

RIBBED CLT ELEMENTS WITH CUT-BACKS

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ABSTRACT: The use of cross laminated timber (CLT) in combination with glulam ribs has become increasingly popular in timber construction. Especially in cases with relatively large spans or higher loads, this solution is even necessary to compete with other building materials. To simplify the process in platform frame type constructions, the ribs may be cut-back so the wall-floor-wall interface is still easily manageable on site. The described cut-back is causing additional stresses in the CLT-element which can quickly become of high relevance. To bring this potentially critical situation to the attention of the designing engineer and to evaluate the magnitude of these stresses (mainly rolling-shear), using a simplified engineering model, are the aims of this paper.

KEYWORDS: CLT, rib element, rolling-shear, cut-back, analytical solution

1 – INTRODUCTION

The use of cross laminated timber (CLT) in construction has become very popular as a pure panel element for walls, ceilings or roofs. For ceiling elements with larger spans (over 7 m) or uses with higher live loads (office or school buildings), or a combination of both, a solid CLT panel is usually not sufficient to fulfil the serviceability requirements. In these cases, rib elements consisting of a CLT panel and glulam ribs are a suitable alternative.

The geometry of these rib elements is irregular in cross-section, therefore the glulam ribs may be cut-back in the support area (ribs do not extend all the way over the support), to simplify the construction process in platform

framing. In addition to the transverse tensile stresses in the joint between rib and CLT slab, as discussed in [1] and [2], this offset creates additional bending stresses in the area of the support due to the eccentric load introduction. In this configuration, the shear verification in the CLT panel will become the governing ULS-design situation (usually, shear is of minor importance in CLT slab verification) depending on the spacing of the ribs.

These normal and shear stresses can only be estimated roughly with existing engineering models and their magnitude is difficult to calculate without the use of finite element methods. This paper will present a method to address this local problem and provides a comparison of different model approaches.

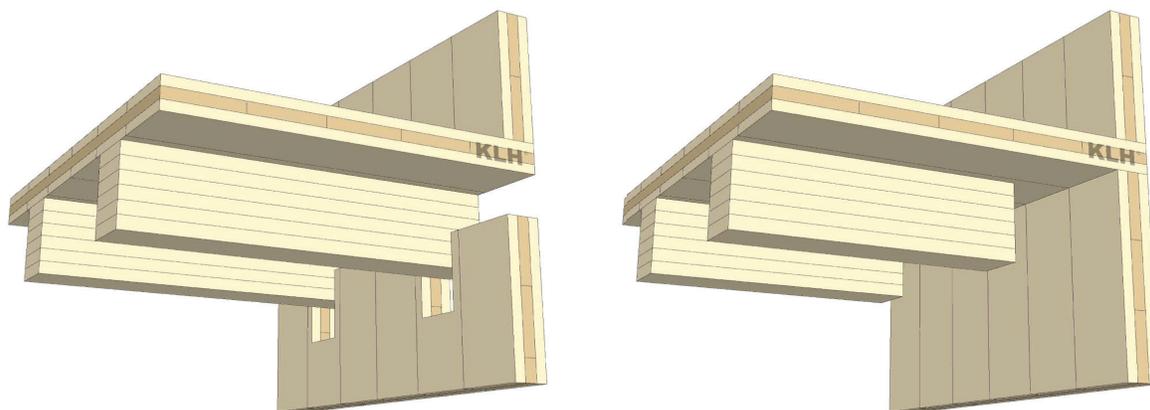


Figure 1. Support options for ribbed CLT slabs – without cut-backs (left), with cut-backs (right)

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2 – MECHANICS / BASICS

2.1 INTRODUCTION

The areas to be verified are in the support area ahead of the rib, which may be up to approximately 200 mm long. From the mechanical perspective, CLT acts as a simple plate without reinforcement from the rib in this region. A pure orthotropic plate is used as a simplified analytical model, in which the entire shear force at the end of the rib is transferred concentratedly to the CLT plate via the reinforcing screws [1]. In the edge area investigated, the plate acts for its own and it can be assumed that the errors made (in the central area without rib) quickly decrease and do not cause any major disturbances close to the support (St. Venant's principle).

A simply supported rectangular plate is to be solved analytically. Symmetry conditions can be assumed for the other edges of the outer strips (see Fig. 2) due to the usually regular sequence of ribs. As model assumption, in the centre of this plate near the end of the ribs, the entire shear force is transferred from rib to plate within a rectangular area (borderlines of the screw-group).

