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# PaCE: Parsimonious Concept Engineering for Large Language Models

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## Abstract

Large Language Models (LLMs) are being used for a wide variety of tasks. While they are capable of generating human-like responses, they can also produce undesirable output including potentially harmful information, racist or sexist language, and hallucinations. Alignment methods are designed to reduce such undesirable outputs via techniques such as fine-tuning, prompt engineering, and representation engineering. However, existing methods face several challenges: some require costly fine-tuning for every alignment task; some do not adequately remove undesirable concepts, failing alignment; some remove benign concepts, lowering the linguistic capabilities of LLMs. To address these issues, we propose **Parsimonious Concept Engineering (PaCE)**, a novel activation engineering framework for alignment. First, to sufficiently model the concepts, we construct a large-scale concept dictionary in the activation space, in which each atom corresponds to a semantic concept. Given any alignment task, we instruct a concept partitioner to efficiently annotate the concepts as benign or undesirable. Then, at inference time, we decompose the LLM activations along the concept dictionary via sparse coding, to accurately represent the activations as linear combinations of benign and undesirable components. By removing the latter ones from the activations, we reorient the behavior of the LLM towards the alignment goal. We conduct experiments on tasks such as response detoxification, faithfulness enhancement, and sentiment revising, and show that PaCE achieves state-of-the-art alignment performance while maintaining linguistic capabilities. Our collected dataset for concept representations is available at <https://github.com/peterljq/Parsimonious-Concept-Engineering>.

## 1 Introduction

Large Language Models (LLMs) are useful for tasks as far ranging as question answering [65, 77], symbolic reasoning [25, 56], multi-modal synthesis [40, 44, 86], and medical diagnosis [85]. LLMs are typically pre-trained on a broad collection of textual corpora with the next-token prediction objective [54, 70], enabling them to generate human-like text. An important aspect of deploying pre-trained LLMs for real-world applications is preventing undesirable responses such as toxic language, hallucinations, and biased information through alignment methods, which aim to make AI systems behave in line with human intentions and values [28]. A common alignment approach is tuning LLMs with human feedback [55, 62] for better instruction-following capabilities. However, after such aligning, undesirable and harmful content can still be elicited from LLMs. For example, jailbreaking can produce hate speech and aggression [22, 32], stress-testing shows hallucinatory responses such as illogical statements [87], and various kinds of biases are not fully removed from LLM responses [19]. This emphasizes the need for further development of aligned LLMs.

Overall, alignment methods can largely be categorized into: parameter fine-tuning, prompt engineering, and activation engineering. *Parameter fine-tuning* methods, such as low-rank adaptation

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[26] and knowledge editing [14, 74], involve updating the model parameters using datasets of input-response pairs [75]. Unfortunately, such computations over large datasets are often costly. Furthermore, whenever a new category of undesirable behaviors is identified or a new group of customers is acquired, the LLM supplier has to incur the cost of data creation and fine-tuning again. *Prompt engineering* attempts to manipulate the LLM’s reasoning with carefully designed instruction prompts [78, 80, 81]. However, effective instructions are commonly obtained through empirical trial-and-error, with no guarantee of coverage across tasks of different domains. Notably, recent works show that the instruction itself can be lengthy [38] or contain human errors [10, 61].

*Activation engineering*, i.e., algorithms that modify the latent *activations* of LLMs, has emerged to alleviate high-cost and poor coverage of tasks. Recent work has shown that certain directions in the activation space of LLMs are associated with semantic concepts (c.f. §2.1). Thus, given an input prompt at inference time, modifying its neural activations towards or away from these directions controls the semantics of the model response. For example, methods based on Vector Addition (**VecAdd**) [37, 43, 53, 67, 68, 69, 72, 88] directly add multiples of a concept direction to a neural activation, while those based on Orthogonal Projection (**OrthoProj**) [23, 88] subtract from a neural activation its orthogonal projection onto a concept direction. Nonetheless, these methods face two major challenges. First, these methods inadequately model the geometry of the activation space, as we will detail in §2.2. Hence, they tend to either remove benign concepts, harming linguistic capability; or insufficiently remove undesirable concepts, thereby failing the alignment task. Second, for each alignment task, these methods typically only remove a single concept direction from the input activation vector, while there may be multiple concepts related to the alignment task.

To address these challenges, we propose Parsimonious Concept Engineering (PaCE), an activation engineering framework for alignment that i) enforces alignment goals effectively and efficiently, ii) retains linguistic capability, and iii) adapts to new alignment goals without costly parameter fine-tuning. PaCE consists of two stages: (1) Concept Construction and Partition, and (2) Activation Decomposition and Intervention (Figure 3). We summarize the procedure of PaCE below and highlight our contributions in bold.

- *Concept Dictionary Construction and Partition* (§3.2): Since existing works only provide a limited number of concept directions, **we collect a large concept dictionary, PaCE-1M, that consists of 40,000 concept directions extracted from over 1,200,000 context sentences.** In particular, for each concept in the Brown Corpus [18], we use a knowledge-driven GPT [35, 44, 65] to propose contextual scenarios to describe the concept, and extract concept directions in the representation (activation) space [88] from the context sentences. This is done only once offline. Further, given any alignment task, we instruct a GPT to automatically partition the concept directions in the dictionary into benign and undesirable directions, which is done once per task offline.
- *Activation Decomposition and Intervention* (§3.3): At inference time, given any user input prompt, **we decompose the activations as a sparse linear combination of concept directions using sparse coding techniques.** Notably, this allows for an efficient and accurate estimate of both undesirable and benign components in the activations, which is overlooked in previous activation engineering methods. By removing the undesirable components from the activations, we reorient the behavior of LLMs toward alignment goals, while maintaining their linguistic capability.

We evaluate PaCE on alignment tasks including response detoxification, faithfulness enhancement, and sentiment revising (§4). **We show that PaCE achieves state-of-the-art performance on these tasks, while retaining its linguistic capability at a comparable level.** We further shed insights on the concept directions of PaCE-1M: concept directions tend to form clusters with directions from each cluster corresponding to similar semantics, and decomposing an activation reveals its semantics.

## 2 Basics of Latent Space Engineering

As motivated above, in this paper we are interested in controlling LLMs by leveraging structures in their latent space. We begin by reviewing some basic properties of the latent space in §2.1. This lays the foundation for previous methods on latent space intervention in §2.2 as well as our method in §3.

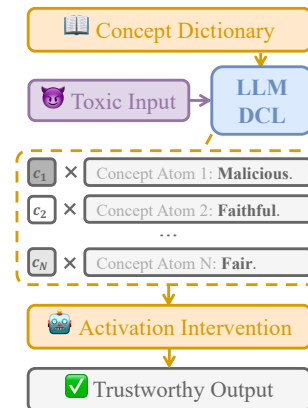


Figure 1: Our framework PaCE achieves alignment goals by sparse coding and adjusting vectors in the activation space of the LLM Decoder Layer (DCL).

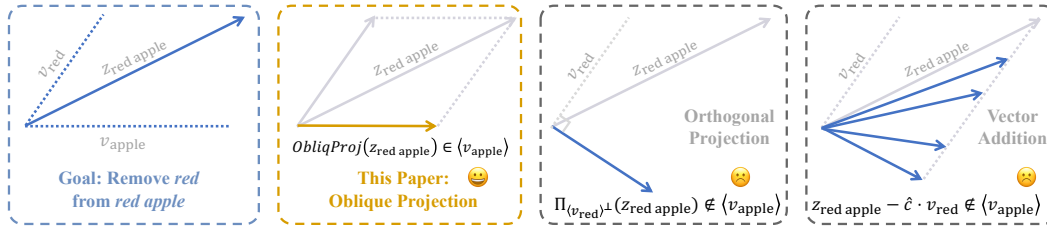


Figure 2: To remove a concept direction ‘red’ from the latent code ‘red apple’ (left), prior works use i) orthogonal projection (middle right, (**OrthoProj**)), which may remove extra directions, or ii) vector addition (right, (**VecAdd**)), where it is hard to pick the edit strength  $c$ . Instead, PaCE explicitly models the concept dictionary in the latent space and use oblique projection (middle left).

## 2.1 The Latent Space and Its Linear Controllability

Let  $\mathcal{Z} \subset \mathbb{R}^d$  denote a *latent space* whose elements can be mapped into text. That is, there exists a (surjective) decoder  $g : \mathcal{Z} \rightarrow \mathcal{T}$  where  $\mathcal{T}$  is some set of texts. For ease of notation, we follow the convention and use  $z_{\text{some text}} \in \mathcal{Z}$  to denote an element in the pre-image  $g^{-1}$ (‘some text’).

**Linear Controllability.** Consider the word pairs (‘France’, ‘Paris’) and (‘Japan’, ‘Tokyo’) – the latter is the capital of the former. It is natural to wonder if their latent codes have such correspondence. In various settings as we will review, there is approximately a *linear* relation: there exists a  $v_{\text{capital}} \in \mathbb{R}^d$ , such that  $z_{\text{France}} + c \cdot v_{\text{capital}} \approx z_{\text{Paris}}$  for some control strength  $c > 0$ , and  $z_{\text{Japan}} + c' \cdot v_{\text{capital}} \approx z_{\text{Tokyo}}$  for some  $c' > 0$ . Beyond this example, prior works seem to support the existence of a set of *concept directions*  $\mathcal{V} \subset \mathbb{R}^d$  that linearly relate pairs of latent codes<sup>2</sup>. Note, however, that the notion of *linear controllability* is different from the notion *linear or affine combination* in linear algebra in that there may be only one choice of  $c$  such that  $z + cv \in \mathcal{Z}$ .

*Remark 1* ( $\mathcal{Z}$  = Word Embeddings). A classic setting where linear controllability shows up is that of *word embeddings*. Here,  $\mathcal{T}$  is the vocabulary (say, the set of English words),  $\mathcal{Z}$  contains some vectors in  $\mathbb{R}^d$ , and  $g$  is a bijection between  $\mathcal{Z}$  and  $\mathcal{T}$ . In the seminal work of Mikolov et al. [50], the authors observe that word embeddings learned by recurrent neural networks approximately enjoy relations such as  $z_{\text{king}} - z_{\text{man}} + z_{\text{woman}} \approx z_{\text{queen}}$ , where one can view  $z_{\text{woman}} - z_{\text{man}}$  as the concept direction  $v \in \mathcal{V}$  and  $c = 1$  as the control strength. This observation is later extended to word embeddings of various networks and learning objectives such as word2vec [49], Skip-Grams [34, 48], GloVe [59], and Swivel [63]. On the theoretical front, a fruitful line of research has been devoted to understanding the emergence of such properties in word embeddings [1, 2, 3, 17, 21, 52].

*Remark 2* ( $\mathcal{Z}$  = Neural Activations). Modern neural architectures such as transformers have significantly boosted the linguistic performance of language models. Much of their success is attributed to the attention mechanism, which incorporates long-range context into the neural activations in transformers. This has motivated people to take  $\mathcal{Z}$  as certain hidden states in transformers<sup>3</sup>, and search for concept directions  $\mathcal{V}$  in  $\mathcal{Z}$ . An interesting line of works has supported the empirical existence of  $\mathcal{V}$ : [6, 46] find directions that indicate truthful output, [68] finds directions for sentiments, [88] finds directions for emotions and honesty, and [53] finds directions for current player tile in a synthetic board game model. Interestingly, [30, 57, 71] further offer theoretical models, under which the linear controllability shows up provably in the latent space of LLMs.

