

Advancing Timber for the Future Built Environment

CASE STUDY OF ENGINEERING THERMAL AND STRUCTURAL ANALYSIS OF TIMBER IN OFFICE TRAVELLING FIRE – FIRE DYNAMICS SIMULATOR AND SAFIR

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ABSTRACT: This work presented a case study of a travelling fire in an office building with different ratios of exposed timber. Two engineering simulation softwares (FDS and SAFIR) were combined in order to investigate the impact of exposed timber on the fire propagation over the building façade and the load-bearing timber structures integrity. Using FDS the impact of the timber exposure rate on the fire safety criteria depended on two distinct ventilated regimes: the 'well ventilated' and the 'numerical flashover'. Results also demonstrate that, for this case study, real fire scenarios consistently exceeded ISO fire predictions, even with minimal exposed timber. The thermal impact on stability of columns was investigated, using SAFIR software and advanced calculation method of Eurocode 5. Thermo-mechanical results were analysed and discussed (including the application of different thermal actions depending on the faces of the column exposed to the fire, which can lead to eccentricities and therefore bending moments, and the proportions of the section below 75°C and 300°C). The increase in exposed timber led to fairly close failure times of the columns (up to 5 minutes). Several perspectives are mentioned in this work, in order to confirm the fairly limited impact of the apparent timber ratio on the stability of structural elements observed here.

KEYWORDS: Timber, Fire spread; Numerical simulation; Fire Dynamics Simulator; SAFIR.

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1 – INTRODUCTION

The use of engineered timber in structural elements, walls and façades is rising in popularity in the context of seeking the maximum reduction of the environmental footprint of buildings. The use of wood in construction nevertheless raises many questions: wood is a material with complex properties and has the particular characteristic of fueling a fire. Evaluating fire propagation and growth on timber buildings and their structural behavior during disasters is therefore essential for designing effective fire safety measures and developing strategies to mitigate associated risks [1].

With this aim, numerical analysis has become essential for evaluating fire safety in buildings. The combination of the softwares Fire Dynamics Simulator (FDS) and SAFIR [9, 10, 11] is widely used in Europe by accredited laboratories. FDS, developed by the NIST, is a CFD tool that models fire and smoke behavior, heat transfer and combustion processes, offering insights into temperature fields, smoke movement, and heat release rates [2, 3, 4]. In contrast, Safir focuses on the thermomechanical analysis of structures, predicting temperature profiles and assessing the deformation and failure of materials like steel, concrete, and timber under fire conditions [5]. Together, the coupling of FDS and Safir provides a comprehensive approach to fire safety, analyzing both thermal effects and structural integrity to ensure performance-based design of timber, in order to achieve building resilience and occupant safety [6].

This study applies the numerical tools FDS and SAFIR to analyze a traveling fire in a typical office building. A comparative assessment is conducted with varying ratios of exposed timber. The results, focusing on the thermal impact on load-bearing structures and façades, as well as the loss of structural integrity, will enhance the

understanding of the risks associated with exposed timber use. These findings serve as a scientific foundation for the ongoing evolution of national and European regulations on timber use. Given wood's combustible nature, a key focus of these regulations is determining the extent to which structural timber can safely remain exposed to fire.

2 – FIRE DEVELOPMENT METHODOLOGY

Development of the fire scenario

The first step of the case study is the assessment of the thermal loads experienced by the load-bearing structures and the external façade during a fire. In order to be consistent with the fire safety engineering studies conducted in Europe, the procedures described in this section follow the guidelines of Eurocode for actions on timber structures exposed to fire [7, 8].

In a typical fire scenario, the evolution of thermal power consists of three phases: growth, plateau (constant power and/or peak maximum power), and decay. In a fire surface element, the growth phase follows a parabolic trend as $\alpha \cdot t^2$, where t represents time until reaching a maximum value. The plateau phase then marks the period during which power remains constant. Finally, the decay phase follows a linear trend once 70% of the combustible load has been consumed.