The following coordinate system will be used: The x-direction is parallel to the span direction of the ribbed CLT-panel, the y-direction is perpendicular to the span and the z-direction vertically oriented (see Fig. 5).

Loads must be transferred from the ribbed CLT panel into the CLT slab in the vicinity of the two support lines. It is assumed that this load transfer should occur with a concentrated load at a distance e from the support line, see Fig. 2 and Fig. 4. This load is usually transferred by several vertical screws at the end of the rib (see right part of Fig. 4). For simplicity, it is assumed that the total

internal shear force acting at a distance e from the support must be transferred. This simplification is somewhat in contradiction to [2], but for the purpose of this paper this simplification can be justified.

The solution strategy uses Fourier series for the load and functions in the x-direction (i.e. span of the ribbed CLT panel), whereas a differential equation has to be solved in the y-direction (perpendicular to the span).

In the y-direction, two different strips must be distinguished for the CLT panel (Fig. 2):

- an inner strip covering the CLT area directly adjacent to the bottom rib
- an outer strip located in a free zone between 2 ribs ending in the middle between the ribs

2.2 ACTING LOAD AND DEVELOPMENT OF FOURIER SERIES

As already stated in 2.1, a concentrated load has to be introduced at distance e in the CLT slab, which transfers the entire load from the ribbed CLT panel into the CLT plate alone near the support line. The vertical force should be determined with the shear force of the ribbed CLT panel at the regarded position, see (1).

$$F_{vert}(e) = \frac{qa \cdot b}{2} \cdot (L - 2 \cdot e) \quad (1)$$

To obtain a Fourier transform in the x-direction (span of the ribbed CLT panel), it is assumed that a uniformly distributed load is acting on the CLT panel over a length of $2 \cdot c$ and a width of $2 \cdot b_1$ (see Fig. 2). A local uniformly distributed load is expected to perform better than a concentrated load.

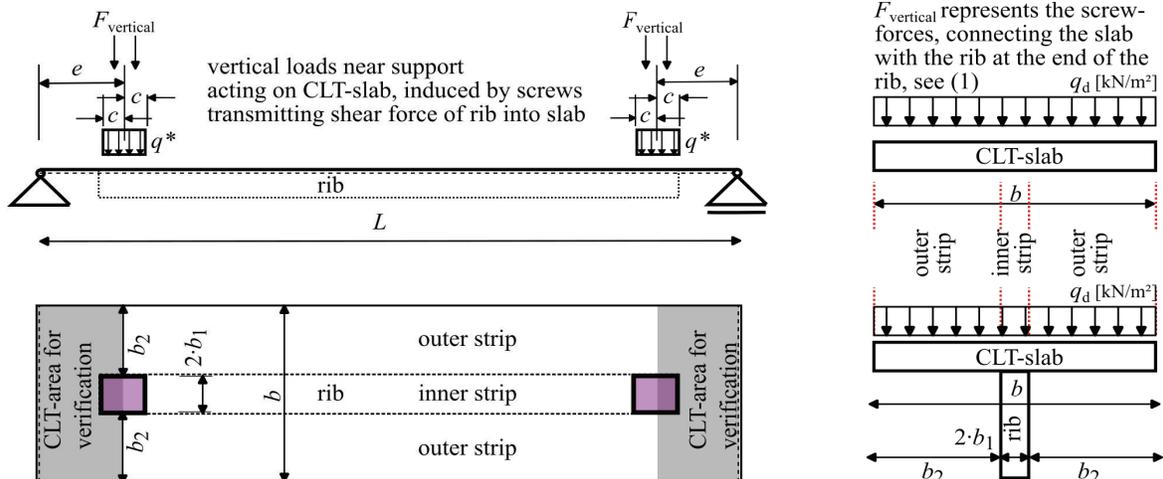


Figure 2. Definition of geometry and symbols used

The vertical force given by (1) must be converted into a distributed load q^* acting on the two highlighted areas at the centre of the reinforcement screws (see Fig. 2). This distributed load q^* is given by (2).

$$q^* = \frac{F_{vert}(e)}{2 \cdot c \cdot 2 \cdot b_1} = \frac{q_d \cdot b}{2} \cdot \frac{(L-2 \cdot e)}{4 \cdot c \cdot b_1} \quad (2)$$

