter energetischen Gesichtspunkten. Aachen; 2012.
[10] Baumgärtner. RiSES4 Rigorous Synthesis of Energy Supply Systems with Seasonal Storage by relaxation and time- series aggregation to typical periods 2019.
[11] Wirtz M, Heleno M, Moreira A, Schreiber T, Müller D. 5th generation district heating and cooling network planning: A Dantzig–Wolfe decomposition approach. Energy Conversion and Management 2023;276:116593.
[12] Binato S, Pereira MVF, Granville S. A new Benders decomposition approach to solve power transmission network design problems. IEEE Transactions on Power Systems 2001;16(2):235–40.
[13] Griset R, Bendotti P, Detienne B, Porcheron M, Şen H, Vanderbeck F. Combining Dantzig-Wolfe and Benders decompositions to solve a large-scale nuclear outage planning problem. European Journal of Operational Research 2022;298(3):1067–83.
[14] Bardow A, Bahl B, Hennen M, Hollermann DE, Chivite A, Wieland L et al. Ganzheitliche Optimierung von Energieversorgungssystemen in der Praxis sOptimo+. Aachen; 2018.
[15] Baumgärtner N, Bahl B, Hennen M, Bardow A. RiSES3: Rigorous Synthesis of Energy Supply and Storage Systems via time-series relaxation and aggregation. Computers & Chemical Engineering 2019;127:127–39.
[16] Bahl B, Söhler T, Hennen MR, Bardow A. Time-series aggregation to typical periods with bounded error in objective function for energy systems synthesis. In: [The 30th International Conference On Efficiency, Cost, Optimization, Simulation And Environmental Impact Of Energy Systems, ECOS 2017, 2017-07-02 - 2017-07-06, San Diego, CA, USA]: The 30th International Conference On Efficiency, Cost, Optimization, Simulation And Environmental Impact Of Energy Systems, San Diego, CA (USA), 2 Jul 2017 - 6 Jul 2017; 2017, p. 1–12.
[17] Hoffmann M, Kotzur L, Stolten D, Robinius M. A Review on Time Series Aggregation Methods for Energy System Models. Energies 2020;13:641.
[18] Kotzur L, Markewitz P, Robinius M, Stolten D. Impact of different time series aggregation methods on optimal energy system design. Renewable Energy 2018;117:474–87.
[19] Teichgraeber H, Brandt AR. Systematic Comparison of Aggregation Methods for Input Data Time Series Aggregation of Energy Systems Optimization Problems. In: Computer Aided Chemical Engineering: Elsevier; 2018, p. 955–960.
[20] Baumgärtner N, Shu D, Bahl B, Hennen M, Bardow A. From peak power prices to seasonal storage: Long-term operational optimization of energy systems by time-series decomposition. In: ; 2019, p. 703–708.
[21] Gabrielli P, Gazzani M, Martelli E, Mazzotti M. Optimal design of multi-energy systems with seasonal storage. Applied Energy 2018;219:408–24.
[22] Bundesamt für Wirtschaft und Ausfuhrkontrolle. Modul 2 - Prozesswärme aus erneuerbaren Energien: Anlage zum Merkblatt Bundesförderung für Energie- und Ressourceneffizienz in der Wirtschaft - Zuschuss 2021.
[23] Furkan Özbeg. Entwicklung einer problembezogenen Relaxierung zur effizienten Beschreibung von MILP-Modellen für Energiesysteme. Bachelorarbeit. Berlin; 2022.
[24] gurobi: gurobi Optimization LLC; Available from: www.gurobi.com.
[25] Miodrag Lovrić, Marina Milanović, Milan Stamenković. Algorithmic methods for segmentation of time series an overview. 1 2014;1(1):31–53.
[26] Kai Mainzer, Russell McKenna, Wolf Fichtner. Rolling Horizon Planning Methods in Long-Term Energy System Analysis MILP Models: Unpublished; 2015.
[27] Bundesnetzagentur | SMARD.de. SMARD | SMARD - Strommarktdaten, Stromhandel und Stromerzeugung in Deutschland. [March 12, 2023]; Available from: https://www.smard.de/home.
[28] Kreidelmeyer S, Dambeck H, Kirschner A, Wünsch M. Kosten und Transformationspfade für strombasierte Energieträger: Endbericht zum Projekt "Transformationspfade und regulatorischer Rahmen für synthentische Brennstoffe" 2020.
[29] Pang Y, Shi M, Zhang L, Sun W, Song X. A multivariate time series segmentation algorithm for analyzing the operating statuses of tunnel boring machines. Knowledge-Based Systems 2022;242:108362.

https://doi.org/10.52202/069564-0249