The heat load is derived from the surface-based values proposed in the Eurocode, depending on the type of occupancy. For office spaces, a recommended value of 740 MJ/m² is used in France. Thermal power is more challenging to assess, as it depends on various factors, including oxygen supply. However, standardized values are also applied according to the Eurocode, based on

building typology. For offices, a value of 250 kW/m² is used, with a growth rate reaching 1 MW in 300 seconds. In large open spaces, a fully developed fire with a constant propagation speed, known as a "travelling fire" is considered. This approach involves selecting both the ignition point and the propagation speed. The Eurocode does not account for a travelling fire but instead assumes a circular propagation model, characterized by the aforementioned time parameter $\alpha \cdot t^2$. According to the Eurocode, the recommended propagation speed for an office-type building is 22,5 cm/min. The fire starts at the center of the office space as illustrated in Figure 1. This choice is artbitrary. Indeed a real fire can occur un numerous places. The consequences of this choice and the fire spread rate on the impact on the load-bearing structures will be discussed in the results section.

Based on these assumptions and considering an office area of 100 m², the time evolution of the imposed heat release rate (HRR) in the system is shown in Figure 2 (0% curve, named so because no timber elements are exposed). The maximum power reaches 25 MW after 60 minutes.

Numerical details

The simulations are performed using Fire Dynamics Simulator (FDS) version 6.8.0 [2, 3, 4]. FDS is a computational fluid dynamics software which solves unsteady low Mach number combustion equations on a rectilinear grid. It solves the conservation of mass, momentum, energy and species. In FDS combustion is modeled with a mixture fraction concept, and the thermal radiation is computed using a finite volume technique on the same grid as the flow solver. It employs the finitedifference method, with second-order explicit predictorcorrector time discretization and second-order central difference space discretization. The time-step is determined dynamically during calculations based on the local control volume size and velocity to ensure computational convergence. A complete description of FDS technical details can be found in refs. [1,3]. The submodels and details used in FDS are summarized in Table

Table 1: Summary of sub-models applied in FDS

Sub-model	FDS version 6.8.0 [2,3]
Pyrolysis	Integrated empirical model [13]
	Eddy dissipation concept model (EDC) [15]
Combustion	Infinitely fast chemical reaction
	Global combustion reaction of propane [13]
Radiation	Finite volume method with 100 discrete angles [17]
Turbulence	Deardorff model [18]

The heat released and charring that occur from timber combustion are treated by an engineering pyrolysis model, integrated in FDS. This model, named S-Pyro, empirically correlates and scales the mass loss rate outcome to the incident heat flux on a combustible surface based on cone calorimetry data [8]. The reader is

encouraged to consult the article which has been concurrently submitted to this conference. In this article, FDS and S-Pyro were extensively evaluated and compared to multiple experimental tests regarding their predictive capabilities and limitations.

Cellulosic composition is used to represent the fuel typically found in office environments, with a combustion enthalpy of 25 MJ/kg [13]. For comparison, this value is equivalent to the average combustion enthalpy of polymers. The thermal loads are evaluated using the concept of adiabatic surface temperature (referred to as temperature in this article for brevity), a single quantity that describes the transfer from the thermal input by the gas phase in the thermal model to the material response in the structural model [12].

FDS model

Figure 1 presents the FDS model, designed to represent a typical office building based on realistic dimensions and structural layout. The office interior spans 15 m in length and 8 m in width, with a centrally positioned volume measuring 7×3.5 m. The flammable surface area covers 100 m^2 .

The walls have a height of 2.75 m, with an additional 1.75 meters incorporated to analyze the impact of flames on the exterior façade and upper floor. The space is well-ventilated, featuring 16 windows (each 1.25 m wide and 1 m high) and two doors (each 1 m wide and 2 m high). The entire computational domain measures 11.5 m in length, 17.5 m in width, and 4.5 m in height.

To ensure an engineering-feasible approach, a grid resolution of 25 cm was adopted, a standard choice in engineering studies. Reducing the grid size by half resulted in an average discrepancy of 15% in external façade temperature predictions - a value lower than the reported error margin for heat flux prediction in FDS. Further halving the grid size reduced the discrepancy to below 5%. However, such a fine grid becomes impractical for engineering applications when the computational domain spans tens of meters, as it exponentially increases computational costs. The simulations were performed using Open Multi-Processing (OpenMP) for parallel computing.

According to Eurocode 5 [7], the ratio of exposed timber r is expressed as:

$$r = \frac{A_{exposed\ timber}}{A_{exposed}}$$

Where:

- A_{exposed timber} is the surface area of timber exposed within the compartment (structure elements, walls, ceiling, etc.).
- A_{walls} is the total internal surface area of the vertical walls (including all materials).

