Different allocation mechanisms to distribute the total profits of the Italian Renewable Energy Community

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Abstract:

Energy communities could benefit from an optimal match between total energy generation and demand, resulting in economic gains. A main issue is the distribution of the optimal economic gain of an energy community among its members. Several works apply a single criterion of cost/profit allocation within an energy community, neglecting the impact of different criteria on different members. This paper aims at evaluating and comparing two different cost/profit allocation mechanisms, i.e., the cooperative "Shapley value", based on the marginal contributions of members to any coalition within the energy community, and the "Uniform pricing", which relies on a uniform price that is independent of the cooperation among members. According to the Italian legislation, a centralized Renewable Energy Community with virtual energy sharing scheme is analysed. The community encompasses one consumer and two prosumers with shiftable electrical demands that are representative of the tertiary, residential and commercial sectors, and prosumers own photovoltaic plants with/without the electrical storage system. A cooperative model of this energy community is presented, and the daily operational profit is maximized by a Mixed-Integer Non-Linear Programming optimization. Subsequently, the mechanisms of Shapley value and Uniform pricing are applied to allocate the optimal profit of the energy community. Uniform pricing leads to daily profits of 1.95€, 0.69€ and 5.71€ for the tertiary, residential and commercial members, respectively. Conversely, results of the Shapley value are fairer towards prosumers, allocating daily profits of 9.57€ and 3.81€ to the residential and commercial prosumers, and a daily cost of 5.03€ to the tertiary consumer.

Keywords:

Renewable Energy Community; Profit allocation; Cooperative game; Shapley mechanism; Fairness.

1. Introduction

Renewable energy communities (RECs), defined by the recast of the Renewable Energy Directive (RED II) [1], represent a regulatory tool to foster the distributed generation and consumption driven by renewable sources. RECs can help achieve the energetic and environmental targets indicated by the "Fit for 55" document of the European Commission [2] (i.e., renewables share of 40%, increase of energy efficiency by 36% in the final energy consumption and emissions reduction by at least 55% compared to the levels of 1990), beyond leading several economic and social benefits to the European citizens. In the Italian context, the regulation about the technical management of RECs has been recently updated by GSE [3], the Italian Energy Services Operator, while the economic incentives have been defined by ARERA [4], the Italian Regulation Agency for Environment, Network and Energy. In such a context, this paper focuses on the economic benefits associated with the configuration of the Italian REC and of its participants. The current and emerging literature on energy communities is very broad and, for this reason, the literature review in this paper summarizes the main recent works in the following identified fields.

Some works in the literature dealt with business models, policies and modelling techniques that can foster the development of Energy Communities (ECs) [5-7]. Lowitzsch et al. [5] presented a holistic analysis of the already existing and still untapped business models for ECs, identifying different structure and design options (e.g., application technology, cooperating partners, geographical location, types of investors and ownerships etc.). Ceglia et al. [6] assessed the energy, environmental and economic performances of an Italian REC, highlighting its higher electricity self-consumption (till 56%) with respect to other energy-sharing models, e.g., the "System of Efficient Users" (till 12% of electricity self-consumption) that is based on directives released before 2018. Gerundo et al. [7] proposed a methodology to identify potential geographical areas that are suitable for the development of RECs, while taking into account the constraints of installation for the generation technologies and social benefits as the reduction of the energy poverty.

Other works focused on the optimization of the design and operation of the generation and storage technologies within ECs [8-10]. Cutore et al. [8] conducted a design and operation optimization of a residential REC in Italy to maximize the net-present value (including investment and operational costs) and, then, calculated different indexes to analyse its energetic, environmental and social performances. The centralized configuration of the REC results more profitable compared to the distributed one, because the former exploits only virtual energy sharing that is more incentivized (in the Italian context) compared to physical selfconsumption (the distributed configuration can exploit both virtual energy sharing and physical selfconsumption). Chang et al. [9] proposed a methodology, based on K-means clustering, to allocate different options of community energy storage among households of an EC, the operational cost of which is minimized by a Mixed-Integer Linear Programming (MILP) optimization. Lazzari et al. [10] developed an optimization framework for the planning and operation of RECs with the aim of increasing their energy sharing. The optimization outcomes, validated with real data of consumption profiles of households in Barcelona, show that a REC with 7 residential users could achieve a self-consumption of 100% and a relevant amount of avoided CO2 emissions till 7 kg/day.

