Overall stability safety factor of K6-type timber reticulated shells

Jiajia OU [1], Di CHEN [2], Weiguo LONG [3], Minjuan HE [4], Yifeng LIU [5]

ABSTRACT: Structural engineers in China are accustomed to use the overall stability safety factor to verify the overall stability of the structure, which is adopted by JGJ 7-2010 "Technical specification for space frame structures" and used for steel reticulated shells, while this study applies this concept to timber reticulated shells. Based on the load partial factor, resistance partial factor and the adjustment factor considering other uncertainties, the elasto-plastic overall stability safety factor is taken to be 2.4, which is larger than the recommended value of 2.0 for steel reticulated shells. In order to obtain the elastic overall stability safety factor, a parametric finite element analysis is carried out to investigate the plasticity reduction factor of timber reticulated shells. Nonlinear spring element was used to simulate the bending semi-rigidity of the joints, and the elastic and elasto-plastic ultimate load bearing capacity factors considering the initial defects, as well as the plasticity reduction factor, were obtained. According to Eurocode 5, the slip stiffness and bearing capacity of dowel-type fasteners are calculated, and the simplified moment-rotation curve of the steel slotted-in plate joint commonly used in practical projects is determined. By varying the parameters, 6480 calculation cases were completed and the plasticity reduction factors under different height-span ratios and joint stiffness reduction factors were calculated, which were all smaller than the recommended value of 0.47 for steel reticulated shells. Then the corresponding elasto-plastic overall stability safety factors were obtained.

KEYWORDS: timber reticulated shell; finite element analysis; semi-rigid joint; plasticity reduction factor; overall stability

1 – INTRODUCTION

Timber reticulated shell structure is a structural form with strong spanning capacity and high structural efficiency, and is widely used in large-span timber structures. Like other thin-shell structures, the overall stability analysis is a key issue in the design of timber reticulated shell structure, which is affected by parameters such as the geometric dimensions of reticulated shell, the cross-section of the members, the load distribution, the stiffness of the joints, the initial defects., etc.

The design of steel reticulated shell structure in China [1] uses stability safety factor to determine the overall stability of the structure, and the overall stability allowable bearing capacity should be equal to the overall stability ultimate bearing capacity divided by the stability safety factor. When the elasto-plastic full-process is analyzed, that is, geometric nonlinearity, material nonlinearity and initial geometric defects are considered, the stability safety factor can be 2.0. When the elastic full-process is analyzed, that is, geometric nonlinearity and initial geometric defects are considered, but the material is elastic, the stability safety factor can be 4.2.

At present, Chinese scholars' researches on the overall stability of timber reticulated shell mainly focus on the experiment or finite element analysis of the semi-rigid joints, to study the bending moment-rotation relationship and failure mode of the joints, and the results are used to study the influence of joint stiffness and other factors on the ultimate bearing capacity of the structure. The parametric study of the stability safety factor as well as the plasticity reduction factor for semi-rigid timber reticulated shell is still imperfect.

In this paper, a parametric analysis is carried out for the overall stability performance of K6-type single-layer spherical timber reticulated shell, which is a common form of Kevitt-type reticulated shell with uniform grids and high stiffness, and is widely used in practical engineerings. In the analysis, the nonlinear spring element is used to

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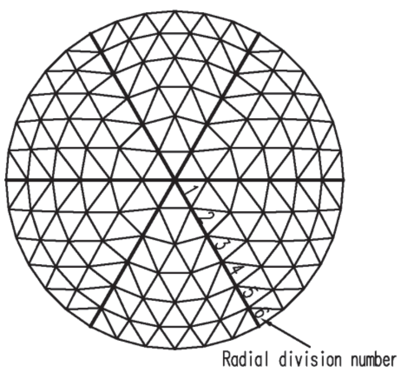
simulate the slotted-in plate semi-rigid joint, and the elastic ultimate bearing capacity factor K_e, the elasto-plastic ultimate bearing capacity factor K_p and the plasticity reduction factor c_p of the structure are calculated, and the design suggestions are given. The ultimate bearing capacity factors, K_e and K_p, are the ratios of the load value at the first critical point obtained from the elastic and elasto-plastic full-process analysis of the structure, respectively, to the characteristic value of the load. They reflect the safety degree of the structure. The plasticity reduction factor c_p is the ratio of the ultimate bearing capacity factors K_p to K_e, which reflects the influence degree of material nonlinearity on the ultimate bearing capacity of the structure.