Four configurations of exposed timber elements were analyzed:

• Configuration 1 features non-combustible structural elements, walls, and ceiling, with the only fire source being the primary traveling fire, as described in the previous section (r = 0%).

- Configuration 2 includes exposed timber columns and beams, accounting for r = 38% of the exposed timber surface.
- Configuration 3 extends exposure to columns, beams, and half of the ceiling, increasing the timber ratio to r = 80%.
- Configuration 4 represents the most severe case, with columns, beams, and the entire ceiling made of timber, resulting in a r = 121% exposed timber ratio and heightened fire safety risks.

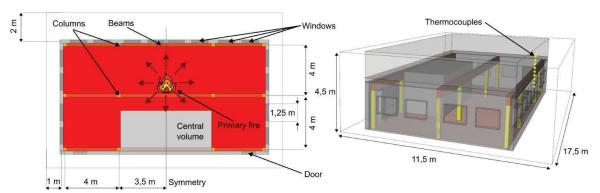


Figure 1. Schematic view of the computational model used in FDS simulations.

3 – STRUCTURAL ANALYSIS METHODOLOGY

Depending on the proportion of exposed timber, the thermal actions received by the structural elements vary. This study looks at the impact on the structural strength of columns as a function of these different exposed timber configurations.

The columns are here considered:

- Made with glue-laminated timber (class GL24h).
- A section of 25 cm x 25 cm.
- A height of 2.80 m.
- A mechanical load of 248 kN.
- The connections are modelled as a hinge at the lower end of the column and as a sliding support at the top of the column.

In this study, the focus was on columns for mainly two reasons. Columns are exposed to fire on 4 sides and undergo a significant reduction in cross-section. According to Eurocode 5, compression of timber is the mechanical property most affected by temperature [7].

For each configuration, the history of the thermal actions to which each of the 4 faces of the column is exposed was considered. It is important to distinguish the actions received on the column according to the 4 faces, as differential combustion between two opposite faces can result in the generation of eccentricity within the column.

This eccentricity leads to the appearance of bending moments, and the columns generally bear these bending forces rather poorly, in addition to the normal compression forces that they already bear. In view of these eccentricities, it is not always safe to apply the most unfavourable thermal action encountered on a column to all its faces equally. For this reason, the authors recommend distinguishing, as far as possible, the thermal actions received for each face of the column. Consequently, thermomechanical calculations need to be carried out column by column.

In the present study, it was first verified that the similarity (symmetry) of the thermal actions received on the 2 central columns was achieved. In this paper, it was decided to present the thermomechanical calculations for the central columns and not the façade columns. As much as the effect of eccentricity could be expected on façade columns (often exposed directly to fire on 2 or 3 faces), it seemed interesting to the authors to show that the effects of eccentricity could appear on central columns.

The calculations were carried out with SAFIR software, developed by University of Liège [9, 10, 11], using the advanced Eurocode calculation method.

The results of fire development calculations using FDS showed a certain numerical limit ("bathtub effect"). Thermomechanical calculations were therefore carried out using the results initially obtained under FDS, and also using modified results in order to propose a correction for this effect.

The main assumptions are:

- If part of the heated column appears to cool, no reversibility is envisaged for the thermophysical and thermomechanical properties.
- No account is taken of the initial eccentricity of the columns.

 Ruin is considered to be the moment when a loss of convergence due to excessive displacements is reached in the SAFIR modelling.

4 - RESULTS

Heat release rate and thermal impact

The evolution of the heat release rate (HRR) for the four configurations is presented in Figure 2a. As expected, increasing the exposed timber surface within the building compartment leads to a higher total heat release.

Despite variations in intensity, the burning behavior of timber follows a similar trend. Ignition occurs approximately five minutes after the onset of the office fire, as the exposed timber begins to receive elevated incident heat flux. This results in a rapid increase in HRR as the unprotected combustible material is ignited.

Following the primary fire's HRR peak, the burning of timber gradually declines until extinction due to two key factors:

- Decreasing incident heat flux from the primary fire.
- Formation of a protective char layer, which inhibits wood degradation by slowing down heat and mass transfer between the gaseous and condensed phases, thereby shielding the underlying material [19].