Another relevant issue to be addressed is the distribution of economic benefits (costs or profits) within an EC [11-14]. Casalicchio et al. [11] allocated the total cost saving of a residential EC by applying the Vickrey-Clarke-Groves mechanism that evaluates the contribution of each member to the cost saving of the whole community. This approach is compared to other business allocation criteria by introducing a fairness index that represents the number of members without an economic gain within the community. Zheng et al. [12] analysed the impacts of demand side flexibility and of a Peer-to-Peer (P2P) energy sharing mechanism within an EC, made of commercial prosumers, finding that the P2P strategy could minimize the operational cost by 24.6% compared to a Peer-to-Grid (P2G) operation where each member exchanges energy with the electric grid only. The profit of the EC is distributed among members by the P2P mechanism, which relies on the definition of internal trading prices that depend on the supply-demand ratio of the community. Zatti et al. [13] optimized the capacities of different generation and storage units within an Italian EC comprised of commercial and residential users. The "Shapley value" mechanism, based on a cooperative game theory approach, is applied to allocate the total benefit of the community by quantifying the contributions of members to the economic revenue of any coalition within the community. Siqin et al. [15] formulated a distributionally-robust optimization for the dispatching of a multi-EC system. An improved Shapley value approach was proposed to allocate higher profits to the participants with higher solar PV consumption. Notice that, in the previous papers, a single cost/profit allocation mechanism (e.g., Shapley value) was applied, neglecting the potential results with other allocation criteria.

Vespermann et al. [16] explored different cooperative allocation mechanisms for an EC, including the Shapley value, and analysed their properties of efficiency, individual rationality and stability. Cremers et al. [14] reviewed different applications of the Shapley value for ECs, proposing also a new method (called "Stratified expected value") to approximate the calculation of the Shapley value in the case of communities of up to 200 prosumers. The authors highlighted the need to further investigate different allocation criteria, other than Shapley, to assess the fairness of the cost/profit distribution within ECs. Li and Okur [17] conducted an operation optimization of an EC and applied Uniform pricing (also called "flat energy pricing method"), time-ofuse energy pricing and segmented energy pricing as cost allocation criteria. However, the authors neglected the possible cooperation between the members of the EC and, thus, cooperative allocation mechanisms as the Shapley value were not applied. It is worth highlighting that the above works did not apply both cooperative allocation mechanisms (e.g., Shapley value) and other mechanisms not derived from cooperative game theory.

In the above literature, works dealing with the allocation of economic costs/profits within an EC *i)* apply one single criterion, without assessing the impact of different allocation mechanisms on the benefits of EC members, and/or *ii)* do not compare cooperative allocation mechanisms (e.g., Shapley value) with other mechanisms not based on cooperative game theory (e.g., Uniform pricing). To the best of authors' knowledge, the application of different allocation criteria for the economic benefit of a REC, according to the Italian framework, represents a first gap in the literature. Moreover, as highlighted in [14], the literature lacks of more in-depth discussions about the fairness of the cost/profit distribution within ECs and, in particular, within RECs. This paper fills in these gaps, and the objective is to present and evaluate two different allocation mechanisms in the context of a REC, i.e., the cooperative "Shapley value" [18] and the "Uniform pricing" criteria [19], with a subsequent insight of the fairness of allocation.

First, a REC with a centralized configuration and a virtual energy sharing scheme, according to the Italian framework, is modelled and its operation is optimized (i.e., the flexible demands of members and the operation of the electrical energy storage system) based on a Mixed-Integer Non-Linear Programming (MINLP) formulation. Subsequently, this work applies the cooperative mechanism Shapley value and the Uniform pricing mechanism to distribute the economic benefits among the members, with the aim of evaluating the differences between these two criteria.

This paper contributes to the current literature with the following novel points:

- Application and comparison of two different allocation criteria, i.e., the cooperative Shapley value and the Uniform pricing, to distribute the total economic profit of an Italian REC among its members;
- Evaluation of how fair is the distribution of economic benefits, based on the Shapley value and the Uniform pricing, within the analysed REC.

The paper is structured as follows. Section 2 describes the Shapley value and Uniform pricing mechanisms. Section 3 reports the mathematical model of the REC, according to the Italian legislation, and the input data for the MINLP optimization. Section 4 discusses the results of the MINLP optimization and subsequent application of Shapley value and Uniform pricing. Conclusions summarize general guidelines retrieved from this work.