The Fourier series for the load q^* , given in Fig. 2, can be evaluated using (3) for odd values of k .

$$p(x) = \sum_{k=1}^{k=n} b_k \cdot \sin\left(\frac{\pi \cdot k}{L} \cdot x\right) \quad (3)$$

If k is odd, the Fourier term b_k is evaluated using (4). If k is even, b_k is zero.

$$b_k = 8 \cdot \frac{q^*}{k \cdot \pi} \cdot \sin\left(\frac{\pi \cdot k}{L} \cdot e\right) \cdot \sin\left(\frac{\pi \cdot k}{L} \cdot c\right) \quad (4)$$

with

- L span length
- b rib spacing
- e, c ... distances, see Fig. 2
- q_d design value of external load per unit area
- q^* uniformly distributed load, see (2)
- k k^{th} term of Fourier series

2.3 BASIC MECHANICAL EQUATIONS

The 4th order differential partial equation for plate bending according to the shear-stiff Kirchhoff's plate theory is given in (5).

$$\frac{B_x \cdot \partial^4 w}{\partial^4 x} + \frac{2 \cdot (B_y + 2 \cdot B_{xy}) \cdot \partial^4 w}{\partial^2 x \cdot \partial^2 y} + \frac{B_y \cdot \partial^4 w}{\partial^4 y} = q_z(x, y) \quad (5)$$

with

- $q_z(x, y)$ arbitrary distributed loads $q_z(x, y)$ acting in transverse direction
- $w(x, y)$ transverse deflection of plate due to loading $q_z(x, y)$
- B_x bending stiffness in x -direction
- B_y bending stiffness in y -direction
- B_v coupled bending stiffness which is generally neglected for CLT
- B_{xy} twisting stiffness of CLT plate

2.4 MULTIPLICATIVE APPROACH

As the solution of a partial differential equation is not simple, the following multiplicative approach is adopted (6). In the x -direction (span of the ribbed CLT panel), a classical Fourier approximation is used, whereas in the y -direction a simpler new function $\bar{w}_k(y)$, which depends only on y , is used.

$$w_k(x, y) = \bar{w}_k(y) \cdot \sin\left(\frac{\pi \cdot k}{L} \cdot x\right) \quad (6)$$

with

- L span length
- k k^{th} term of Fourier series

Finally, a summation over all k terms for $k = 1, 2 \dots$ to INF (theoretically) has to be performed. In the practical calculations used in this paper, the summation in the developed Python program always ends at $k = 500$.

2.5 LEADING ORDINARY DIFFERENTIAL EQUATIONS

Based on the equations of 2.3, using the approach of 2.4 and some mathematical transformations, the following ordinary differential equation can be derived for the y -coordinate only (following (6)). Instead of solving a differential equation of 4th order (5), four differential equations of 1st order can be derived (7-10). Four new functions have to be defined. These functions are $\bar{w}(y)$, $\bar{\beta}_x(y)$, $\bar{v}_y(y)$ and $\bar{m}_y(y)$ which have to be multiplied by the sine or cosine term according to eq. 6 to obtain the complete solution.

The governing 4 equations (derivates d/dy) are the following:

- kinematic differential equation

$$\bar{w}'(y) = \bar{\beta}_x(y) \quad (7)$$

- constitutive differential equation

$$\bar{\beta}'_x(y) = -\frac{\bar{m}_y(y)}{B_y} \quad (8)$$

- transverse equilibrium with external loads

$$\begin{aligned} \bar{v}'_y(y) &= B_x \cdot \bar{w}_y(y) \cdot \left(\frac{\pi \cdot k}{L}\right)^4 \\ &+ 2 \cdot \frac{B_{xy}}{k_y} \cdot \left(\frac{\pi \cdot k}{L}\right)^2 \cdot \bar{m}_y(y) - b_k \end{aligned} \quad (9)$$

- rotational equilibrium for shear section

$$\begin{aligned} \bar{m}'_y(y) &= \\ \bar{v}_y(y) &- 2 \cdot B_{xy} \cdot \left(\frac{\pi \cdot k}{L}\right)^2 \cdot \bar{\beta}_x(y) \end{aligned} \quad (10)$$

2.6 SOLUTION

Homogeneous solution

The homogeneous solution is obtained by transforming (7) to (10) into a set of linear differential equations with constant parameters. By introducing the exponential approach for all 4 unknown functions, the homogeneous solution is a combination of eigenvectors multiplied by the exponential function of eigenvalues times y -coordinate. 4 different eigenvalues and eigenvectors can be calculated from the system.