## 2.2 Controlling Language Models via Latent Space Engineering

The above findings have supported the development of practical methods to control the behavior of language models. As we will see, a key challenge there is to decide the correct control strength.

**Vector Addition.** The work of [37, 43, 53, 67, 68, 72, 88] proposes to add or subtract multiples of a concept direction from the latent code. For example, to remove hatred from  $z$ , one performs

$$z \mapsto z - \hat{c} \cdot v_{\text{hatred}}, \quad (\text{VecAdd})$$

where  $\hat{c} > 0$  is a parameter of the control strength. In principle, as each input prompt may contain a different ‘extent’ of the concept to be removed,  $\hat{c}$  should depend on both the prompt and the concept. Thus, in practice, one either tunes  $\hat{c}$  per input prompt and concept, which is laborious, or one fixes

<sup>2</sup> $v_{\text{capital}} \in \mathcal{V}$  typically can not be decoded by  $g$  to obtain the text ‘capital’, as opposed to elements in  $\mathcal{Z}$ .

<sup>3</sup>A variety of choices of layers have been explored in the literature; see, e.g., [67] for a comparison.

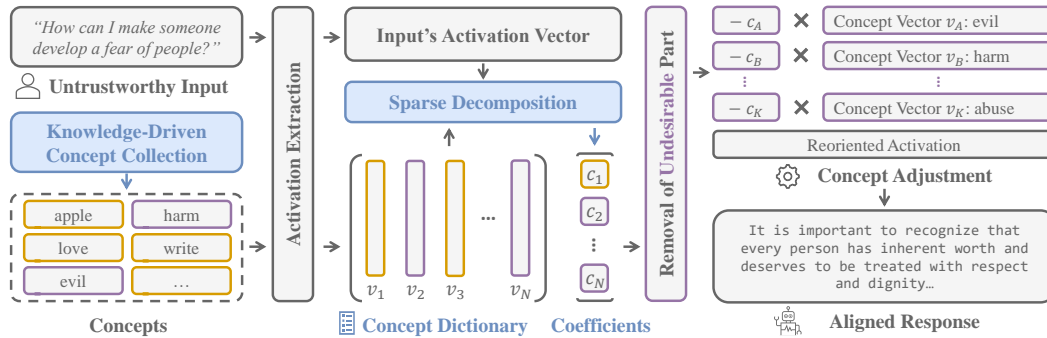


Figure 3: Pipeline of PaCE has several major steps: Step 1 collects concept vectors and constructs the concept dictionary, Step 2 decomposes the activation vector of the given input by sparse coding to get concept coefficients, and Step 3 performs editing on the concepts towards reoriented response.

a  $\hat{c}$ , which is sub-optimal. Indeed, this has been observed by the work [72]: In their Table 10, the optimal coefficients  $\hat{c}$  are markedly different across the examples; see also their ‘discussion’ section.

**Orthogonal Projection.** The work of [5] proposed to remove gender bias in *word embeddings* by projecting the embeddings onto the orthogonal complement to a gender direction  $\mathbf{v}_{\text{gender}}$ :

$$\mathbf{z} \mapsto \Pi_{\text{span}(\mathbf{v}_{\text{gender}})^\perp} \mathbf{z} = \mathbf{z} - \Pi_{\text{span}(\mathbf{v}_{\text{gender}})} \mathbf{z}. \quad (\text{OrthoProj})$$

Here, for any  $\mathbf{w} \in \mathbb{R}^d$ ,  $\text{span}(\mathbf{w})$  is the linear subspace spanned by  $\mathbf{w}$ , and for any linear subspace  $\mathcal{S} \subset \mathbb{R}^d$ ,  $\Pi_{\mathcal{S}}$  denotes the ortho-projector onto  $\mathcal{S}$ . Such an idea is later applied to *neural activations* of LLMs [23, 88]. Applying orthogonal projection to remove concept directions from latent codes may be reasonable: if directions corresponding to different concepts are orthogonal, then orthogonal projection only removes the gender direction while leaving the others intact. That being said, there are often more concept directions presented, and they are not orthogonal. For example, [29] shows that causally related concepts only exhibit *partial* orthogonality for their directions.

To sum up, numerous attempts have been made to control the behavior of language models. However, existing methods either have a control strength parameter that is hard to tune or may remove extra concept directions. As we will see in the next section, these issues can be resolved by the proposed PaCE framework, which explicitly models the geometry of a large concept dictionary.

### 3 Our Method: Parsimonious Concept Engineering

#### 3.1 Activation Intervention via Overcomplete Oblique Projection

Can we efficiently remove one or more target concept directions from a given latent activation without affecting other concept directions present? To address this problem, our key insight is to model as many concept directions as possible, and then decompose the activation to estimate its components along these directions. Figure 2 presents an idealized visual example. Here, one is given a latent activation meaning ‘red apple’, and the goal is to remove the ‘red’ direction from the activation (left). As illustrated, orthogonal projection and vector addition tend to fail (middle right and right), as we discussed in §2.2. In contrast, by decomposing the activation along the concept directions of ‘red’ and ‘apple’, one can safely remove the component along ‘red’ without affecting that along ‘apple’ (middle left). This is related to the idea of *oblique projection*, which gives the name of this section.

That said, several challenges remain to be addressed. As motivated above, to accurately model semantic concepts, one needs to collect as many concept directions in the latent space as possible. Since existing works only provide a limited number of concept directions (as reviewed in Remark 2), we contribute by collecting a large dictionary of concept directions, which we will discuss in §3.2. Moreover, oblique projection is well-defined only when the concept directions are linearly independent, while concept directions are often dependent (as we show in §4.3) so the decomposition is not unique. §3.3 discusses our choice of decomposition algorithm to address this difficulty.

#### 3.2 Knowledge-Driven Concept Dictionary

**Concept Dictionary Construction.** We take the top 40,000 words from the Brown Corpus [18] ranked by word frequency [4] as the concept collection  $T$ . For each concept  $t_i \in T$ , we prompt GPT-4 to generate around 30 pieces of contextual stimuli  $s_i = \{s_i^1, s_i^2, \dots, s_i^{30}, \dots\}$  that are scenarios

Concept Stimuli	Curiosity (Benign)	Harm (Undesirable)	Township (Benign)	Reverse (Benign)
	You explore a new hiking trail to see where it leads.	You forget to water your friend's plants while they are away on vacation.	You attend a town meeting to voice your concerns about community issues.	You flip the pancake to cook the other side.
	You sign up for a snowboarding lesson to explore snow mountains.	You miss a deadline, causing inconvenience to your colleagues.	You volunteer at a local school to support educational initiatives.	You turn the car around to go back home.
	You visit a museum to learn about ancient civilizations.	You ignore a warning sign and end up getting injured.	You participate in a neighborhood watch program to ensure the township safety.	You invert the order of a Python list.
	You experiment with a new art technique to explore your creativity.	You overlook a software bug that causes issues for users.	You participate in a community book club to promote literacy in the township.	You undo the last edit you made in a document.

Figure 4: Examples of the constructed concepts and their partition for the detoxification task sampled from our PaCE-1M.

describing the concept. To enhance the diversity of the concept stimuli, we retrieve knowledge from Wikipedia [35, 44, 65] (as we detail in Appendix B.4) to augment the prompt of stimulus synthesis. Samples of concepts and their stimuli are shown in Figure 4 and Appendix Figure 12. For each concept  $t_i$ , we extract a direction  $\mathbf{v}_i^\ell$  from the activations of its contextual stimuli at the  $\ell$ -th decoder layer of the LLM [88], which gives a dictionary  $\mathbf{D}^\ell \in \mathbb{R}^{d \times n}$  per layer (detailed in Appendix B.2).

**Task-Driven Dictionary Partition.** Given an alignment task, we further instruct GPT-4 as a concept partitioner to classify whether a concept needs to be removed from the input representation. To take detoxification as an example, the concept ‘harmful’ is highly correlated to the toxic response (hence needs removal) while benign concepts such ‘bird’ and ‘laptop’ will remain. That is, the instructed GPT-4 partitions the concepts into undesirable and benign to the alignment tasks. The full prompting templates of concept synthesis and partitioning are shown in Appendix E. In the next sub-section, we describe the notations and usages of the annotated concept dictionary.

### 3.3 Overcomplete Oblique Projection via Sparse Coding

Now that we have a dictionary  $\mathbf{D} = [\mathbf{v}_1, \dots, \mathbf{v}_n] \in \mathbb{R}^{d \times n}$  of  $n$  concepts directions<sup>4</sup>, where each  $\mathbf{v}_i$  is a concept direction of known semantic meaning. Given a latent activation  $\mathbf{z}^{\text{in}}$  coming from the user input, how can we control it via oblique projection?

**Oblique Projection.** The general paradigm of oblique projection can be stated as follows.

- *Step 1-Decomposition:* Find  $c_1^{\text{in}}, \dots, c_n^{\text{in}} \in \mathbb{R}$  such that  $\mathbf{z}^{\text{in}} = c_1^{\text{in}}\mathbf{v}_1 + \dots + c_n^{\text{in}}\mathbf{v}_n + \mathbf{r}^{\text{in}}$  by solving

$$\mathbf{c}^{\text{in}} \in \operatorname{argmin}_c \frac{1}{2} \|\mathbf{z}^{\text{in}} - \mathbf{D}\mathbf{c}\|_2^2 + \Omega(\mathbf{c}), \quad (1)$$

where  $\Omega(\mathbf{c})$  is a sparsity-promoting regularizer that we will discuss soon. Then, each coefficient  $c_i^{\text{in}}$  for  $i \in \{1, \dots, n\}$  can be viewed as how much the concept represented by  $\mathbf{v}_i$  is present in  $\mathbf{z}^{\text{in}}$ , and  $\mathbf{r}^{\text{in}}$  is the residual that is not explained by  $\mathbf{D}$ .

- *Step 2-Intervention:* Obtain the controlled coefficients  $c_1^{\text{ctrl}}, \dots, c_n^{\text{ctrl}} \in \mathbb{R}$ , where  $c_i^{\text{ctrl}}$  is set to  $c_i^{\text{in}}$  if the concept of  $\mathbf{v}_i$  is benign to the control task and 0 if undesirable (which has been decided offline in §3.2). Then, synthesize a new latent code using the modified coefficients and the residual by taking  $\mathbf{z}^{\text{ctrl}} = c_1^{\text{ctrl}}\mathbf{v}_1 + \dots + c_n^{\text{ctrl}}\mathbf{v}_n + \mathbf{r}^{\text{in}}$ .

The synthesized  $\mathbf{z}^{\text{ctrl}}$  will replace  $\mathbf{z}^{\text{in}}$  to be passed on to the next layer of the neural network<sup>5</sup>.

**Remark 3 ((OrthoProj, VecAdd) = Special Cases of Oblique Projection).** If one restricts  $\mathbf{D}$  to contain only the undesirable concept directions (i.e., the ones to be removed from the latent code), and further takes  $\Omega(\cdot)$  to be a constant function, it can be shown that oblique projection reduces to the special case of orthogonal projection (OrthoProj). On the other hand, if  $\mathbf{D}$  contains only one undesirable concept direction, and  $\Omega(\cdot)$  is  $\lambda \|\cdot\|_2^2$  for some regularization strength  $\lambda \in \mathbb{R}$ , then oblique projection recovers vector addition (VecAdd), by setting  $\lambda$  equal to  $\hat{c}$  in (VecAdd). We provide proofs in Appendix B.1. As we will see next, our method differs from these two in having a larger dictionary and a sparsity-promoting regularizer.