2. Materials and method

This Section describes the two cost/profit allocation mechanisms considered in this work, i.e., the cooperative Shapley value (Section 2.1) and the Uniform pricing (Section 2.2). As reported by Berka and Creamer [20] and Gjorgievski et al. [21], the cost/profit allocation within an Energy Community (EC) is an important aspect that is still not exhaustively addressed in the literature. Different allocation mechanisms of the costs/profits of ECs are available in the literature, and they usually refer to approaches from the cooperative game theory or to approaches that define prices for the EC members in relation to their profiles of energy demand [17].

2.1. Shapley value

Game theory is a large field that encompasses techniques to model a group of interacting players [22] under a cooperative or non-cooperative framework. Cooperative games assume that cooperation exists among the players, for example the members of an EC, thus leading to a "grand coalition" of members (e.g., the energy community), with the aim of achieving a common objective (e.g., minimization of the total cost of the EC, improvement of the total renewable self-consumption or self-sufficiency, etc.). On the contrary, the game is non-cooperative when the players pursue independent and, sometimes, conflicting objectives, thus leading to a Nash equilibrium solution. However, this work assumes that the members of an EC can cooperate and, therefore, the focus here is on cooperative games. A cooperative game is defined by a pair *(N,v)*, where *N* refers to the number of players participating into the game, and forming the "grand coalition", while *v(S)* is the "value" function of any possible coalition *S* of players within the grand coalition [23]. The "value" function of a coalition *S* is a mathematical function that takes real values:

$$
v(S) \colon 2^N \to R \tag{1}
$$

where 2^N represents the set of all coalitions within the grand coalition, included the grand coalition itself and the empty coalition (i.e., the coalition without players). The "value" function represents the "value" in forming the coalition *S*, e.g., the total cost/profit associated with players cooperating in the coalition *S*. In the framework of ECs, notice that the value of the grand coalition corresponds to the total cost/profit of an EC, while the value of the empty coalition is zero. To clarify, consider an EC with three members (players *1,2,3*) that cooperate to minimize the total operational cost. The value of the coalition *{1,2}*, given by the players *1* and *2*, is the total cost of the cooperation between these members, thus neglecting player *3*.

This work adopts the Shapley value as cooperative allocation mechanism because it can lead to a fair distribution of economic gains in a cooperative game. Indeed, this approach allows to allocate higher gains (e.g., higher profits or lower costs) to players that contribute the most to the "value" of all coalitions they could take part in within the grand coalition. According to the Shapley value, the gain, or payoff x_i , allocated to member *i* is calculated as follows:

$$
x_i = \sum_{S \subseteq N, i \in S} \frac{(|S| - 1)!(N - |S|)!}{N!} \big(v(S) - v(S \setminus \{i\}) \big) \tag{2}
$$

where $|S|$ refers to the size of the coalition *S* and $(v(S) - v(S \setminus \{i\}))$ is the contribution of member *i* to the value $v(S)$ of coalition *S*. Hence, the Shapley value assigns to each player a payoff that represents his weighted average marginal contribution to the value of any coalition, within the grand coalition, he could take part in. However, the main drawback of the Shapley value, which requires to consider 2^N coalitions, is the computational complexity arising with a high number of players *N*. In the following, a basic example of the Shapley calculation is provided. Let us consider a grand coalition with three players, assuming the values of the 8 (2^N) possible coalitions, where each of these values could be derived by solving a specific cooperative optimization problem with the players involved. Table 1 reports the coalitions analysed and their corresponding assumed "values", where {} represents the empty coalition.

Table 1. Example of a grand coalition with three players, with the "values" of all coalitions.

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r m Value ∼							\sim . <u>.</u>

Eq. (2) is applied to compute the Shapley payoff for each of the three players. Player 1 appears in coalitions {1}, {1,2}, {1,3} and {1,2,3} and, therefore, its payoff considers its weighted average marginal contribution to the values of these four coalitions:

$$
x_1 = \frac{(1-1)!(3-1)!}{3!}(v({1}) - v({1}) + \frac{(2-1)!(3-2)!}{3!}(v({1},2)) - v({2}))
$$

+
$$
\frac{(2-1)!(3-2)!}{3!}(v({1},3)) - v({3})) + \frac{(3-1)!(3-3)!}{3!}(v({1},2,3)) - v({2},3))
$$

=
$$
\frac{1}{3}(0) + \frac{1}{6}(3-2) + \frac{1}{6}(5-3) + \frac{1}{3}(12-6) = \frac{15}{6}
$$

Player 2 can participate in coalitions $\{2\}$, $\{1,2\}$, $\{2,3\}$ and $\{1,2,3\}$, and its payoff is:

$$
x_2 = \frac{(1-1)!(3-1)!}{3!}(\nu({2}) - \nu({3})) + \frac{(2-1)!(3-2)!}{3!}(\nu({1,2}) - \nu({1}))
$$