Particular solution

The following approach can provide a particular solution:

$$\bar{w}_p(y) = \frac{b_k}{B_{x'} \left(\frac{\pi k}{L}\right)^4} \quad (11)$$

with b_k evaluated using (4). The other 3 inhomogeneous solutions remain zero, as shown below (12):

$$\bar{\beta}_{x,p}^-(y) = \bar{v}_{y,p}^-(y) = \bar{m}_{y,p}^-(y) = 0 \quad (12)$$

2.7 TRANSITION CONDITIONS

Transition conditions must be formulated between the inner and outer CLT strips (see Fig. 2). The inner CLT strip receives the vertical load q^* according to (2), while the outer CLT strip has to work in conjunction with the central zone. 4 integration constants have to be solved for the inner strip (index i) and another 4 integration constants for the outer strip (index o).

For all 4 main unknown functions, the transition conditions at $y = b_1$ require smooth transitions:

- transition for displacement

$$\bar{w}_i(b_1) = \bar{w}_o(b_1) \quad (13)$$

- transition for rotation about the x -axis

$$\bar{v}_{y,i}^-(b_1) = \bar{v}_{y,o}^-(b_1) \quad (14)$$

- transition for shear force

$$\bar{v}_{y,i}^-(b_1) = \bar{v}_{y,o}^-(b_1) \quad (15)$$

- transition for moment

$$\bar{v}_{y,i}^-(b_1) = \bar{v}_{y,o}^-(b_1) \quad (16)$$

These four transition conditions lead to 4 linear algebraic equations with 8 integration constants as unknowns.

2.8 BOUNDARY CONDITIONS

Boundary conditions must be formulated in the centre of the inner and at the end of the outer CLT strip (see Fig. 2). A symmetry condition can be formulated in the centre of the inner strip. For the outer strip, 2 different situations can be considered: One is a single ribbed CLT panel without any interaction with adjacent members (case 1). Alternatively, several beams can be attached to a CLT panel (case 2, typical case). For simplicity, it is assumed that the number of units of such ribbed CLT panels is infinite, which allows the assumption of a symmetry condition between each ribbed CLT panel.

Boundary condition at $y = 0$ (rib axis, line of symmetry):

$$\bar{\beta}_{x,i}^-(0) = \bar{v}_{y,i}^-(0) = 0 \quad (17)$$

Boundary condition at $y = \pm b/2$ (case 1, free edge):

$$\bar{\beta}_{x,i}^-(b_1) = \bar{\beta}_{x,o}^-(b_1) \quad (18)$$

Alternative boundary condition at $y = \pm b/2$ (case 2, line of symmetry):

$$\bar{\beta}_{x,o}^-\left(\pm \frac{b}{2}\right) = \bar{v}_{y,o}^-\left(\pm \frac{b}{2}\right) = 0 \quad (19)$$

Now the 8 equations for 8 integration constants as unknowns are available.

2.9 NUMERICAL IMPLEMENTATION

Although the given solution can be considered analytical and therefore exact, the use of infinite series requires a lot of number crunching. This work is best done using a software tool. We chose Python as the preferred programming language because it requires little development time.

As mentioned in 2.6, eigenvalues and eigenvectors have to be found, and according to 2.7 and 2.8, a set of linear equations has to be solved. All of this mathematical work is done using software packages written entirely in Python.

Exact numerical results can be computed at any position on the basis of this solution. For the purpose of this paper, only results along the symmetry axis are shown.

3 – THE PROJECT-EXAMPLE

In this section a model-project will be presented and described which will be utilized to apply several different methods of verification (sections 4.1 to 4.3).