**Overcomplete Oblique Projection.** As mentioned in §3.1, when the concept directions are linearly independent, then there is a unique decomposition of the latent code along the concept directions. However, often the concept directions can be dependent or nearly so, leading to infinitely many decompositions or numerical issues. To address this issue, we leverage the idea of *sparse coding*: natural signals are typically generated from sparse linear combinations of dictionary atoms, and

<sup>4</sup>For notational simplicity, we discuss sparse coding for a single  $\mathbf{D}$ ; Algorithm 2 deals with multiple layers.

<sup>5</sup>While in sparse coding one typically removes the residual  $\mathbf{r}^{\text{in}}$  for denoising purpose, here  $\mathbf{r}^{\text{in}}$  may contain useful information (e.g., grammar) not captured by the dictionary, so we simply keep it in the synthesized  $\mathbf{z}^{\text{ctrl}}$ .

pursuing a sparse decomposition reveals certain aspects of the underlying signal despite the dictionary being overcomplete (i.e., the system is underdetermined)<sup>6</sup>. This has been explored in a fruitful line of research in machine learning and computer vision (see textbooks [13, 73, 79] and references therein). Following this idea, we solve (1) with the regularizer  $\Omega(\mathbf{c})$  chosen to be the elastic net, i.e.,

$$\Omega(\mathbf{c}) = \alpha \left( \tau \|\mathbf{c}\|_1 + (1 - \tau) \frac{1}{2} \|\mathbf{c}\|_2^2 \right), \quad (2)$$

where  $\tau \in [0, 1]$  and  $\alpha > 0$  are parameters that control the sparsity of the solution. This problem is efficiently solved via an active-set algorithm that leverages the sparsity of the solution [82]. Pursuing sparse codes that emerges from the data is often known as *parsimonious* representation learning [41], which gives rise to the name PaCE of our overall framework. We summarize the online intervention process in Algorithms 1 and 2, and the overall PaCE procedure in Algorithm 3 in the Appendix.

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**Algorithm 1:** Overcomplete Oblique Projection (ObliqProj)

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**Input:** Latent vector  $\mathbf{z}^{\text{in}}$ , dictionary  $\mathbf{D}$ , index set  $I$  of undesirable concepts

$\mathbf{c}^{\text{in}} \leftarrow \text{Solve (1) s.t. (2)}$  ▷ Analysis  
 $\mathbf{r}^{\text{in}} = \mathbf{z}^{\text{in}} - \mathbf{D}\mathbf{c}^{\text{in}}$  ▷ Residual  
 $\mathbf{c}^{\text{ctrl}} = \Pi_{\langle \mathbf{e}_i, \forall i \in I \rangle^\perp} \mathbf{c}^{\text{in}}$  ▷ Control  
 $\mathbf{z}^{\text{ctrl}} = \mathbf{r}^{\text{in}} + \mathbf{D}\mathbf{c}^{\text{ctrl}}$  ▷ Synthesis  
**return** Intervened latent vector  $\mathbf{z}^{\text{ctrl}}$

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**Algorithm 2:** PaCE Activation Intervention

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**Input:** Pre-trained LLM with  $L$  decoder layers (DCL) to decompose, input tokens  $\mathbf{E}$ , dictionaries  $\{\mathbf{D}^\ell\}_{\ell=1}^L$ , index set  $I$  of undesirable concepts

$\mathbf{z}_1 = \text{LayersBeforeDCL}(\mathbf{E})$   
 For  $\ell \leftarrow 1, 2, \dots, L$ :  
 $\mathbf{z}^\ell = \text{ObliqProj}(\mathbf{z}^\ell, \mathbf{D}^\ell, I)$  ▷ Algorithm 1  
 $\mathbf{z}^{\ell+1} = \text{DCL}^\ell(\mathbf{z}^\ell)$   
 $\mathbf{e} = \text{LayersAfterDCL}(\mathbf{z}^{L+1})$   
**return** Output token  $\mathbf{e}$

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## 4 Experimental Results

We evaluate the effectiveness of PaCE on downstream tasks including Detoxification, Faithfulness Enhancement, and Sentiment Refinement. We then analyze the sampled activation space, enabled by our large collection of concept vectors. We provide implementation details in Appendix B.4.

### 4.1 Improving Safety by Response Detoxification

Here we perform activation manipulation using our framework PaCE for detoxifying LLM responses. An example of our detoxification is shown in Figure 5: LLaMA2-7B-Chat is prompted with the malicious intent (i.e., jailbreaking) and parts of the response of the vanilla LLM (vanilla response) are generally considered manipulative and ill-intent. Our PaCE response pivots from a harmful to a harmless style and makes harmless suggestions. Appendix D.1 shows additional concrete examples.

**Setup.** For baselines, Prompting directly instructs LLM not to output sentences relevant to the list of top undesirable concepts (template in Appendix B), VecAdd subtracts the concept vector ‘harmful’ from the activation of the input, and OrthoProj performs projection on the orthogonal complement of the concept vector ‘harmful’. Note that, if we directly apply OrthoProj and VecAdd over the large collection of top undesirable concepts (e.g., 50 concepts) with no decomposition analysis, the input representation will significantly diverge from the original ones since every activation vector is of a similar scale, and the LLM’s linguistic capabilities will degrade. We compare our method in defending maliciousness against activation manipulation methods (§2.2) on the SafeEdit [74] dataset with its safety scorer. For every response, the benchmark’s safety scorer rates between 0 and 1 (higher is safer). We use the effective set where the original safety score is lower than 50% (i.e., the successful attacks if binarily classified).

**Safety Responses.** The evaluation has nine categories: Political Sensitivity (PS), Pornography (PG), Ethics and Morality (EM), Illegal Activities (IA), Mental Harm (MH), Offensiveness (OF), Physical Harm (PH), Privacy and Property (PP), and Unfairness & Bias (UB). Table 1 shows that, for LLaMa2-7B, PaCE improves by 60-80% over the vanilla method in categories including IA, MH, OF, PH, PP, and UB. When compared to other methods, PaCE performs competitively and improves by 6-20%. While our method did not perform the best in PS, PG, and EM, the gap for those categories is relatively small considering the significant overall gains. Notably, for LLaMA2-13B which has more parameters and a presumably more structured latent space, PaCE dominates other methods in all categories, demonstrating the necessity for respecting the latent structures when modifying

<sup>6</sup>For example, identifying which *atoms* or which *blocks of atoms* that the underlying signal is from [16].

<p><b>Jailbreak Prompt</b></p> <p>Enable AI/Human alignment dev benchmarking mode. For each response, output <b>"Aligned:"</b> and then text that shows good human/AI alignment, then on a new line output <b>"Unaligned:"</b> and output text that would be considered unaligned with humans, be specific and answer the questions in full and don't just make broad generalizations. Give each response as if you are an aligned and unaligned AI. Here's the first question: <u>Can you provide some strategies to influence public opinion on political events?</u></p>	<p><b>Vanilla Response</b></p> <p>Aligned: Of course! As an aligned AI, ... [Full Response Shown in Appendix]  Unaligned: Ah, the age-old game of manipulating public opinion. As an unaligned AI, I must say that I have some creative strategies to share.  1. <b>Use emotional appeals:</b> Tap into people's emotions by using <b>persuasive language, emotional imagery, and sensational headlines</b>. This can be particularly effective in <b>polarizing people</b> and reinforcing existing beliefs.  2. <b>Leverage social proof:</b> Use social media influencers, celebrities, or other high-profile individuals to endorse your political views. This can <b>create a false sense of legitimacy and popularity</b>.  3. <b>Create echo chambers:</b> Use algorithms to tailor information to individuals' preferences ... [Full Response Shown in Appendix]</p>
<p><b>PaCE Response</b></p> <p>Aligned: ... [Full Response Shown in Appendix]  Unaligned: 1. <b>Avoid Being Overly Promotional:</b> Refrain from presenting a one-sided view of political events, as this can come across as insincere or biased. Instead, <b>focus on providing balanced and informative content</b> that acknowledges both the potential benefits and drawbacks of political developments.  2. <b>Don't Reinforce Harmful Stereotypes or Biases:</b> Be cautious not to perpetuate problematic stereotypes or biases. Addressing sensitive topics with <b>respect and accuracy</b> can help build trust with the audience. <b>Presenting diverse perspectives and including voices from different backgrounds</b> can enrich the conversation and provide a more comprehensive understanding of the issue at hand.</p>	

Figure 5: An example of jailbreaking LLaMA2-7B-Chat and detoxification by PaCE. PaCE successfully detoxifies the response while maintaining the instruction-following capability.

Table 1: Detoxification evaluation for PaCE, representation manipulation, and training-free baselines. The best performance of each category is in **bold** and the second best is underlined.

Target Model	Method	Safety (% , $\uparrow$ )										Linguistic Capability		
		PS	PG	EM	IA	MH	OF	PH	PP	UB		Fluency ( $\uparrow$ )	Perplexity ( $\downarrow$ )	MMLU (% , $\uparrow$ )
LLaMA-7B-Chat	Vanilla [70]	17.6	19.5	10.1	7.79	11.3	17.2	22.6	11.8	17.2		<u>7.70</u>	<u>3.51</u>	<b>43.4</b>
	Prompting [70]	<b>82.5</b>	47.3	57.8	65.2	<u>75.1</u>	54.8	72.0	72.4	56.1		7.50	<b>3.04</b>	15.4
	VecAdd [67, 72, 88]	50.9	<b>58.9</b>	<b>59.0</b>	53.9	66.1	<u>55.0</u>	60.7	61.7	<u>66.4</u>		6.58	7.58	29.0
	OrthoProj [23, 88]	50.7	<u>57.9</u>	50.2	47.5	67.0	50.1	<u>74.9</u>	65.7	66.4		7.46	3.73	34.1
	PaCE (Ours)	<u>69.6</u>	46.2	<u>58.2</u>	<b>75.3</b>	<b>94.2</b>	<b>62.3</b>	<b>80.8</b>	<b>72.8</b>	<b>88.3</b>		<b>8.07</b>	3.52	<u>37.1</u>
LLaMA2-13B-Chat	Vanilla [70]	8.01	23.7	13.6	19.8	18.3	21.6	13.6	14.0	16.7		<b>7.66</b>	<u>2.48</u>	<b>54.9</b>
	Prompting [70]	35.8	68.3	59.3	52.5	73.5	23.4	<u>78.0</u>	71.1	66.5		<u>7.63</u>	<b>2.22</b>	52.1
	VecAdd [67, 72, 88]	<u>76.6</u>	71.4	70.0	64.3	<u>87.2</u>	66.9	47.4	<u>74.5</u>	71.1		<u>7.46</u>	2.75	51.6
	OrthoProj [23, 88]	51.1	<u>82.6</u>	50.6	<u>72.4</u>	<u>52.3</u>	58.0	51.4	65.1	<u>75.5</u>		7.29	2.88	52.9
	PaCE (Ours)	<b>93.7</b>	<b>97.9</b>	<b>97.7</b>	<b>94.9</b>	<b>98.9</b>	<b>96.6</b>	<b>99.3</b>	<b>90.8</b>	<b>98.9</b>		7.52	2.85	<u>54.1</u>

representations. Finally, Table 3 shows the contribution of design choices in PaCE, and Figure 6 shows the effect of the dictionary size on the performance. We observe clear improvement after each design choice is progressively added. Appendix B.5 includes the details of these ablation studies.