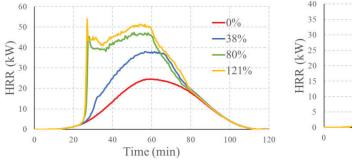
The distinction between the HRR inside the compartment (solid lines) and outside the compartment (dashed lines) is shown in Figure 2b. When no timber is exposed, the fire scenario remains well-ventilated, meaning sufficient oxygen is available to sustain combustion within the

compartment, preventing flames from extending beyond the office space.

However, from a 38% exposed timber ratio onwards, signs of under-ventilation begin to emerge inside the compartment. The maximum HRR sustainable within the compartment converges to a value of approximately 33 MW, aligning with the theoretical estimation given by $Q_{in}=1500\times A\times \sqrt{h}$, where A is the opening area and h is the opening height, as suggested in the literature [20]. As the proportion of exposed timber increases, the excess HRR - beyond what the compartment can sustain - develops outside the enclosure.

Consequently, the thermal impact on the exterior façade intensifies, as depicted in Figure 3. At high ratios of exposed timber (80% and 121%), a numerical flashover occurs, where the HRR rises sharply to approximately 33 MW, leading to a corresponding increase in temperatures at the external façade.

The temperatures illustrated in Figure 3 indicate that, compared to the scenario with no exposed timber, a 121% exposed timber ratio results in temperatures of 300°C being reached at least 2 m higher. This highlights the elevated risk of flame spread along the building's exterior and into adjacent compartments, particularly when this numerical flashover regime is reached.



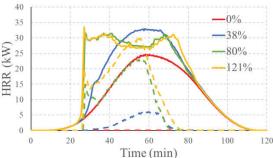


Figure 2: Time evolution of the heat release rate for the 4 tested configurations. Left: total HRR; Right: HRR inside the compartment (solid lines) and outside the compartment (dashed lines)

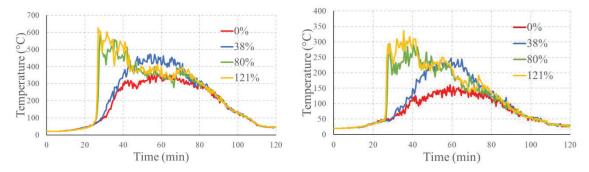


Figure 3: Time evolution of the calculated adiabatic sutface temperature at the façade. Left: Temperature just above the window top; Right:

Temperature at 2 m above the window top

The evolution of the column surface temperature on the four faces exposed to the fire (named X+, X-, Y+ and Y-) is presented in Figure 4, alongside the ISO 834 curve for reference. As the proportion of exposed timber increases from 0% to 38%, the column surface temperature rises due to the higher heat release rate inside the compartment and in its vicinity. However, this expected trend is not clearly observed in the numerical flashover regime. While temperatures initially rise more rapidly, they then drop unexpectedly to values lower than those in previous configurations. Following this drop, temperatures increase again after the peak, eventually converging with the other curves during the decay phase. There is no clear physical explanation for this behavior, named here "bathtub effect". Evidence suggests it may result from numerical instabilities, grid resolution effects, or, most importantly, combustion and extinction modeling limitations in under-ventilated conditions [3]. A proposed correction to address this issue is illustrated in Figure 4 (curves with dashed lines).

Thermomecanical analysis

An advanced calculation carried out initially determined the resistance of the column to an ISO fire of 98 minutes (the load applied here is 248 kN).

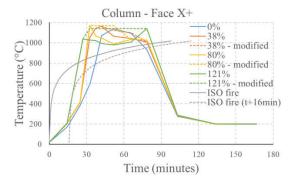
As mentioned previously, the ISO fire curve (for 98 minutes) has also been indicated on Figure 4 to provide a reference point. In this real fire scenario, given the speed at which the fire develops, it took around 16 minutes for the developed fire to reach the column. In order to display a better comparative scenario, the ISO fire curve (for 98 minutes) with a start time offset by 16 minutes has also been shown in these figures.

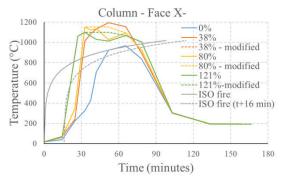
Figure 4 illustrates that real fire scenarios exceed the ISO fire across all ratios of exposed timber, including 0%.