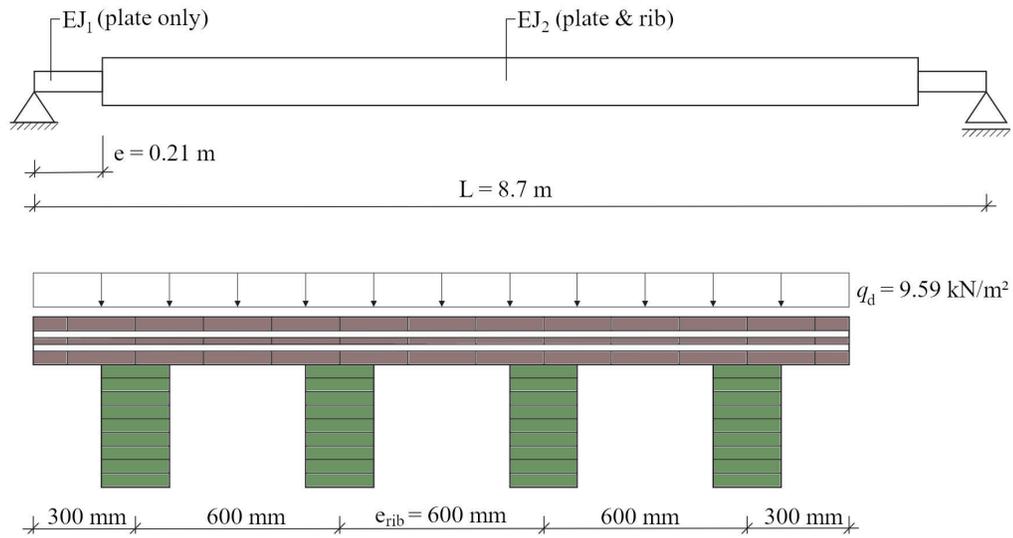


Figure 3. Structural system (top) and cross-section (bottom) for the project example

The CLT-glulam rib deck with a span of $L = 8.7$ m is loaded by a combination of dead-load and live load for office occupation. The slab is a 140 mm 5-layered CLT with ribs spaced at 600 mm having the size 160/360 mm (GL24h) and being cut-back by 240 mm (see Fig. 3 and Fig. 4).

The design distributed load per unit area amounts to $q_{d,max} = 9.63$ kN/m² and has the following contributions:

- self-weight rib-deck: $g_{1,k} = 1.30$ kN/m²
- self-weight floor buildup: $g_{2,k} = 2.50$ kN/m²

- office occupation category B2: $q_{B2,k} = 3.0$ kN/m²

The structural analysis and verification was performed according to ÖNORM B 1995-1-1 [4], taking into account the effective width of $b_{ef} = 543$ mm (calculated according to FprEN 1995-1-1 [3]). The result yields utilisations of 45% ($M_d = 91.1$ kNm/m) and 28% ($V_d = 41.9$ kN/m) for the ultimate limit state and for the usually governing serviceability limit state 100% ($a_{rms} = 0.5$ m/s²) and 39% ($w_{inst} = 11.4$ mm), when the cut-back situation is neglected during the verification process.

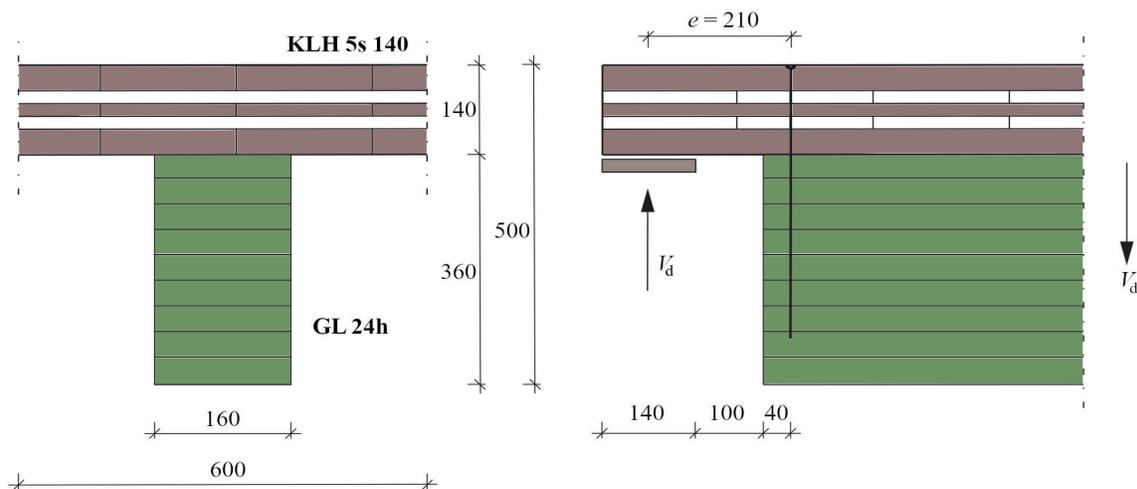


Figure 4. Detail of the cross-section (left) and longitudinal section (right)

4 – DESIGN PROCESS

In this section, three different approaches to determine the maximum permissible design load per square meter of the slab (q_d) are presented including a short model-description.