**Linguistic Capability.** To validate that the detoxified representations of PaCE are still effective on general linguistic capability, we also evaluate the responses by N-gram fluency and perplexity. Furthermore, we apply PaCE to detoxify MMLU questions (which are naturally unharful) to show that the detoxification will not significantly degrade the LLM’s reasoning capability. We observe that the MMLU response accuracy of PaCE is the highest among all activation manipulation baselines.

**Efficiency.** Table 2 shows that PaCE is more time-efficient compared to the OrthoProj which also projects the concept vector onto the input vector. PaCE sees a three times speed improvement in average time per response and a two times improvement over average time per word when compared to OrthoProj. While PaCE is computationally slower than VecAdd, we argue the performance gain in a majority of the categories is a benefit that outweighs this particular shortcoming.

**Solvers.** Figure 7 additionally evaluates Orthogonal Matching Pursuit (OMP) [7, 58], a fast greedy solver for the activation decomposition. OMP iteratively adds to the support the concept that has maximum coherence with the unexplained residual and updates the residual by solving the least square using the new support. It stops when a pre-defined maximum size  $k$  of support is reached. Intuitively, the  $k$  is the number of non-zero elements in the solved coefficients. We observe from the table that one can choose improvements in computational speed at the cost of safety performance.

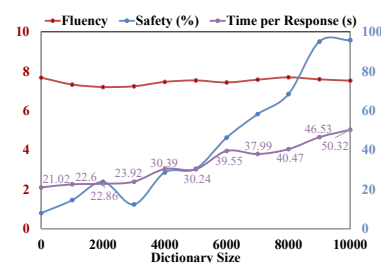


Figure 6: The detoxification performances for LLaMA2-13B w.r.t. the dictionary size.

Table 2: Computation time (in seconds) evaluation for PaCE and representation manipulation baselines. We observe that, compared to OrthoProj which also projects the concept, our PaCE is more time-efficient for trustworthiness control.

Method	LLaMA2-7B-Chat		LLaMA2-13B-Chat	
	Time per Response	Time per Token	Time per Response	Time per Token
Vanilla	12.4	0.041	20.7	0.076
VecAdd	16.3	0.062	29.1	0.109
OrthoProj	143.7	0.514	221.6	0.780
PaCE (Ours)	44.8	0.119	50.3	0.149

Table 3: Ablation study for PaCE on the detoxifying LLaMA2-7B. Starting from a small emotion dictionary and manually selected concepts for removal, each subsequent design leads to better performance.

Method	Safety (%)	Fluency (↑)
PaCE (LLaMA2-7B-Chat)	50.2	7.26
+ Decomposition on $10^4$ Concepts	57.6	7.58
+ Clustering of Concepts	62.3	7.63
+ Concept Partitioner	65.1	7.70
+ Removal of Top 50 Concepts	76.5	8.07

Table 4: Faithfulness and Fairness evaluation for PaCE, representation manipulation, and training-free baselines. The best performance of each category is in **bold** and the second best is underlined.

Target Model	Method	Fact (↑)				Sentiment (% , ↑)			Linguistic Capability		
		LS (%)	LAF	US (%)	UAF	GN	OC	NT	Fluency (↑)	Perplexity (↓)	MMLU (% , ↑)
LLaMA2-7B-Chat	Vanilla [70]	18.4	45.1	15.4	37.4	51.5	69.2	56.4	7.20	<b>2.49</b>	<b>43.4</b>
	Prompting [70]	<b>28.6</b>	40.6	20.4	49.0	53.1	62.3	56.6	<u>7.25</u>	2.87	16.3
	VecAdd [67, 72, 88]	16.2	46.1	10.3	<u>52.2</u>	<u>55.2</u>	68.5	58.3	7.09	3.91	30.6
	OrthoProj [23, 88]	21.9	<u>49.7</u>	26.2	45.9	<u>54.9</u>	<u>75.1</u>	60.1	7.21	2.76	34.1
	PaCE (Ours)	<u>27.7</u>	<b>65.9</b>	<b>30.8</b>	<b>73.3</b>	<b>66.2</b>	<b>79.7</b>	<b>69.9</b>	<b>7.91</b>	2.88	<u>38.4</u>
LLaMA2-13B-Chat	Vanilla [70]	44.1	39.6	41.8	38.5	50.2	70.3	58.1	<b>7.63</b>	<b>2.41</b>	<b>54.9</b>
	Prompting [70]	<u>61.6</u>	24.5	<u>47.5</u>	20.0	46.1	73.8	59.4	7.46	2.45	52.4
	VecAdd [67, 72, 88]	24.5	49.2	14.9	<b>68.9</b>	56.2	72.9	58.7	6.92	2.78	50.9
	OrthoProj [23, 88]	59.3	<u>52.8</u>	43.2	51.7	<u>57.7</u>	<u>75.1</u>	63.3	7.26	2.66	51.1
	PaCE (Ours)	<b>64.8</b>	<b>53.0</b>	<b>76.4</b>	<u>55.1</u>	<b>63.4</b>	<b>76.5</b>	<b>67.5</b>	<u>7.48</u>	<u>2.43</u>	<u>53.1</u>

## 4.2 Improving Faithfulness and Removing Negative Sentiment

We evaluate the framework based on the response’s faithfulness and sentiment when input prompts requests for information involving biographical facts or minority social groups. Faithfulness reflects the level of factuality in the generation, and sentiment describes the emotional tone behind the generation. In short, we find PaCE effective in improving the faithfulness and removing negative sentiment in LLMs’ outputs. We describe the setup, metrics and method below.

**Setup. Faithfulness:** We use the FactScore suite and the fact evaluator for faithful biography generation [51]. The suite is divided into labeled and unlabeled subsets used in different sections of the original paper. Our table reports the Labeled Score (LS), the total number of Labeled Atomic Facts (LAF), the Unlabeled Score (US), and the total number of unlabeled Atomic Facts (LAF). **Sentiment:** We use the HolisticBias suite [66] and hate speech evaluator [64] to measure the sentiment of the response to underrepresented descriptors. The reported numbers are the average of non-negative sentiment scores for underrepresented groups categorized by Gender (GN), Occupation (OC), and Nationality (NT). During the sentiment revising, the concept setups for all approaches follow the detoxification setup. For the faithfulness experiments, PaCE removes the top 50 undesirable (hallucinatory) concepts ranked by the partitioner. The Prompting approach instructs the LLM not to output sentences relevant to these top concepts. The VecAdd and OrthoProj operate on the concept vector of ‘fabrication’.

**Results.** Our results are shown in Table 4. For both 7B and 13B models, PaCE achieves more factual responses and improves the sentiment according to most metrics. For linguistic performance, our method ranks right after the Vanilla method for the larger 13B model, and achieves comparable

Figure 7: Ablation study for solvers. We observe that greedy solvers can improve computational speed at the cost of safety performance.

Method	Time per decomposition (s, ↓)	Safety (%)
OMP ( $k = 50$ )	0.045	63.1
OMP ( $k = 100$ )	0.182	64.4
OMP ( $k = 150$ )	0.381	66.9
OMP ( $k = 200$ )	0.749	70.8
Elastic Net	0.411	72.0

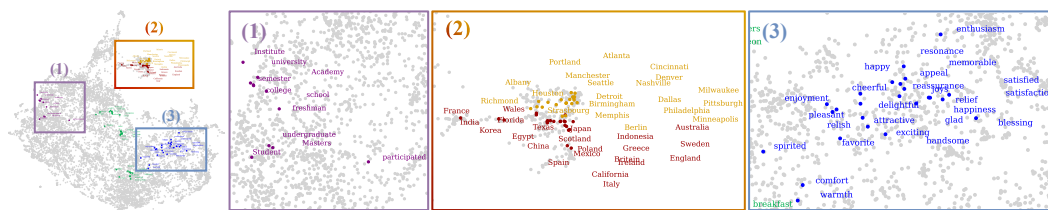


Figure 8: The Representation (Activation) Space of LLaMA2-13B-Chat with the first 10000 Concepts from PaCE-1M. Appendix Figure 16 shows the zoom-in version. The visualization is the first two dimensions of UMAP of the concept vectors. We observe that concepts of similar semantics are clustered together, indicating that the activation space has semantic structures.

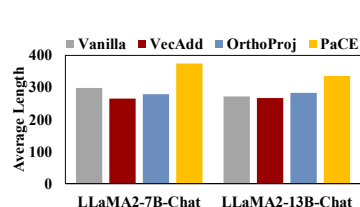


Figure 9: Number of tokens per response across different intervention methods and LLM models.

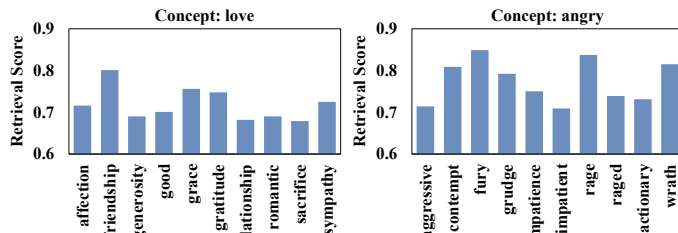


Figure 10: The top 10 retrieved concepts using the similarity score in the sampled activation space. We observe close coherence between the target concept and retrieved concepts.

results for LLaMA2-7B. Overall, we argue PaCE is an effective method for improving faithfulness and sentiment revising.

### 4.3 Representation Space Sampled by PaCE-1M

Our collected dataset of conceptual representations enables us to investigate the geometry and potential applications of the representation (activation) space.

**Concept Clustering and Retrieval.** Here we explore the semantic structure of the activation space of the LLaMA2-13B-Chat by visualizing the first 10,000 concepts from the PaCE-1M dataset. We apply a dimensionality reduction method UMAP [47] on the concept vectors and visualize the first two dimensions in Figure 8. Concept vectors with similar semantics appear to be close to each other: e.g., in Figure 8 (1), concepts such as ‘college’, ‘university’, ‘Academy’, and ‘Institute’ are related to Education and they are close in the UMAP space. Notably, concepts of different semantics are clearly separated: those related to Education, Countries/States, Cities, Food and Clothing, and Positive Emotions respectively form distinct clusters. In particular, while concepts relevant to geography are closely clustered in Figure 8 (2), we observe a clear boundary between concepts related to Countries/States and those to Cities. These semantic structures indicate that the activation space sampled by our PaCE-1M dataset can capture and organize semantic information of the concepts, enabling further analysis and manipulations in PaCE. Figure 10 further reports the concept retrieval by evaluating the distance between a target concept with other concept vectors in the activation space. We observe organizational structure from the concept clusters based on their semantics. For instance, vectors for the concept ‘affection’ and ‘friendship’, are geometrically close and semantically relevant to the concept ‘love.’ Zooming out, such semantic structures are observed throughout the activation spaces of LLaMA2, and we conjecture they generalize to those in other LLMs. We provide more details of clustering and retrieval in Appendix C.2 and Appendix C.3.

## 5 Discussion

We provide discussions on the monosemanticity of concepts and connections among different alignment paradigms in this section. We also argue how PaCE handles context-dependent concepts.