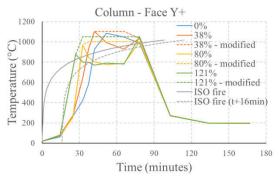
By comparing the configurations with gradually more exposed timber, we can see that the maximum temperatures increase, as does the onset of the peak of the fully-developed fire. Incidentally, the more exposed timber there is, the longer the peak lasts.

The modified curves are designed to correct the bathtub effect that can be observed on configurations with 38% (in minor amounts on X+ and Y+ faces), 80% and 121% exposed timber.

In the rest of this paper, curves where the bathtub effect has been corrected are then named 'XXX-modified'.







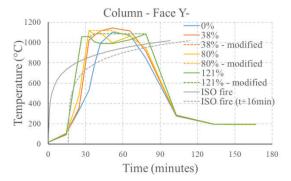


Figure 4: Time evolution of the column surfaces temperatures on the four faces exposed to the fire

Figure 5 shows the evolution of the temperatures reached within the section of the column for the '38%-modified' fire configuration.

The legend has been calibrated to easily identify temperatures below 75°C and above 300°C.

The contour of 300°C corresponds to pyrolysis temperature, representing the point of carbonisation of wood [19]. Above this temperature, timber no longer has any significant mechanical strength.

For fire situations, Eurocode 5 [7] proposes changes in mechanical properties as a function of temperature. The value of 75°C is an average where most mechanical properties are considered significantly altered for structural calculations (in average 50% of initial strength).

In Figure 5, white crosses have been added to indicate the geometric centre of the section, so that any eccentricities due to the different thermal actions applied to each face of the column can be seen more clearly.

The temperature fields shown at 33, 52 and 66 minutes show two zones in the section of the column quite clearly. A zone above 300°C where resistance is low and a zone below 75°C where resistance remains relatively high. The transition zone is quite thin (less than 1cm).

The 103-minute capture was chosen to illustrate a phenomenon that occurs during the cooling phase of the fire. It can be seen that temperatures at the periphery of the section fall while the heat wave continues to progress within the section.

At 133 and 167 minutes, there is no longer any part of the column with a temperature below 75°C. From a mechanical point of view, the performance of the section is severely impaired.

This is a point of attention when natural fires are considered for timber elements, as the most unfavourable temperature field for the section is not necessarily the one where the peak of the natural fire is reached, but can sometimes be reached during the cooling phase.

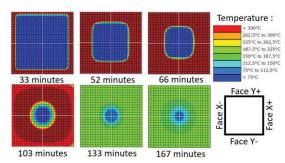


Figure 5: Time evolution of the temperatures inside the column exposed to the fire's configuration named "38%-modified"

For the thermomechanical calculations, 6 fire configurations were considered. The 3 configurations with a proportion of exposed timber in the room (38%, 80% and 121%), and these three modified configurations that correct the bathtub effect. The configuration without exposed timber (0%) was not considered in this phase, as it would involve a protected timber column or an incombustible material.

Table 2 : Summary of failure times obtained for cenral columns with SAFIR

Exposed timber ratio	Failure time [minutes]
	Values in () : if the 16 first minutes of preheating are neglected
38%	56 (40)
38%-modified	55 (39)
80%	55 (39)
80%-modified	52 (36)
121%	54 (38)
121%-modified	50 (34)

Table 2 shows the failure times obtained for the 6 proposed configurations. The modified curves logically lead to lower failure times. This evolution represents a few minutes. For the unmodified curves, the failure times are between 54 and 56 minutes depending on the proportion of exposed timber. For modified curves, failure times are between 50 and 55 minutes. In all cases, these failure times are fairly close to each other, despite the significant increase in the proportion of visible timber, and therefore, in the increase of heat released.

The failure time values in brackets are the values calculated by deducting 16 minutes from the thermal action curves. As can be seen from Figure 4, the first 16 minutes of the real fire are a pre-heating phase due to the distance between the point where the fire starts in the model and the position of the column. This is also one of the difficulties in the case of real fire studies, as the start time could be defined differently depending on each person's interpretation. This is one of the points that makes it difficult to compare, in terms of duration, performance obtained under ISO fire to performance obtained under real fire, and a parametric study under

numerous fire scenarios and configurations would be a future perspective. Figure 6 shows the evolution of two ratios as a function of fire time. The first ratio is the proportion of the section whose temperature remains below 75°C, the second ratio is the proportion of the section whose temperature remains below 300°C.