+
$$
\frac{(2-1)!(3-2)!}{3!}(\nu({2,3}) - \nu({3})) + \frac{(3-1)!(3-3)!}{3!}(\nu({1,2,3}) - \nu({1,3}))
$$

=
$$
\frac{1}{3}(2) + \frac{1}{6}(3-0) + \frac{1}{6}(6-3) + \frac{1}{3}(12-5) = 4
$$

Player 3 can participate in coalitions $\{3\}$, $\{1,3\}$, $\{2,3\}$ and $\{1,2,3\}$, and its payoff is:

$$
x_3 = \frac{(1-1)!(3-1)!}{3!}(\nu({3}) - \nu({0}) + \frac{(2-1)!(3-2)!}{3!}(\nu({1,3}) - \nu({1}))
$$

+
$$
\frac{(2-1)!(3-2)!}{3!}(\nu({2,3}) - \nu({2})) + \frac{(3-1)!(3-3)!}{3!}(\nu({1,2,3}) - \nu({1,2}))
$$

=
$$
\frac{1}{3}(3) + \frac{1}{6}(5-0) + \frac{1}{6}(6-2) + \frac{1}{3}(12-3) = \frac{11}{2}
$$

According to cooperative game theory, this payoff allocation is defined as an "imputation" because it guarantees simultaneously the properties of "efficiency" and "individual rationality". The property of efficiency ensures that the "value" of the grand coalition is allocated to all players, while the property of individual rationality means that each player benefits from participating into the grand coalition compared to operating as an independent player. In the example above, efficiency is ensured since the sum of payoffs is equal to the "value" of the grand coalition (i.e., 12 €). Moreover, the payoff allocation satisfies the property of individual rationality because the payoff allocated to each player is higher than the "value" of the coalition including only that player (e.g., for player 2, the payoff in the grand coalition is $4 \in \text{while its value } v({2})$, as an independent player, is $2 \notin$).

2.2. Uniform pricing

Time-of-use energy pricing, segmented energy pricing and Uniform pricing (also called "flat energy pricing method") are cost/profit allocation mechanisms, reported by Li et al. [19] and Li and Okur [17], that do not consider the cooperation among EC members. These mechanisms allow to allocate the economic benefits of the EC by defining different prices in relation to the individual energy demands. Time-of-use energy pricing sets high prices in peak hours (i.e., when peak demand occurs) and low prices in off-peak hours (i.e., when low demand occurs), thus promoting load shifting of the energy demands from peak to off-peak hours. Segmented energy pricing establishes a threshold of the energy demand, under which a low price is charged, while a high price is charged above this threshold. Uniform pricing is based on a unique price calculated as the ratio between the total cost (or profit) and the total demand of the EC:

$$
c_u = \frac{TC}{\sum_{i=1}^n \sum_{t=1}^T E_{i,t}}\tag{3}
$$

where TC is the total cost (or profit) of the EC and E_i , is the hourly energy demand of member *i*. Uniform pricing mechanism is simple to be implemented and well accepted by network regulators [24].

3. Case study

The Renewable Energy Community (REC) under analysis comprises of three members, one consumer of the tertiary sector and two prosumers that are representative of the residential and commercial sectors. It is assumed that the residential prosumer owns only a Photovoltaic plant (PV), whereas the commercial prosumer owns a PV plant and a battery as electrical storage system. Section 3.1 reports the main equations and constraints associated with the Mixed-Integer Non-Linear Programming (MINLP) optimization model of the Italian REC, developed from a previous work of the authors [25]. Section 3.2 lists the input data of the model.

3.1. Mathematical model of the Renewable Energy Community

The operation of the PV plant of a member *i* is described by Eq. (4):

$$
P_{i,t}^{PV} = \eta^{PV} \cdot A_i^{PV} \cdot I_t^{PV}
$$
\n⁽⁴⁾

where $P_{i,t}^{PV}$ [kW] is the power generated by the PV plant in time step t , η^{PV} [-] is the average efficiency of the PV plant, A^{PV}_i [m²] is the available PV area and I^{PV}_t [kW/m²] is the global solar irradiance (on a tilted surface) in time step *t*. In the following, energy and power variables have, respectively, [kWh] and [kW] as units of measurements.