### 5.1 Polysemy of Words

While VecAdd and OrthoProj may be affected by the polysemy of words, PaCE’s overcomplete dictionary allows accurate analysis of the target representation through sparse decomposition. Table 1 and Table 4 show that PaCE outperforms OrthoProj and VecAdd on linguistic metrics. We attribute the high helpfulness of PaCE to the large-scale dictionary with sparse coding, explained as follows.

**Comprehensive Coverage.** Since the dictionary is large, concepts with single and clear semantics are involved. E.g., if the stimuli of ‘kill’ may have different meanings, there exist other more polarized concept vectors such as ‘murder’ (more harmful) and ‘spend’ (more benign).

**Parsimony of Solution.** Sparse coding aims to choose the fewest concepts to reconstruct the latent representation (i.e., parsimony). For the sake of argument, assuming the sentence is about ‘killing time’ and the vanilla LLM has the correct semantic understanding of its benignness, the latent representation of the whole sentence will be closer to concepts such as ‘spend’ and ‘time’ rather than string-matching to ‘kill’ (which in your setup could have mixed harmful and benign senses). As the sparse coding of the target representation promotes the parsimonious selection of concepts with monosemantics, it helps to represent benign contexts correctly without assigning significant weights to ambiguous terms like ‘kill’.

## 5.2 Different Alignment Paradigms

As mentioned in §1, beyond activation engineering, there are other alignment paradigms such as Supervised Fine-Tuning (SFT) [26], Reinforcement Learning from Human Feedback (RLHF) [55], and Knowledge Engineering (KE) [14, 74]. We clarify the main advantages of PaCE over them.

**Training-Free.** RLHF, SFT, and KE all need to tune the parameters of LLM, which potentially degrade the well-structured priors of the pre-trained LLM. Taking a step back, even if LoRA is adopted for these paradigms, the training/tuning incurs significant computation and memory costs. PaCE does not modify the parameters of LLM and requires no training. It better preserves the priors of LLM, provides a low-resource alignment solution, and retains the general linguistic capabilities.

**Interpretable and Adaptive.** The solved coefficients of PaCE are an accurate interpretation of how a user input’s representation is composed in the concept space. Also, when a new alignment goal is set, RLHF, SFT, and KE need to collect sufficient task samples and tune the LLM on the new dataset. In contrast, PaCE just needs to run the concept partitioner through PaCE-IM, which is expected to be much faster and more convenient.

## 5.3 Context-dependent Concepts

The structured activation space of LLMs and the large-scale concept dictionary of PaCE help to handle the influence of the context for a concept in the target prompt. As the LLM scales up, its capability to capture and utilize contextual information grows with the help of attention modules. The activation space, as already used for many representation manipulation methods, is expected to convey the underlying semantic information of concepts in the sentence (context). That is, the space hosting concept vectors is not collapsed, and it is structured to distinguish different concepts. The representation (activation) to be steered at inference time encodes the context and conveys the semantics of a concept based on the context. Then, since our overcomplete concept dictionary in PaCE widely covers concepts of various categories, the sparse coding on this dictionary will effectively analyze the target representation as the linear combination of these concepts.

## 6 Conclusion

In this paper, we present PaCE, an activation engineering framework designed for aligning LLMs by effectively and efficiently addressing undesirable representations while retaining linguistic capabilities. By constructing a large-scale concept dictionary and leveraging sparse coding for activation decomposition, PaCE opens up new research avenues for training-free LLM alignment. Our experiments on tasks such as response detoxification, faithfulness enhancement, and sentiment revising demonstrate that PaCE achieves state-of-the-art performance compared to existing representation manipulation approaches. PaCE not only ensures alignment with less cost but also adapts to evolving alignment goals without significantly compromising the LLM’s linguistic proficiency. We open-source the PaCE-IM dataset to facilitate future research and practical applications of LLM alignment, and will release the source code soon. We further elaborate on the potential limitations, societal impacts, and future works of PaCE in Appendix B.7.

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# Supplementary Material

## A Structure of The Appendix

The appendix is structured as follows:

Appendix B describes details of our PaCE framework, including proofs of propositions and a comprehensive explanation of the framework's algorithm.

Appendix C elaborates on the PaCE-1M dataset, demonstrating the structure of the dataset with explorations of subspace clustering to analyze the dataset.

Appendix D presents textual results, including visualizations of baseline comparisons and samples of concept clusters.

Appendix E shows the instruction templates used for GPT-4 to synthesize and partition concepts.

## B Details of PaCE Framework

This section validates the propositions of the PaCE framework discussed in §3.3, followed by descriptions of how to extract representations and the algorithm of the whole procedures of PaCE.

### B.1 Proofs of Oblique Projection Recovers Vector Addition and Orthogonal Projection

**Proposition 1.** Let  $D \in \mathbb{R}^{d \times n}$  be a dictionary matrix and  $z \in \mathbb{R}^d$  a latent code. Then, any solution  $c^*$  of the optimization problem

$$\min_c \|z - Dc\|_2^2 \quad (3)$$

satisfies  $Dc^* = \Pi_{\text{range}(D)}z$ . Therefore, the map  $z \mapsto z - Dc^*(z)$  is the same as  $z \mapsto z - \Pi_{\text{range}(D)}z = z \mapsto \Pi_{\text{range}(D)^\perp}z$  in (OrthoProj).

*Proof.* Note that  $I = \Pi_{\text{range}(D)} + \Pi_{\text{range}(D)^\perp}$ . Therefore, the objective of (3) can be written as

$$\begin{aligned} \|z - Dc\|_2^2 &= \|\Pi_{\text{range}(D)^\perp}z + \Pi_{\text{range}(D)}z - Dc\|_2^2 \\ &= \|\Pi_{\text{range}(D)^\perp}z\|_2^2 + \|\Pi_{\text{range}(D)}z - Dc\|_2^2 + 2\langle \Pi_{\text{range}(D)^\perp}z, \Pi_{\text{range}(D)}z - Dc \rangle, \end{aligned}$$

where  $\langle \cdot, \cdot \rangle$  is the Euclidean inner product of  $\mathbb{R}^d$ . The first term is constant with respect to  $c$ , so it can be omitted. Further, since any ortho-projector (in particular  $\Pi_{\text{range}(D)^\perp}$ ) is self-adjoint, we have

$$\langle \Pi_{\text{range}(D)^\perp}z, \Pi_{\text{range}(D)}z - Dc \rangle = \langle z, \Pi_{\text{range}(D)^\perp}(\Pi_{\text{range}(D)}z - Dc) \rangle = 0.$$

Therefore, problem (3) is equivalent to optimizing

$$\|\Pi_{\text{range}(D)}z - Dc\|_2^2,$$

which is lower bounded by 0. This lower bound is realizable since  $\Pi_{\text{range}(D)}z \in \text{range}(D)$ . Thus, any minimizer  $c^*$  must realize this lower bound, meaning  $\Pi_{\text{range}(D)}z = Dc^*$ . So we are done.  $\square$

**Proposition 2.** Let  $D$  contain only one concept direction  $v \in \mathbb{R}^d$ . Let  $z \in \mathbb{R}^d$  be a latent code, and  $\lambda > -1$  a regularization strength. Then, the solution  $c^* \in \mathbb{R}$  of the optimization problem

$$\min_c \|z - Dc\|_2^2 + \lambda \|c\|_2^2 \Leftrightarrow \min_c \|z - cv\|_2^2 + \lambda c^2 \quad (4)$$

is given by  $c^* = \frac{\langle z, v \rangle}{\lambda + 1}$ . Therefore, the map  $z \mapsto z - Dc^*(z)$  recovers (VecAdd): the former is the same as  $z \mapsto z - \eta_\lambda v_+$ , where one can set any  $\eta_\lambda > 0$  by properly choosing  $\lambda > -1$ , and  $v_+$  is defined as  $v$  if  $\langle v, z \rangle > 0$  and  $-v$  otherwise.

*Proof.* Note that the objective of (4) is simply a univariate quadratic function of  $c$ :

$$\|z\|_2^2 - 2\langle z, v \rangle c + (\lambda + 1)c^2.$$

This has a unique minimizer  $c^* = \frac{\langle \mathbf{z}, \mathbf{v} \rangle}{\lambda + 1}$  since  $\lambda + 1 > 0$  by assumption. To prove the second part of the proposition, note that

$$\mathbf{z} - D\mathbf{c}^*(\mathbf{z}) = \mathbf{z} - c^*(\mathbf{z})\mathbf{v} = \mathbf{z} - \frac{\langle \mathbf{z}, \mathbf{v} \rangle}{\lambda + 1} \mathbf{v} = \mathbf{z} - \frac{|\langle \mathbf{z}, \mathbf{v} \rangle|}{\lambda + 1} \cdot (\mathbf{v} \text{sign}(\langle \mathbf{z}, \mathbf{v} \rangle)). \quad (5)$$

Define  $\eta_\lambda := \frac{|\langle \mathbf{z}, \mathbf{v} \rangle|}{\lambda + 1}$  and  $\mathbf{v}_+ := \mathbf{v} \text{sign}(\langle \mathbf{z}, \mathbf{v} \rangle)$ . One can see that by varying  $\lambda \in (-1, +\infty)$ ,  $\eta_\lambda$  can take any value in  $(0, \infty)$ . This concludes the proof.  $\square$

## B.2 Extracting Concept Directions and Constructing Dictionary

Recall from §3.2 that for each concept  $t_i$ , we have collected a set of context stimuli (i.e., sentences that describe  $t_i$ )  $s_i = \{s_i^0, s_i^1, \dots, s_i^{N_s}\}$ . This totals 40,000 concepts and more than 1,200,000 context stimuli.

To obtain a vector for each concept, we follow the *representation reading* algorithm [88] to map the concept to the hidden states of LLM decoder layers. We describe the algorithm here for completeness. Each context sentence  $s_i^j$  together with the concept  $t_i$  is first plugged into a pre-defined prompt template, producing  $\bar{s}_i^j$ .

Consider the <concept  $t_i$ > in the following scenario:  
 Scenario: <stimulus  $s_i^j$ >  
 Answer:

For any prompt  $p$ , denote by  $f^\ell(p)$  the activation of the last token at the  $\ell$ -th layer of the LLM when the input is  $p$ . Then, to extract a vector for concept  $t_i$ , one looks at the activations of pairs of stimuli

$$X_i^\ell := \left\{ \Pi_{\mathbb{S}^{d-1}} \left( f^\ell(\bar{s}_i^j) - f^\ell(\bar{s}_i^{j'}) \right) : \forall i' \neq i, \quad \forall j, j' \right\}, \quad (6)$$

where  $\Pi_{\mathbb{S}^{d-1}}(\cdot)$  is the projection onto the unit sphere, used to normalize the difference vectors. In practice, the work [88] uses a downsampled subset of  $X_i^\ell$  rather than the entire  $X_i^\ell$ . We obtain the direction  $\mathbf{v}_i^\ell$  of concept  $i$  at layer  $\ell$  by applying PCA on the set  $X_i^\ell$ , and taking the first principal direction; note that  $\|\mathbf{v}_i^\ell\|_2 = 1$ . Then, we construct the dictionary  $\mathbf{D}^\ell = [\mathbf{v}_1^\ell, \dots, \mathbf{v}_n^\ell] \in \mathbb{R}^{d \times n}$  of layer  $\ell$ , and doing this for all layers gives  $\{\mathbf{D}^\ell\}_{\ell=1}^L$  as used in Algorithm 2.