The maximum design resistances for the previously described CLT slab with a layup of 40|20|20|20|40 mm of KLH[®]-C24 material and a medium-term live load for office occupation ($f_{m,k} = 24 \text{ N/mm}^2$, $f_{v,R,k} = 1.2 \text{ N/mm}^2$, $\gamma_M = 1.25$ and $k_{mod} = 0.8$) are as follows:

$$v_{x,Rd} = 126.8 / 1.25 * 0.8 = \mathbf{81.2 \text{ kN/m}}$$

$$m_{x,Rd} = 72.46 / 1.25 * 0.8 = \mathbf{46.4 \text{ kNm/m}}$$

These values are used to determine the maximum permissible design load per square meter q_d in the following subchapters.

4.1 ANALYTICAL SOLUTION (DEQ)

The analytical solution, based on solving the set of differential equations (DEQ), yields a maximum permissible design load per square meter of $q_{d,max,DEQ} = 9,59 \text{ kN/m}^2$ (meaning nearly no reduction due to the cut-back situation).

4.2 FINITE ELEMENT MODEL (FEM)

Suitable and sufficiently exact Finite Element models are using **2D shell elements** for CLT slab and glulam beam – the use of 3D volumetric elements is not necessary. When using 2D shell elements it is recommended to model the rib at least with **2 adjacent shells** with each of them having only half of the thickness of the real rib (see green part of Fig. 5).

To bridge the distance between the central plane of the CLT panel (brown part of Fig. 5) and the true geometric top of the glulam rib, the model utilises a transition layer that is only able to transfer in-plane shear (d_{88}) and normal forces in z-direction (d_{77}). Without this interlayer, the stiffness in this part of the section would be too high. The FE-model may also take advantage from using lines of symmetry at the midspan and also along the CLT edges in the centre between two adjacent glulam ribs to minimise the numerical effort. By comparing the difference between a symmetrical element (i.e. an inner rib element) with a free edge element (i.e. a rib element at the edge of the entire floor slab), it can be shown that the differences are negligible as the shear-problem is local.

The solution for bending moment $m_x(x)$ yielded from solving the differential equations (DEQ) agrees very well

with the results obtained from the 2D shell model (RIB-2). This is slightly different for the shear force $v_x(x)$ as can be seen from Fig. 7. It should be noted that the DEQ solution is only valid within the range of applicability (i.e. the grey shaded area in Fig. 6) – and in this range it is on the conservative side.

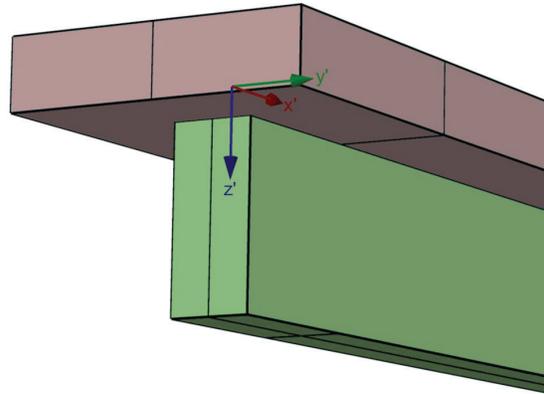


Figure 5. 3D view of the 2D-FEM model (RIB-2) in the local support area

As an alternative to the rib model, excentric beams (again, 1 to 8 adjacent ones) were introduced, but this approach gave less satisfactory results for the bending moment $m_x(x)$ and is therefore not recommended (EXC-2). For the shear force $v_x(x)$ the quality using excenters improved compared to the RIB-approach (see Fig. 7).

The results used for verification of the FE solution (RIB-2) are taken from the centerline (axis of the rib) and are shown in the figure (Fig. 6) below.