The operation of the battery of a member *i* is given by the following equations and constraints:

$$
E_{i,t}^{bat} = E_{i,t-1}^{bat} + P_{i,t}^{bat,c} \cdot \eta^{bat,c} \cdot \Delta t - \frac{P_{i,t}^{bat,d} \cdot \Delta t}{\eta^{bat,d}}
$$
(5)

$$
E_{i,t}^{bat} \le E_i^{bat,cap} \tag{6}
$$

$$
P_{i,t}^{bat,c} \le P_i^{bat,peak} \cdot \delta_{i,t}^{bat,c} \tag{7}
$$

$$
P_{i,t}^{bat,d} \le P_i^{bat,peak} \cdot \delta_{i,t}^{bat,d} \tag{8}
$$

$$
\delta_{i,t}^{bat,c} + \delta_{i,t}^{bat,d} \le 1 \tag{9}
$$

$$
E_{i,1}^{bat} = E^{start, end} \cdot E_i^{bat, cap} \tag{10}
$$

$$
E_{i,24}^{bat} = E_{i,1}^{bat} \tag{11}
$$

Eq. (5) describes the energy balance of the battery, where $E_{i,t}^{bat}$, $P_{i,t}^{bat,c}$ and $P_{i,t}^{bat,d}$, $\eta^{bat,d}$ and $\eta^{bat,d}$, and Δt are, respectively, the hourly energy stored in the battery, the hourly charging (*c*) and discharging (*d*) power, the battery charging (*c*) and discharging (*d*) efficiency, and the time step of the MINLP optimization model, i.e., one hour. Constraint (6) states that the hourly energy stored in the battery is bounded by the battery capacity $E^{bat,cap}_i$. Constraints (7)-(8) bound the hourly charging and discharging power of the battery by the maximum value $P_i^{bat,peak}$. The binary variables $\delta_{i,t}^{bat,c}$ and $\delta_{i,t}^{bat,d}$, which make the model non-convex, avoid charging and discharging in the same time step *t*, as indicated by constraint (9). The initial and final energy levels of the battery ($E_{i,1}^{bat}$ and $E_{i,24}^{bat}$) are fixed as ratio of the battery capacity by parameter $E^{start,end}$, as shown in Eqs. (10)-(11).

The shifted electrical demand of each member *i* is subject to the following constraints:

$$
\sum_{t=1}^{24} E_{i,t}^{el,shift} = \sum_{t=1}^{24} E_{i,t}^{el}
$$
 (12)

$$
E_{i}^{el,min} \le E_{i,t}^{el,shift} \le E_{i}^{el,max} \tag{13}
$$

$$
(1 - D^{var}) \cdot E_{i,t}^{el} \le E_{i,t}^{el,shift} \le (1 + D^{var}) \cdot E_{i,t}^{el}
$$
\n
$$
(14)
$$

Eq. (12) states that shifting the hourly electrical demand of a member does not change its total daily electrical demand, where $E_{i,t}^{el,shift}$ and $E_{i,t}^{el}$ are, respectively, the shifted and input electrical demands in time step t . Constraints (13) and (14) bound $E_{i,t}^{el,shift}$, where $E_i^{el,min}$, $E_i^{el,max}$ and D^{var} are the hourly minimum and maximum of the input electrical demand and the hourly maximum fraction of the load that can be shifted.

According to the Italian legislation, the members of the centralized REC are directly and separately connected to the same low-medium voltage distribution grid. The energy shared among the members occurs within the distribution grid, in conformity with the concept of "virtual energy sharing" of the Italian REC*.* The legislation defines two hourly energy balances for the total energy withdrawn from (E_t^{imp}) and injected to (E_t^{exp}) the grid as reported by Eq. (15) and Eq. (16), respectively:

$$
E_t^{imp} = \sum_{i=1}^{n} (E_{i,t}^{el,shift} + E_{i,t}^{bat,c})
$$
 (15)

$$
E_t^{exp} = \sum_{i=1}^{i=1} (E_{i,t}^{PV} + E_{i,t}^{bat,d})
$$
 (16)

where $E^{PV}_{i,t}$, $E^{bat,c}_{i,t}$ and $E^{bat,d}_{i,t}$ are, respectively, the energy generated by PV, the energy charged and discharged of the battery, for each member *i* in each time step *t*.

The total energy withdrawn from and injected to the grid are limited by the following constraints:

$$
E_t^{imp} \le E^{grid,max} \tag{17}
$$

$$
E_t^{exp} \le E^{grid,max} \tag{18}
$$

where $E^{grid, max}$ is the maximum allowed energy exchanged with the grid.