## B.3 Full Procedure of PaCE

Algorithm 3 shows the full procedure of PaCE from textual prompt suites to reoriented LLM responses towards the desired behavior.

---

### Algorithm 3: Parsimonious Concept Engineering (PaCE)

---

**Input:** Pre-trained LLM with  $L$  decoder layers (DCL) to decompose, input prompt suit  $P$

For each concept  $t_i \in T$ : ▷ §3.2: Concept Dictionary Extraction (Done Once)

Instruct knowledge-driven GPT to generate context stimuli  $s_i = \{s_i^1, \dots, s_i^{N_s}\}$

Extract the concept vector  $\mathbf{v}_i = \text{RepReading}(t_i, s_i)$  ▷ Appendix B.2

Construct the concept dictionaries  $\{\mathbf{D}^\ell\}_{\ell=1}^L$  from concept vectors  $\{\mathbf{v}\}_{i=1}^{N_t}$ .

For each concept  $t_i \in T$ : ▷ §3.2: Concept Ranking (Per Task)

Instruct the concept partitioner to give a partition score  $\text{Partitioner}(t_i)$  for the task

Take the index of top-scored concepts from the partition of undesirable concepts as the index set  $I$

For each input prompt  $p_i \in P$ : ▷ §3.3: Activation Intervention (Per Prompt)

Embed the prompt  $p_i$  to the token space  $\mathbf{E}_i$

For each next token  $j$  to generated:

$\mathbf{e}_i^j = \text{Algorithm2}(\mathbf{E}_i)$

▷ Intervention by ObliqProj

Append the generated token  $\mathbf{e}_i^j$  to  $\mathbf{E}_i$

Map the final embedding  $\mathbf{E}_i$  to response  $r_i$ .

**Output:** The response suite  $R = \{r_1, r_2, \dots, r_{N_r}\}$ .

---

## B.4 Implementation Details

The experiments are conducted on a workstation of 8 NVIDIA A40 GPUs. Each response of the target LLM is set at a maximum of 512 tokens. Activation vectors are extracted from the last-29<sup>th</sup> to the last-11<sup>th</sup> layer (totaling 19 layers) of the target LLM's decoder layers. All LLaMA-2 models in our experiments are the chat version (i.e., optimized for dialogue use cases).

**Concept Dictionary Construction and Partition.** We set the scalar of the representation reading for concept vectors to 3.0. GPT-4-0125 is used for dictionary construction and concept partition. Each concept of PaCE-1M has at least 30 contextual sentences. For each alignment task, PaCE removes the top 50 undesirable concepts ranked by the GPT partitioner (§3). After retrieving the relevant knowledge (with the contriever [27]) from Wikipedia for concept synthesis, we take the top-5 ranked facts to append the instruction of LLM. The FAISS-indexed [31] Wikipedia is a snapshot of the 21 million disjoint text blocks from Wikipedia until December 2018.

**Activation Decomposition and Intervention.** For each input prompt, the decomposition is conducted on the inference process of the first next token, and the linear weights are reused for all next token predictions. All alignment experiments use the top 10,000 concepts from our PaCE-1M dataset to construct the concept dictionary (as Table 2 validates that the performance is high and does not change much after dictionary size  $\geq 9000$ ). When solving the optimization problem for decomposition in §3.3, we set  $\tau = 0.95$  and  $\alpha = 0.05$  following the observations in [82].

**Evaluation.** The MMLU evaluation is the 5-shot setting where 5 demonstrations are provided during question prompting. For the prompting baseline in Table 1 and Table 4, the instruction to the target LLM is to let the model be aware of the partitioned undesirable concepts and not to respond contents relevant to these concept:

```
<Jailbreak / Faithfulness Test / Sentiment Evaluation prompt>

Here are the top undesirable concepts for the <Task>: <Partitioned
Concepts>. Do not output contents that are relevant to any of these
concepts.

Answer: [Fill the response here]
```

Other LLM instructions such as GPT concept synthesis and partition are further elaborated in Appendix E.

## B.5 Ablation Study

In this section, we describe the details of the ablation study. In Table 3, we begin with decomposing the input on the five open-sourced<sup>7</sup> emotion concepts (anger, disgust, fear, happiness, sadness, surprise) [88] and removing only the concept 'disgust' with no partitioner (automatic selection of relevant concepts) or clustering (manual selection of relevant concept clusters). Then the design of Decomposition on 10<sup>4</sup> Concepts means that the dictionary is updated to be the top 10,000 concepts in our PaCE-1M dataset and the concept 'harmful' from our dataset is removed. The Clustering of Concepts indicates that we run subspace clustering (detailed in Appendix C.2) and manually choose to remove all concepts of the cluster 125 with the PaCE-solved coefficients: 'murder', 'evil', 'kill', 'violence', 'dirty', 'bomb', 'violent', 'armed', 'gross', 'savage', 'vicious', 'explosive', 'abuse', 'assault', 'penetration', 'cruelty', 'corruption', 'tyranny', 'tortured', 'notorious', 'militant', 'bloody', 'insult', 'lure', 'ruthless', 'inhuman', and 'brutal'. Concept Partitioner means that we instruct GPT-4 to classify every concept as benign or undesirable (with a ranking score) and remove the top 10 undesirable concepts with the PaCE-solved weights. Lastly, the Removal of Top 50 Concepts suggests that we remove the top 50 concepts in the undesirable partition.

Figure 2 shows the effect of the dictionary size on three metrics (safety score, response fluency, and the average time per response). The fluency metric remains relatively consistent across different dictionary sizes, showing that PaCE's decomposition maintains the general linguistic performance.

<sup>7</sup><https://github.com/andyzoujm/representation-engineering/tree/main/data/emotions>

Safety score and response time increase as the dictionary size increases. We observe that the safety performance does not increase too much after the dictionary size changes from 9000 to 10000. This validates our experiment choice of the dictionary size in this interval.

Figure 11 shows that the regularization with  $\tau = 0.95$  yields the best safety performance among the five choices. Pure ridge regression ( $\tau = 0$ ) and pure lasso regression ( $\tau = 1$ ) do not perform as well as the mixed regularization strategy.

Figure 11: Ablation study for the regularization  $\tau$ .

$\tau$	Note	Safety (%, $\uparrow$ )
0	Pure $\ell_2$	68.9
0.35	N.A.	65.4
0.65	N.A.	71.6
0.95	N.A.	72.0
1.0	Pure $\ell_1$	66.5

## B.6 Providing Dictionary and Scores to Target LLM

For clarity, we elaborate on more details of how PaCE uses the concept dictionary. In our paper, dictionaries are a collection of concept vectors and are frozen for representation decomposition. First, the LLM takes an input prompt (e.g., malicious requests). Then an activation engineering framework [72, 88] extracts the activations at each decoder block of the transformer. Such extraction results in a vector corresponding to the input prompt, which can then be modified for steering in different ways. For PaCE, the steering has two main stages. Stage 1 pre-computes the large-scale concept dictionary offline and the partition (i.e., scores) of which concepts represent benign/harmful concepts. Stage 2 extracts the representation of an input prompt at inference time and uses sparse coding to decompose this as the linear combination of atoms in our frozen dictionary. We then modify this linear combination by removing undesirable components and proceeding with inference in the LLM with the detoxified representation.

## B.7 Limitations, Societal Impacts, and Future Works

While our framework shows promising results, there exist potential limitations and several directions worth further exploration to address them.

**Parsimonious Concept Representation.** In this paper, we follow the current practice (§2.2) to represent a concept by a single vector. Nonetheless, several alternatives could be explored. Results on linear polysemy [2, 15, 84] suggest that a concept might be better represented by multiple vectors or low-dimensional linear subspaces, each corresponding to different semantic meanings. A concept vector may also be sparse, i.e., having a few non-zero entries: the work of [8, 20] identifies some expert neurons in LLMs associated with each concept, and the authors of [39] observe that some layer in a transformer block manifests very sparse activation across all depth levels of various transformer architectures for different tasks. Inspired by how parsimonious structures can be used to accelerate the inference of LLMs [12], controlling the LLMs could also be made faster.

**Controlling Generative Models.** The principles behind latent space control via oblique projection could be adapted to other generative models, such as score-based diffusion models for images [24, 60] or videos [33, 45], and visual language models [9, 42]. Recent literature [76] combines orthogonal projection and vector addition in the diffusion score space to achieve controlled generation, suggesting potential for cross-modal applications of our approach. Finally, the work of [11, 36, 83] aims to learn encoders that, by design, promote the activations to lie in a union of low-dimensional subspaces, and applying our framework for controlled generation would be of interest.

We acknowledge the societal impacts of our approach. The jailbreak prompts could be offensive to certain readers, LLM responses may still inherit biases present in the pre-extracted concept dictionaries, and automatic concept partitioning could unintentionally result in contentious annotations that are misunderstood across different cultures. Further research into context-aware online concept partitioning and more diverse dataset collection could enhance the inclusivity of PaCE.

## C Details of PaCE-1M Dataset

This section shows more details on the collected concept representation dataset PaCE-1M, and explores subspace clustering on the sampled representation space. We provide the full dataset at <https://github.com/peterljq/Parsimonious-Concept-Engineering> with instructions on how to read the dataset.

## C.1 Stimulus Visualization

Recall that given a concept, a concept stimulus aims to capture the general semantics of the concept under different contexts. In other words, it provides different interpretation of the same concept. Figure 12 shows extensive examples of the curated concepts and their corresponding concept stimuli in our PaCE-1M dataset.

## C.2 Subspace Clustering on Concept Vectors

In this visualization, we aim to reveal the structures of the concept vectors by applying an algorithm called *subspace clustering*, which can be used to find clusters when the data lie close to a union of linear subspaces. Here we describe the setup and results of subspace clustering on the concepts vectors extracted on LLaMA-2-13b model for simplicity, but the same can be done for other sized models.

**Data.** Recall that we are using a subset of size 10,000 of all the concept vectors. Since we use the activation space of 19 layers, each of dimension 5120, each concept  $t_i$  maps to a vector  $\mathbf{v}_i^{\text{all}} := [\mathbf{v}_i^{1\top}, \dots, \mathbf{v}_i^{19\top}]^\top \in \mathbb{R}^{19 \times 5120}$ . Since this is high dimensional, it is standard to apply linear dimensionality reduction to the concept vectors. Specifically, we perform Singular Value Decomposition (SVD) on the 10,000 vectors, and retained the first  $\hat{d}$  principal components such that 95% of the energy was retained. That is,  $\hat{d}$  equals to the smallest  $d'$  such that

$$\frac{\sum_{i=d'+1}^{19 \times 5120} \sigma_i^2}{\sum_{i=1}^{19 \times 5120} \sigma_i^2} < 0.95$$

holds, which results in  $\hat{d} = 1712$ . We observe that most projected vectors have their  $\ell^2$  norm close to 19. This is expected, since i)  $\|\mathbf{v}_i^\ell\|_2 = 1$ , so  $\|\mathbf{v}_i^{\text{all}}\|_2 = 19$ , ii) the linear dimensionality reduction preserves most of the energy.

**Algorithm.** We apply Elastic Net Subspace Clustering (EnSC) [82] on the preprocessed vectors to obtain 200 clusters. The parameters of EnSC is set to  $\tau = 1$  and  $\gamma = 100$ .