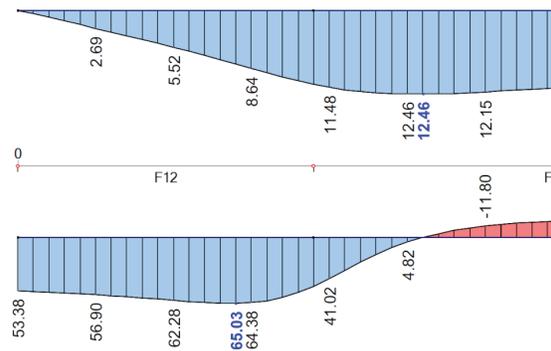


Figure 6. Bending moment $m_{x,Ed}$ [kNm/m] and shear force $v_{x,Ed}$ [kN/m] along the centerline from the RIB-2 model.

From the FE 2D-shell model (RIB-2) a maximum permissible design load of $q_{d,max,FEM} = 11.97 \text{ kN/m}^2$ could be determined, while also considerable smaller values can be obtained for other modelling approaches (e.g variation in number of adjacent sub-ribs or excenters).

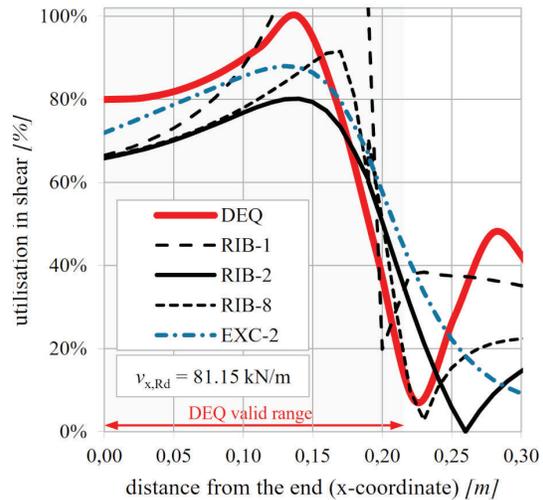
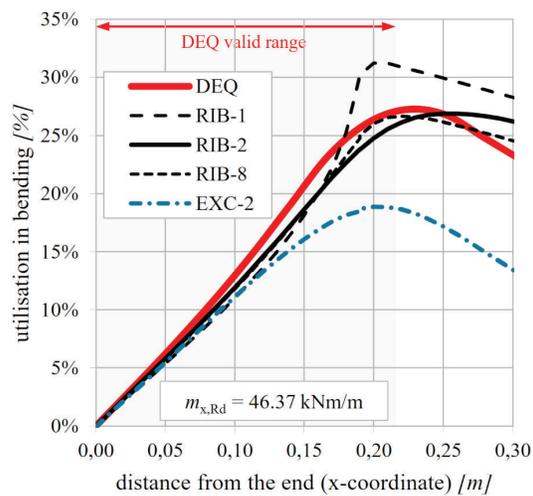


Figure 7. Comparison of bending moment capacity m_x (left) and shear capacity v_x (right) calculated using different models (differential equation DEQ, 2D shell model RIB, plate model using excentric beams EXC) – plots are valid for a load of 9.59 kN/m^2 meaning 100% in shear for DEQ

4.3 ENGINEERING MODEL (ENG)

A very simplified method to verify the shear stresses in a ribbed CLT slab is presented in [5], see Fig. 8 below.

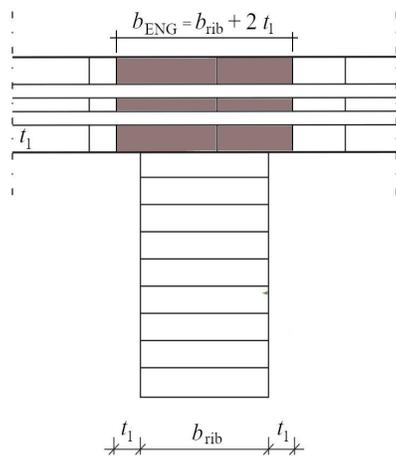


Figure 8. Cross-section for verification of the resistance in the simplified engineering model in the support area.