The Italian REC benefits from an economic incentive for the "virtual energy shared", $E_{s,t}$, among its members, that is defined as the hourly minimum between E_t^{imp} and $E_t^{exp}\colon$

$$
E_{s,t} = min(E_t^{imp}, E_t^{exp})
$$
\n(19)

where constraint (19) is non-linear and, therefore, makes the model non-linear.

The REC under analysis aims at maximizing its total daily profit and, therefore, the objective function to be maximized is:

$$
c^{REC} = \sum_{t=1}^{24} (E_t^{exp} \cdot c_t^{exp} - E_t^{imp} \cdot c_t^{imp}) + inc_{REC} \cdot \sum_{t=1}^{24} E_{s,t}
$$
 (20)

where c_t^{exp} , c_t^{imp} and inc_{REC} are, respectively, the grid sale price, the grid purchase price and the incentive of the REC (that consists of the sum between a feed-in premium and a feed-in tariff, the latter linked to the avoided network losses within the REC). The first summation represents the difference between the revenue for the energy sold to the grid $(E_t^{exp} \cdot c_t^{exp})$ and the cost for the energy purchased from the grid $(E_t^{imp} \cdot c_t^{imp}).$ The last term is the revenue due to the incentive for the virtual energy shared. Notice that the optimal profit of the REC (i.e., the optimal value of the objective function c^{REC}) corresponds to the value of the grand coalition including, as players, all the members of the REC, as explained in Section 2.

3.2. Input data

Figure 1 (a) and (b) show, respectively, the profile of the global solar irradiance on an inclined surface (optimal tilted angle of 38°) for the location of Padua (Italy) derived from the PVGIS database [26], and the profiles of the grid purchase and sale prices [27], in a characteristic day of the spring season. Figure 2 reports the daily electrical demands of the energy users, within the analysed REC, that are representative of the tertiary (consumer "Ter"), residential (prosumer "Res") and commercial (prosumer "Com") sectors [28]. Given the chosen electrical demands, it is worth highlighting that the REC presents a heterogeneous composition that improves its operational flexibility. Table 2 lists the values of other input parameters for the mathematical model described in Section 3.1.

Figure 1. a) Global solar irradiance on an inclined surface (optimal tilted angle of 38°) in Padua (Italy) and b) grid purchase and sale prices, in a characteristic day of spring.

Figure 2. Electrical demands of the tertiary consumer (Ter), residential (Res) and commercial (Com) prosumers.

Parameter	Value	Source
η^{PV} [-]	0.17	assumed
A_i^{PV} [m ²]	60 (Res), 250 (Com)	assumed
$\eta^{bat,c}$, $\eta^{bat,d}$ [-]	0.95	[9]
$E_i^{bat,cap}$ [kWh]	50 (Com)	$[29]$
$P_i^{bat,peak}$ [kW]	7 (Com)	[29]
$E^{start, end}$ $\left\lceil - \right\rceil$	0.5	assumed
D^{var} [-]	0.1	assumed
$E^{grid,max}$ [kWh]	70	assumed
<i>inc_{REC}</i> [€/kWh]	0.12	[3]

Table 2. Values of the input parameters in the MINLP optimization.

4. Analysis and discussion of results

Section 4.1 reports the results of the Mixed-Integer Non-Linear Programming (MINLP) optimization, conducted on the mathematical model described in Section 3.1, and solved by the Gurobi software [30]. Subsequently, Section 4.2 presents the outcomes of the distribution of the optimal total profit for the REC among the members, by application of the mechanisms of Shapley value and Uniform pricing.

4.1. Optimization results

Figure 3 (a) shows the optimal shifted electrical demands of the tertiary consumer ("Ter"), residential ("Res") and commercial ("Com") prosumers, as well as the total PV power generation for the two prosumers, within the REC. The optimal load shifting of the electrical demand of the commercial prosumer results more evident compared to the other optimal demands, since the commercial prosumer can exploit the electrical battery storage and, thus, has a higher flexibility compared to the other members of the REC. Figure 3 (b) exhibits the optimal operation of the electrical battery storage of the commercial prosumer within the REC, in terms of power charging (red line), power discharging (green line) and energy stored (blue line). Notice that all the shifted electrical demands increase compared to the input demands during hours 7-9 (Figure 3 (a)), when the PV power generation is still low, thus requiring the power discharging of the battery in the same period (Figure 3 (b)). In the middle of the day, during hours 10-15, the total PV power generated is high (maximum value of 39 kW at hour 12), and this allows to charge the battery till almost the maximum capacity (50 kWh) at hour 15. After hour 15 the available PV power decreases and, therefore, power discharging of the battery helps meet the total electrical demand. Figure 4 displays the daily profile of the virtual energy shared within the REC. According to constraint (19) in Section 3.1, the virtual energy shared is defined as the hourly minimum between the total energy withdrawn from the grid (i.e., sum of the total shifted electrical demand and energy charged into the battery) and the total energy injected into the grid (i.e., sum of the total PV energy generation and energy discharged from the battery). At hour 2, the virtual energy shared corresponds to the energy discharged from the battery (7 kWh) that is lower compared to the value of the total shifted electrical demand (17 kWh). At hour 12, the virtual energy shared is the maximum PV energy generated (39 kWh) that is lower compared to the sum between the values of total shifted electrical demand (37 kWh) and energy charged into the battery (7 kWh). Hence, these outcomes demonstrate that a REC with renewable energy plants (as PV) and energy storage systems can effectively exploit the energy sharing and, in turn, achieve higher economic revenues due to the incentive for the energy shared according to the Italian legislation.