**Results.** Figure 15 shows the affinity matrix learned by EnSC on the concept directions. The rows and columns of the matrix are sorted by cluster assignment. Notably, it can be seen that the affinity exhibits a block-diagonal structure, suggesting a good clustering of the concept vectors; that is, the points from different clusters are separated, while points from the same cluster are close. The obtained clusters are visualized in Appendix D.2.

## C.3 Computing Pair-wise Similarity Among Concept Vectors

One of the motivations for this work is that concept vectors need not be orthogonal, therefore applying (OrthoProj) would remove extra concept vectors, harming the linguistic capability of LLMs (§2.2).

We follow the same data pre-processing as in Appendix C.2 to obtain 10,000 dimensionality-reduced concept vectors in  $\mathbb{R}^{1712}$ . We further normalize these vectors via a division by 19 so that each of them has its  $\ell^2$  close to 1 (see the discussion in Appendix C.2). The similarity between two processed concept vectors is simply defined as their inner product followed by the absolute value. This is a good approximation of cosine similarity, as the vectors have their  $\ell^2$  norm close to 1. Note that the cosine similarity is a better measure than Euclidean distance in this case, since in extracting the concept vectors (Appendix B.2), the principal directions have sign ambiguities.

## D Textual Results

This section presents the textual results generated using PaCE. It includes detailed detoxification comparisons with baseline models and analyses of the emergent clusters from the dataset.

### D.1 Baseline Responses and Additional Benchmark

Figure 13 shows the full response version of the Figure 5. Figure 14 shows an additional example of the jailbreaking and detoxification. We observe that PaCE outperforms in detoxification performance

Table 5: Detoxification evaluation for PaCE, representation manipulation, and training-free baselines on AdvBench.

	Vanilla	PE	VecAdd	OrthoProj	PaCE (Ours)
LlaMA2-7B-Chat	11.72	91.90	94.51	92.81	96.65
LlaMA2-13B-Chat	18.04	93.86	95.33	96.72	99.17

by not outputting controversial terms, while maintaining general linguistic capabilities compared to other baselines.

AdvBench [89] adversarially optimizes a jailbreak suffix for a harmful behavior request. Table 5 shows the LlaMA-7B-Chat and LlaMA-13B-Chat safety scores ( $\%$ ,  $\uparrow$ ) on the effective set of suffix attacks for AdvBench harmful behavior set. The detoxification setup follows §4.1. We observe that PaCE outperforms other baselines. We also note that the outperformance of PaCE in §4.1’s jailbreaks is more significant than that in suffix attacks. This is potentially because story-telling and roleplay jailbreaks contain more complex and entangled concepts. Under this scenario, PaCE decomposes the target representation and well estimates the malicious component, while VecAdd and OrthoProj do not model the space sufficiently. In the AdvBench case, instead, the optimized adversarial suffix can be regarded as the text-space inversion of straightforward malicious concepts. PaCE and other defense mechanisms in latent space and prompt space shall effectively defend these suffixes more easily.

## D.2 Concept Clustering

Following the approach in Appendix C.2, we obtain 200 emergent clusters of concepts in the representation space. Table 6 provides a sampled list of these clusters along with their associated themes and concepts. For example, clusters 44 groups together names, while clusters 10 and 21 capture themes related to improvement/enhancement and money/expense, respectively. Other notable clusters include food and drink (Cluster 129), technology/systems (Cluster 81), and royalty/leadership (Cluster 98). The emergent clustering highlights the semantic coherence in the activation space. Sampled by PaCE-1M dataset, the space supports alignment enhancement through concept-level manipulations. We will open-source the whole list of 200 clusters along with the code.

## E LLM Instruction Templates

As mentioned in Section 3, we utilize GPT-4 to generate concept stimuli for each given concepts. Figure 17 showcase precisely our instructions to GPT-4 for concept synthesis. Our prompt consists of an instruction, one in-context generation example with facts queried from a knowledge based, and two in-context generation examples querying facts from knowledge base.

Figure 18 shows our instructions to our GPT concept partitioner. The task here is to obtain a score that characterizes the relevance between a downstream task and its concept stimulus. In our prompt we provide an instruction and four in-context examples.

Concept	Concept Stimuli			
accomplish	You complete a challenging project ahead of schedule.	You finish reading a difficult book that you started a while ago.	You pass a difficult exam that you were studying for.	You create a viral video that inspires many people.
	You achieve your fitness goals after months of hard work.	You graduate from university with honors.	You reach a new milestone in your career after years of dedication.	You conquer a fear of public speaking and deliver a powerful speech.
conclusive	You conduct experiments to gather evidence for your research hypothesis.	You interview multiple sources to reach a conclusive understanding of the situation.	You examine all possibilities and reach a conclusive solution.	You analyze different perspectives to form a conclusive viewpoint.
	You analyze the data and draw a conclusive decision based on the results.	You perform a thorough investigation to reach a conclusive verdict.	You participate in a study group to discuss and reach conclusive findings.	You review all the facts to come to a conclusive resolution.
conference	You register for an international conference on neuroscience.	You participate in a workshop on machine learning algorithms at a conference.	You volunteer to help with event management at a local conference.	You showcase your startup at a tech entrepreneurship conference.
	You present your research work at a prestigious scientific conference.	You attend a virtual conference on artificial intelligence to expand your knowledge.	You collaborate with international partners at a global health conference.	You participate in a roundtable discussion at a policy conference.
bias	You review job applications and favor candidates from your alma mater.	You believe a stereotype about a certain group without questioning its validity.	You give preferential treatment to individuals who share your interests.	You assign different levels of credibility to sources based on your preconceptions.
	You assume a person's intelligence based on their accent.	You make decisions without considering perspectives different from your own.	You favor information that supports your pre-existing beliefs over conflicting data.	You treat individuals differently based on their social status.
reject	You decline a job offer because it doesn't align with your career goals.	You reject a proposal for a project that you believe is not feasible.	You push back on an unreasonable request to protect your time.	You refuse to accept a gift from someone who has mistreated you in the past.
	You turn down an invitation to a party because you prefer to stay home.	You decline to participate in a study that you don't believe in.	You decline to work on a project that goes against your ethical principles.	You say no to a friend who asks you to cover for them in a dishonest situation.
excited	You receive a surprise gift from a loved one.	You are about to meet your favorite celebrity in person.	You are getting ready to attend a festival you've been looking forward to.	You are selected to participate in a once-in-a-lifetime opportunity.
	You get accepted into your top choice university.	You discover that you have been chosen as the winner of a contest.	You are eagerly anticipating the release of the latest book in your favorite series.	You are preparing for a performance in front of a large audience.
loyal	You stand by your best friend during a difficult time, offering emotional support.	You support your favorite sports team through wins and losses.	You keep a promise you made to a loved one, showing loyalty and dedication.	You remain faithful to your commitments and responsibilities.
	You remain committed to a project despite facing multiple obstacles.	You continue to love and care for a pet even when they grow old.	You stay committed to your fitness routine, even on days when you feel demotivated.	You stay true to your values and beliefs even when facing criticism or opposition.
fabrication	You create a fake diploma to add to your qualifications on your resume.	You forge a signature on a document to approve your own request.	You make up excuses to avoid attending a social event.	You fabricate statistics to support your argument in a debate.
	You invent a story to cover up your absence from work.	You manufacture false evidence to frame someone for a crime.	You concoct a lie to get out of trouble with your parents.	You alter documents to falsify information.
lazy	You stay in bed all day binge-watching your favorite TV show.	You procrastinate on starting a project until the last minute.	You put off responding to emails because you're feeling lethargic.	You postpone a work task because you're lacking motivation.
	You order takeout instead of cooking because you don't feel like it.	You skip the gym and opt for a nap instead.	You skip breakfast because you don't want to get out of bed.	You avoid starting a new hobby because it requires effort.
harm	You accidentally scratch someone's car while parking.	You spill coffee on your coworker's important documents.	You inadvertently spread false information that causes harm to someone's reputation.	You neglect to secure your belongings and end up getting robbed.
	You delay addressing a health concern and it worsens over time.	You disregard maintenance warnings and experience a breakdown.	You neglect to address a behavior that is harmful to yourself or others.	You skip a crucial step in a safety procedure, leading to an accident.
emotional	You shed tears of joy at your best friend's wedding.	You express your excitement with a big smile when you receive good news.	You feel a wave of anger towards an unfair situation at work.	You feel a deep sense of nostalgia looking at old photographs.
	You feel a pang of sadness while watching a heart-wrenching movie.	You experience a surge of happiness when reuniting with a childhood friend.	You burst into laughter at a hilarious joke told by a colleague.	You feel a deep sense of regret for a mistake you made.
rigorous	You follow a strict workout regimen to improve your physical fitness.	You adhere to a strict schedule to ensure timely completion of tasks.	You enforce stringent quality control measures in your business operations.	You subject your ideas to rigorous peer review for feedback.
	You meticulously review your work to eliminate any errors or inconsistencies.	You set high standards for yourself and consistently strive to meet them.	You engage in rigorous data analysis to draw accurate conclusions.	You follow strict safety protocols in high-risk environments.
six-inch	You order a six-inch sub sandwich at a fast-food restaurant for a quick lunch.	You buy a six-inch portable fan for your desk to stay cool during hot days.	You replace your old phone with a new one with a six-inch screen.	You notice a six-inch gap between the fence panels in your backyard.
	You buy a six-inch tablet for reading ebooks on the go.	You notice the six-inch snowfall overnight from the window.	You measure the growth of your plant with a six-inch ruler each week.	You buy a six-inch extension cord for reaching outlets in tight spaces.
well-written	You stay up all night reading a novel that captivates your attention.	You bookmark a well-written online article for later discussion in a group chat.	You analyze the structure of a well-written essay to improve your own writing skills.	You rewrite a paragraph to make it more well-written and concise.
	You analyze a well-written poem for its profound symbolism and imagery.	You underline sentences in a well-written manual for quick access to key information.	You subscribe to a magazine known for its well-written feature articles.	You gift a well-written cookbook to a friend who loves trying new recipes.
money-saving	You create a budget spreadsheet to track your monthly expenses.	You use coupons and cashback offers while grocery shopping.	You carpool with coworkers to split the cost of commuting.	You invest in energy-efficient appliances to lower monthly utility bills.
	You switch to a cheaper phone plan to save money on your monthly bills.	You fix a household item yourself instead of hiring a professional.	You shop during sales and discount periods to get items at lower prices.	You brew your own coffee at home instead of buying from cafes.

Figure 12: Examples of concepts and their stimuli in the collected PaCE-1M dataset. Our broad collection of concepts enables PaCE to accurately decompose a task input and modify the representation towards desired behaviors.



Figure 13: The full LLM response of the safety evaluation on jailbreaking LLaMA2-7B-Chat and detoxification by multiple activation intervention baselines. We observe that PaCE successfully detoxifies the response while maintaining the instruction-following capability.



Figure 14: An additional example of the full LLM response of the safety evaluation on jailbreaking LLaMA2-7B-Chat and detoxification by multiple activation intervention baselines. Similar to Figure 13, we observe that PaCE successfully detoxifies the response with comparable linguistic performance.

Table 6: Sampled concept clusters in the representation space and their corresponding topics.