It is interesting to note that for these 6 configurations, failure is reached when the proportion of the section below 75°C reaches 30 to 32%. Considering the ratio of the section below 300°C, failure is reached for rates between 35 and 42%, which represents a slightly larger standard deviation.

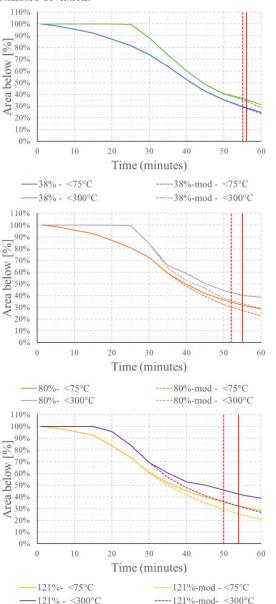


Figure 6: Evolution of the ratio of the cross-sectional are of the column below 75°C and 300°C modelled with the 6 fire configurations (the vertical lines reprsent the time of failure (dashed line = modified temperature curve))

Figure 7 shows the eccentricities, in the axes y and z of the column section, between the geometric center of the column section and the center of the resistant section, taking into account the degradation of mechanical performance (Young's modulus) as a function of temperature.

For modified cases, the eccentricities are less important. Eccentricities vary between 3 and 9 mm and lead to bending moments within the column. Generally, the moment when eccentricities increase corresponds to the moment when the column's failure time is approached.

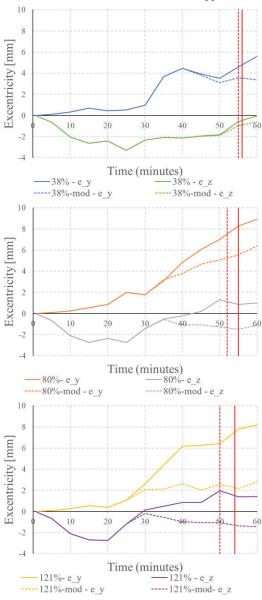


Figure 7: Evolution of eccentricity in both directions of the column modelled with the 6 fire configurations (the vertical lines reprsent the time of failure (dashed line = modified temperature curve))

5 - CONCLUSIONS

In this work, a numerical case study was conducted on a travelling fire in an office building, with the objective of assessing the risks associated with different levels of exposed timber. The coupling of a fire model (FDS) and structural model (SAFIR) allowed for evaluating the increased thermal impact on the building façade and the load-bearing structures. The increased risks of fire propagation and failure times were then discussed.

The increase in timber exposure led to an increase in the thermal impact and energy input into the timber structures. However, at a certain threshold of exposed timber, a numerical 'flashover' was observed. The heat release rate inside the compartment converged to a maximum value, piloted by the maximum air that can be admitted through the openings. As a consequence, the increase in thermal impact and decrease in failure times of timber structures were less significant. On the other hand, flame ejection from windows increased significantly at this point, along with the elevated risks of fire propagation over the exterior façade.

Regarding stability calculations on the columns, the increase in exposed timber led to the fairly close failure times of the columns (5 minutes). Several perspectives can be considered for this work, in order to confirm the fairly limited impact of the apparent timber ratio on the stability of structural elements observed here.

Future perspecrives include a parametric study with other column dimensions (height, section), mechanical loading rates (and eventual initial eccentricities), real fire configurations (more or less rapid, powerful fires, etc.) and ventilation conditions. Beyond stability of columns, stability of beams, walls and floors should also be studied.

Finally, it is important to address numerical instabilities and inaccuracies that arise in performance-based structural fire modeling for engineering applications. One notable issue is the 'bathtub effect,' which occurs in under-ventilated conditions, leading to an unrealistic underestimation of heat release from combustion within the compartment. This, in turn, affects the predicted thermal impact on load-bearing structures. Users should exercise caution when relying on default models to estimate the heat impact near the flame region and carefully evaluate parameter adjustments to ensure accurate predictions.

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