Figure 3. a) Input and optimal shifted electrical demands of REC members and total PV power generation and b) optimal operation of the battery.

Figure 4. Optimal virtual energy shared in the analysed Italian REC.

4.2. Results of the profit allocation

The Mixed-Integer Non-Linear Programming (MINLP) optimization leads to an optimal daily profit of the REC (optimal value of the objective function, Eq. (20) in Section 3.1) equal to 8.35 €. This total profit is allocated to the members of the REC by applying, separately, the cooperative Shapley value and the Uniform pricing mechanisms, described in Section 2.1 and 2.2, respectively.

The calculation of the Shapley value, according to Eq. (2) in Section 2.1, requires the identification of all coalitions of members within the REC and the computation of their "values" (i.e., the total costs/profits of coalitions, see Section 2.1). In the analysed REC with three members, the 8 possible coalitions are *{}*, *{Ter}*, *{Res}*, *{Com}*, *{Ter, Res}*, *{Ter, Com}*, *{Res, Com}* and *{Ter, Res, Com}*, where the first and last coalitions are, respectively, the empty coalition and the grand coalition that constitutes the REC. *Ter*, *Res* and *Com* refer, respectively, to the tertiary consumer, the residential and commercial prosumers. Notice that the value of the empty coalition is 0, while the value of the grand coalition is the optimal daily profit of the REC (8.35 ϵ). To obtain the value of a specific coalition, it is sufficient to solve the model in Section 3.1 considering only the equations and constraints associated with the members involved in that coalition. For instance, coalition *{Ter, Res}* is made of one consumer (Ter) and one prosumer with only a PV plant (Res), thus the equations and constraints from (5) to (11) (in section 3.1) of the battery energy storage are not used in solving the optimization problem, and the variables of the battery in Eqs. (15)-(16) are not considered as well. Moreover, for coalitions *{Ter}*, *{Res}*, *{Com}*, representing the energy users operating independently, and without cooperation with other community members, the term of energy shared in the objective function (20) is not included. Table 3 reports the "values" calculated for all coalitions within the REC, where negative and positive values refer to net costs and net profits, respectively. For example, the coalition *{Ter}* includes only one consumer, leading to a daily cost of 12.85 €. On the other hand, the coalition *{Res, Com}* is based on the cooperation between the residential and commercial prosumers and, thus, presents the highest daily profit of 20.09 €. Once obtained the values of all coalitions, the formula of the Shapley value (Eq. (2) in Section 2.1) is applied to compute the Shapley payoff of each member of the REC.

The Uniform pricing mechanism is implemented by calculating a uniform price within the REC, as reported in Eq. (3) of Section 2.2, where the numerator is represented by the optimal daily profit of the REC (8.35 ϵ) and the denominator is the optimal shifted demand of the REC. According to Uniform pricing, the payoff of each member is found by multiplying the uniform price with its optimal daily demand.