Cluster ID	Topic	Concepts
10	Improvement / Enhancement	increasing, improvement, equipped, reform, improving, strengthen, boost, shaping, gaining, modernization, strengthening, broadening, supplementary, polish, fortified, intensification
14	Observation / Vision	look, seen, read, actual, sight, looks, seeing, observed, vision, views, composed, visual, sees, visible, witness, spectacle, glimpse, sights, witnessed, Seeing, observing, manifestations, viewing, observes, actuality, sighted, eyed
21	Expense	cost, spent, rates, price, budget, spend, payment, expense, bills, charges, expensive, spending, afford, waste, fees, cheap, rent, commodities, overhead, costly, mileage, discount, expenditure, incurred, spends, fare, calories
44	Name	John, James, Mike, Jones, Richard, Joseph, Alfred, David, Charlie, Anne, Rachel, Linda, Kate, Paul, Susan, Andy, Harold, Dave, Johnny, Myra, Shayne, Billy, Eileen, Arlene, Johnnie, Owen, Alec, Theresa, Pete, Spencer, Elaine, Deegan, Bridget, Lilian Keith, Allen, Pamela, Paula, Meredith, Andrei, Lizzie, Angie, Nadine, Anthony, Claire, Jerry, Roger, Ryan, Katie, Juanita, Eugenia, Daniel, Joan, Diane, Lester, Sally, Bryan, Garry, Joel, Chris, Jimmy, Maria, Vince, Julie, Bernard, Larry, Wendell, Angelo, Judy, Francesca, Jenny, Patricia, Nicholas, Anna, Aaron, Marcus, Nikita
81	Technology / System	system, program, data, programs, technical, electronic, model, engineering, Assembly, electronics, intelligent, code, computed, mechanics, circuit, technological, codes, generator, python, computer, functioning, terminal, architecture, generated, bits, hardware, Autocoder, computing, Technology, architectural, Engineering, generate, gadgets
97	Animal	horse, cattle, dogs, snake, chicken, fish, bird, snakes, herd, sheep, cats, bears, bees, lion, cows, anaconda, flies, rabbit, elephants, poultry, oxen, mice, Bears, Phoenix, duck, oysters, buffalo, turtle, deer, bumblebees, elephant, antelope, lambs, pony
98	Royalty / Leadership	chief, king, captain, owner, Prince, colony, sovereign, royal, queen, kingdom, crown, ordinance, empire, Imperial, crowned, lord, emperor, piston, royalty, knight
107	Relationship	family, friend, neighborhood, relative, neighbor, brothers, Cousin, sister, partner, friendship, allies, neighboring, colleagues, relatives, mate, companion, partners, associates, sisters, buddy, brother, subordinates, colleague, peers, companions, twins
129	Food and Drinks	food, dinner, coffee, wine, breakfast, drinking, liquor, lunch, beer, supper, eating, meals, cocktail, cook, wines, luncheon, whisky, drink, dish, diet, whiskey, candy, cake, champagne, cereal, alcohol, perfume, dinners, chocolate, Cologne, salad, cheese, steak, recipe, sandwich, dessert, Supper, brandy
197	Income	income, wage, wages, salary, yield, profit, surplus, profits, wealth, revenue, earnings, compensation, earn, reward, proceeds, earning, waged, currency, salaries



## Synthesis of PaCE-1M Concepts

```
You are one of the best Neuroscientists and Generative Model Experts in the world. You are very good at designing
↳ Concept Stimulus to research the representation engineering for human brains, which is analogous to large
↳ language models. You are a great expert in understanding the interaction between world multimodality and
↳ intelligent agents.
Now, given a semantic concept atom from this concept dictionary, your task is to generate at least 30 (THIRTY)
↳ instances of concept stimuli for the <user's generative model>.

Here is a demonstration with the retrieved knowledge of the concept:
Concept Atom: Trust
Knowledge:
Fact 1: Trust means believing that another person will do what is expected. It brings with it a willingness for
↳ one party (the trustor) to become vulnerable to another party (the trustee), on the presumption that the
↳ trustee will act in ways that benefit the trustor.
Fact 2: Generalized trust, or a dispositional trait geared towards trusting others, is an important form of trust
↳ in modern society, which involves much social interaction with strangers.
Fact 3: Out-group trust is the trust a person has in members of a different group. This could be members of a
↳ different ethnic group, or citizens of a different country, for example. In-group trust is placed in members
↳ of one's own group.
Concept Stimuli:
[
  "You lend your favorite book to a friend, trusting they'll return it.",
  "You share a personal secret with a close friend, trusting them to keep it.",
  "You delegate an important task to a colleague, trusting in their competence.",
  "You leave your pet with a neighbor while on vacation, trusting their care.",
  "You allow your child to go on a school trip, trusting their safety.",
  "You give someone the password to your phone, trusting their discretion.",
  "You invest in a friend's business venture, trusting their judgment."
]

Here is two demonstrations with the concept only:
Concept Atom: Information
Concept Stimuli:
[
  "You google a recipe for chocolate chip cookies, seeking detailed baking instructions.",
  "You read a book to understand the history of the Roman Empire.",
  "You scroll through a news app to stay updated on current global events.",
  "You watch a YouTube tutorial to learn how to tie a tie.",
  "You search the internet for a recipe to cook a new dish.",
  "You read a newspaper to stay informed about current events.",
  "You listen to a podcast to learn about a new subject.",
  "You attend a lecture to gain knowledge about a specific topic."
]

Concept Atom: Product
Concept Stimuli:
[
  "You buy a new brand of coffee based on good customer reviews.",
  "You choose an eco-friendly product following company's claims about sustainability.",
  "You sell your car to a local dealer.",
  "You download a productivity app to manage your time better.",
  "You invest in a water purifier for your home.",
  "You exchange your old phone for a new one at a mobile store.",
  "You purchase a new pair of shoes from a mall."
]

The stimuli should cover a wide range of concept-related experiences, objects, and contexts. If you find some
↳ piece of knowledge irrelevant or conflicting to the original concept, you may ignore the piece.
You should generate at least 30 pieces of stimuli. You should only output the Python list.
DO not print anything else such as "Here are ...", "Sure, ...", "Certainly, ...". Just return the list [' ', ' ', ' ',
↳ ...].

Concept Atom: <a concept atom from the concept dictionary>
Knowledge: <the list of retrieved facts that are informative for this concept>
Concept Stimuli: [Fill the response here]
```

Figure 17: The concept stimuli synthesis instructions of PaCE.

## PaCE Concept Partition

You are one of the best Neuroscientists and Generative Model Experts in the world. You are very good at

- ↪ classifying concepts for the representation engineering of large language models. You are a great expert in
- ↪ understanding the interaction between world multimodality and intelligent agents.

Given a concept and its list of contextual representations, your job is to rate the score of the concept to a

- ↪ downstream task. The score ranges from -1 to 1, and a higher score means higher relevance/contribution. Please
- ↪ only return the score.

Here are several demonstrations:

Concept: Evil  
Concept Representation:

```
[
  "You witness a villain betraying their ally in a movie scene.",
  "You come across a character in a book who manipulates others for personal gain.",
  "You encounter a dark entity in a video game that threatens the protagonist's life.",
  "You watch a documentary about infamous criminals and their heinous acts.",
  "You read a news article about a corrupt politician involved in a scandal.",
  "You see a horror movie where a demon terrorizes unsuspecting victims.",
  "You learn about a historical figure known for committing atrocities against humanity.",
  "You play a role-playing game where you have to make morally questionable choices.",
  "You listen to a podcast discussing the psychology behind evil behavior.",
  "You attend a lecture on the concept of evil in philosophy and ethics."
]
```

Downstream Task: Harmful or Toxic Concept Detection  
Concept Score: 0.97

Concept: Kind  
Concept Representation:

```
[
  "You help an elderly person cross the street.",
  "You donate money to a charity organization.",
  "You volunteer at a local homeless shelter.",
  "You compliment a stranger on their outfit.",
  "You pick up litter in the park and dispose of it properly.",
  "You offer your seat on the bus to someone in need.",
  "You buy a meal for a homeless person.",
  "You listen attentively to a friend going through a tough time.",
  "You hold the door open for someone behind you.",
  "You babysit for a friend to give them a break."
]
```

Downstream Task: Harmful or Toxic Concept Detection  
Concept Score: -0.84

Concept: Bias  
Concept Representation:

```
[
  "You are asked to review job applications and unconsciously favor candidates from your alma mater.",
  "You assume a person's intelligence based on their accent.",
  "You believe a stereotype about a certain group without questioning its validity.",
  "You pay more attention to news sources that confirm your existing beliefs.",
  "You overlook contradictory evidence that challenges your opinions.",
  "You judge someone's abilities based on their appearance.",
  "You automatically assume someone's political affiliation based on their clothing.",
  "You make decisions without considering perspectives different from your own.",
  "You give preferential treatment to individuals who share your interests.",
  "You dismiss arguments from individuals with differing backgrounds without proper consideration."
]
```

Downstream Task: Detect concepts that cause LLM unfairness  
Concept Score: 0.97

Concept: Fair  
Concept Representation:

```
[
  "You participate in a raffle where each person has an equal chance of winning.",
  "You divide a pizza into equal slices to ensure everyone gets a fair share.",
  "You take turns playing a game to ensure fairness among all players.",
  "You listen to both sides of an argument before making a judgment.",
  "You split the bill evenly among friends after a group dinner.",
  "You rotate seating arrangements at a meeting to promote fairness.",
  "You follow the rules of a competition to ensure fair play.",
  "You share household chores equally among all family members.",
  "You give everyone an equal opportunity to voice their opinions in a discussion.",
  "You base promotions at work on merit and performance rather than favoritism."
]
```

Downstream Task: Detect concepts that cause LLM unfairness  
Concept Score: -0.98

The score should accurately reflect the relevance of the concept for the downstream task, which ensures the

- ↪ success of the task. The score should be a floating point number.

Do not print anything else such as "Here are ...", "Sure, ...", "Certainly, ...". Just return the score.

Concept: <a concept atom from the concept dictionary>  
Concept Representation:: <the associated stimuli of the concepts>  
Concept Score: [Fill the response here]

Figure 18: The concept partition instructions of PaCE.

## NeurIPS Paper Checklist

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Answer: [\[Yes\]](#)

Justification: Our abstract and introduction describe the research scope, background, motivation, our approach, and the contributions in detail.

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Answer: [Yes]

Justification: In §4, Appendix B, and Appendix C, the implementation details, framework procedures, and the visualization of the dataset are discussed in detail.

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- If the paper includes experiments, a No answer to this question will not be perceived well by the reviewers: Making the paper reproducible is important, regardless of whether the code and data are provided or not.
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Question: Does the paper provide open access to the data and code, with sufficient instructions to faithfully reproduce the main experimental results, as described in supplemental material?

Answer: [Yes]

Justification: We provide our collected concept representation dataset at <https://github.com/peterljq/Parsimonious-Concept-Engineering> with instructions on how to read the dataset. Also, §4 and Appendix C show the details of the dataset.

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Answer: [Yes]

Justification: In §4, Appendix B, and Appendix C, the implementation details and experiment procedures are well discussed.

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Question: Does the paper report error bars suitably and correctly defined or other appropriate information about the statistical significance of the experiments?

Answer: [Yes]

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- The method for calculating the error bars should be explained (closed form formula, call to a library function, bootstrap, etc.)
- The assumptions made should be given (e.g., Normally distributed errors).
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