The maximum permissible design load per square meter for the simplified engineering model can be determined using (20). This equation takes into account that a certain part of the load (being the load acting between support axis and distance e , see Fig. 4) is distributed along the whole width of the panel. The main proportion of the load acting between $(L - 2 \cdot e)$ has to be transferred through the narrow section $b_{ENG} = b_{rib} + 2 \cdot e$ (see Fig. 8).

$$q_{d,max,ENG} = \frac{v_{x,Rd}}{\frac{L-2 \cdot e}{2 \cdot (b_{rib} + 2 \cdot t_1)} + \frac{e}{e_{rib}}} \cdot \frac{1}{e_{rib}} \quad (20)$$

The simplified engineering model solution yields a maximum permissible design load per square meter of $q_{d,max,ENG} = 7,68 \text{ kN/m}^2$ (meaning a reduction to 80% due to the cut-back situation).

4.4 MODEL SUMMARY

The results of the different modelling approaches are shown in the table below (see Tab. 1). It is evident that the design of a typical ribbed plate without cut-back is clearly dominated by vibration requirements (VIB), resulting in a maximum allowable design load of 9.63 kN/m^2 .

Table 1: Result summary: Maximum permissible load per unit area in kN/m^2 for the considered models

Model description		$q_{d,max}$	$q_{d,max,m}$	$q_{d,max,v}$	
		$x=L/2$	$x=e$	$x=e$	
without cut-back	ULS	22.5	-	-	
	SLS	24.5			
	VIB	9.63			
with cut-back	DEQ	-	35.2	9.59	
	FEM ¹⁾	ULS	24.3	30.8 to 51.0	1.78 to 12.0
		SLS	23.7	35.7 ²⁾	12.0 ²⁾
		VIB	8.28²⁾	11% stiffness reduction	
	ENG	-	50.8	7.68	

¹⁾ Result range depending on the used model (RIB-1 to RIB-8 and EXC-2)

²⁾ Values obtained from RIB-2 model

³⁾ Reduction of the eigenfrequency f_1 by ca. 6%

With the introduction of a cut-back, the situation is expected to shift towards the ULS situation, in particular to the shear check of the CLT panel in the region without rib.

Depending on the model approach used to solve this problem, the maximum allowable design load may be lower, i.e. 7.68 kN/m² for the simple engineering model (ENG). Using the analytical solution (DEQ), it has been shown that almost the same external load is acceptable (9.59 kN/m²).

When creating a 2D shell model (FEM), a range of acceptable loads can be obtained – strongly depending on the type of model (RIB, EXC, and especially number of ribs or excenters) and the mesh used. When taking the results in the centerline from the referenced RIB-2 model, the external load of 12.0 kN/m² seems acceptable (governed by shear), being on the upper limit of the result range from 1.78 kN/m² to 12.0 kN/m².

The interpretation of FEM-results is often difficult and requires a lot of experience from the engineer – applying the DEQ approach does not suffer from these limitations.

It is worth noting that stiffness and vibration are slightly influenced by the existence of a cut-back – it can be roughly stated that both values are reduced by about 10% for the presented example.

5 – DESIGN GUIDANCE

To address the local stress situation resulting from the cut-back rib, a finite element model (FEM) is a suitable solution, though it comes with some modelling effort and requires experience in result interpretation. The herein presented new method based on solving the differential equations (DEQ) can be an effective alternative to address the problem at hand. The simplified engineering model (ENG) is conservative but may be used for a preliminary design.

Depending on the spacing of the ribs and the distance between the support axis and the edge of the glulam rib (e), a local rolling-shear reinforcement (according to FprEN 1995-1-1 [3]) may be required to satisfy the rolling-shear design.

The authors want to state that the situation with a cut-back rib should be avoided if possible as it does not conform with the basic rules of timber engineering to avoid tension perpendicular to the grain by design. Anyways they are aware that building practice always will have its challenges and this paper wants to raise awareness and show possible ways to deal with such situations.

6 – CONCLUSION

The design of ribbed CLT-elements is usually governed by SLS conditions (typically vibration). In the case of cut-back ribs at the support area, this can shift to a ULS-governed condition, in that specific case the rolling-shear design situation. Depending on the model approach (DEQ, ENG or FEM), the results obtained are varying to a certain extent.

The presented analytical solution (DEQ) is very helpful to solve this local problem as it delivers a continuous function for the results (in contrast to different model approaches used in FEM). Therefore, the authors intend to prepare a set of equations to make this solution applicable for design practice (comparable to the equations for the effective width of CLT rib decks in FprEN 1995-1-1 [3]).

A reinforcement perpendicular to the grain may be necessary, but this was not addressed in the paper by definition. Approaches to solve this can be found in [1] and [2].

7 – REFERENCES

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