Table 4 displays the payoffs (positive for profits and negative for costs) allocated to the members of the REC under the Shapley value and Uniform pricing mechanisms. A first outcome is that both allocation mechanisms are efficient (see Section 2.1) because the sum of their payoffs is equal to the optimal daily profit of the REC (8.35ϵ) , i.e., the value of the grand coalition. Both Shapley value and Uniform pricing also quarantee the property of individual rationality and, therefore, all members of the REC have higher gains (higher profits or lower costs) within the REC compared to the case without cooperation. For instance, the tertiary, residential and commercial members have higher daily profits within the REC according to the uniform pricing allocation (profits of 1.95 ϵ , 0.69 ϵ and 5.71 ϵ , respectively, in Table 4) compared to the case in which they operate independently in separate coalitions (costs of -12.85 €, -1.03 €, -22.20 €, respectively, in Table 3). With Uniform pricing all members have daily profits, included the tertiary consumer, while the Shapley allocation shows daily profits for prosumers and a daily cost for the tertiary consumer. Hence, it seems that Uniform pricing is more economically profitable than Shapley value, at least for the tertiary consumer. However, the application of the Shapley value mechanism favours the prosumers of the REC that provide a major contribution in increasing the total daily economic profits, derived from the energy sold to the grid and the incentivised energy shared within the REC. For example, Table 4 shows that the residential prosumer receives the highest payoff, according to Shapley, equal to 9.57 €, highlighting the relevant contribution of this prosumer to the total profit of the REC. Hence, the Shapley value can be considered a fair allocation mechanism in the sense that prosumers, who promote self-consumption and virtual energy sharing within the REC, receive higher payoffs compared to consumers (in any case the latter have economic convenience to be part of the REC instead of operating independently). On the contrary, with the Uniform pricing mechanism, the resulting payoffs are more homogeneous (i.e., all members obtain daily profits) within the REC, without higher rewards to prosumers with respect to consumers.

Table 3. Coalitions of the REC and their calculated "values", which represent daily operational profits (positive) or costs (negative).

Table 4. Payoffs of all members of the REC according to Shapley value and Uniform pricing.

5. Conclusions

This paper focuses on the issue of costs/profits allocation within the emerging Renewable Energy Communities (RECs). In the current literature, most of the works apply one single allocation mechanism or does not apply and compare different allocation mechanisms that belong to the field of cooperative game theory or other fields. Contrary to the current literature, this work applies two different allocation mechanisms, i.e., the "Shapley value" based on cooperative game theory and the "Uniform pricing", with the aim of evaluating the impact of these mechanisms on the economic benefits of the members of a REC. The cooperative mechanism Shapley value allows to allocate the optimal cost/profit of the community, considering the average marginal contributions of members to the "value" (i.e., total cost or profit) of any coalition of the community in which they could participate. On the other hand, the mechanism of Uniform pricing distributes the optimal cost/profit of the REC by defining a uniform price that is the ratio of the optimal cost/profit and the total electrical demand of the community. This work considers a REC, according to the Italian framework, as case study. The analysed REC includes three members belonging to different sectors, i.e., a tertiary consumer, one residential and one commercial prosumers. The residential prosumer owns only a Photovoltaic (PV) plant (available area of 60 $m²$), whereas the commercial prosumer owns a PV plant (available area of 250 m²) and an electrical battery

storage (capacity of 50 kWh). A Mixed-Integer Non-Linear Programming (MINLP) optimization is carried out to maximize the daily operational profit of the REC under analysis, considering the Italian framework and the related economic incentive for the energy shared. The optimal daily operational profit of 8.35 ϵ of the REC is allocated to the members by applying, separately, the "Shapley value" and "Uniform pricing" mechanisms. Given the payoffs distributed by both Shapley value and Uniform pricing, the three members of the REC always find economically convenient the participation into the community (therefore, higher profits and lower costs) compared to their independent operation, which is characterized by daily costs of 12.85 €, 1.03 € and 22.20 € for the tertiary consumer, residential and commercial prosumers, respectively. Uniform pricing allows to achieve daily profits of 1.95 €, 0.69 € and 5.71 € for the tertiary consumer, residential and commercial prosumers, respectively. On the other hand, the allocation by Shapley value leads to a daily cost of 5.03 € for the tertiary consumer and to daily profits of $9.57 \in \text{and } 3.81 \in \text{for the residual and commercial prosumers}$, respectively. Eventually, the outcomes of the allocation mechanisms highlight that Uniform pricing can provide more homogeneous payoffs among the members of the REC, whereas Shapley value is fairer towards prosumers (in particular, the residential one), in the sense that "awards" the higher contributions of the prosumers to the optimal daily profit of the REC by giving them higher payoffs than those of consumers. A future research direction of this work could cover the analysis of the economic stability of the REC, strictly dependent on the costs/profits distributed by the implemented allocation mechanisms, to avoid the case of members willing to exit from the REC.

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Nomenclature

- Com Commercial
- **MINLP Mixed-Integer Non-Linear Programming**
- **REC** Renewable Energy Community
- Residential
- Ter Tertiary

Subscripts and superscripts

- bat Battery
- i Member of the REC
- PV Photovoltaic

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