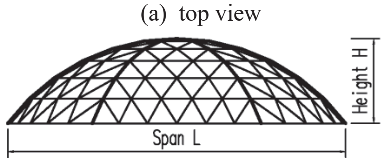
2 – NUMERICAL MODEL

2.1 BASIC COMPONENT

Fig. 1 shows a K6-type single-layer spherical reticulated shell with span L and height H.



(a) top view



(b) front view

Fig.1 K6-type single-layer spherical reticulated shell

In this paper, ANSYS software is used for modeling and analysis. Fig. 2 shows the basic component, which consists of rigid beams at both ends, a timber beam in the middle, and connections between the two.

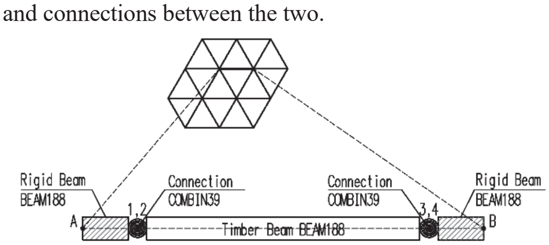


Fig.2 Basic component of reticulated shell

The rigid beams and timber beam are simulated using beam element BEAM188, while the connections are simulated using nonlinear spring element COMBIN39. Nodes 1 and 4 of the rigid beams coincide with nodes 2 and 3 of the timber beam, respectively. Adjust the nodal coordinate system of the above nodes so that the nodal local z-axis is perpendicular to the OAB plane, as shown in Fig. 3, ensuring that the nodal local z-axis aligns with the timber beam's curvature central axis within the plane of the steel soltted-in plate. Set the COMBIN39 element description option KEYOPT(3) to 6, meaning the spring action direction is along the nodal local z-axis, used to simulate the the semi-rigid bending behavior of the joint within the plane of the steel soltted-in plate. Couple the remaining degrees of freedom of nodes 1 and 4 and nodes 3 and 4, respectively.

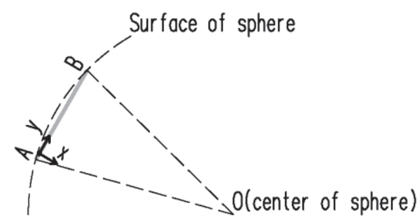


Fig.3 Nodal coordinate system of COMBIN39 element

Structural instability includes both member instability and overall instability, which are addressed through member stability verification and overall stability verification to ensure safety. According to JGJ 7-2010 "Technical specification for space frame structures" [1], since the members have already been verified to meet the requirements, it is unnecessary to repeatedly consider member-level instability during the overall stability analysis.

In order to avoid the impact of member instability on the nonlinear full-process analysis, the middle timber beams are not segmented. The roof loads are applied to the intersection nodes based on subordinate areas, and support constraints are applied to the outermost nodes. The established finite element model is shown in Fig. 4. It should be noted that in order to show the elements, the length of the rigid beam in Fig. 4 is greater than the value taken in the actual model.

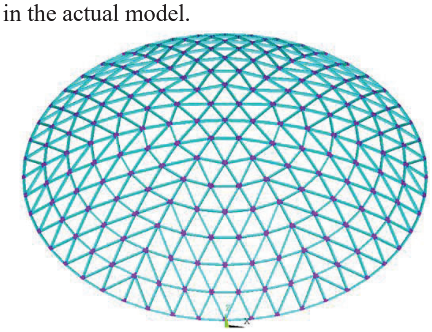


Fig.4 Geometry model of reticulated shell

2.2 MATERIAL

In this study, the glulam is treated as a transversely isotropic material. The material parameters parallel to grain are taken according to GB 50005-2017 "Standard for design of timber structure" [2]. The parameters perpendicular to grain are based on the recommended values provided in the "Manual for Timber Structure Design" [3]. In ANSYS, the Hill yield criterion is

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employed to simulate the transversely isotropy behavior of the glulam, while the constitutive curve parallel to grain is modeled using a bilinear shape as shown in Fig. 5. f_c is the compressive strength, ε_{c0} is the corresponding compressive strain, and ε_{cu} is the ultimate compressive strain; f_t is the tensile strength, and ε_{t0} is the corresponding tensile strain.

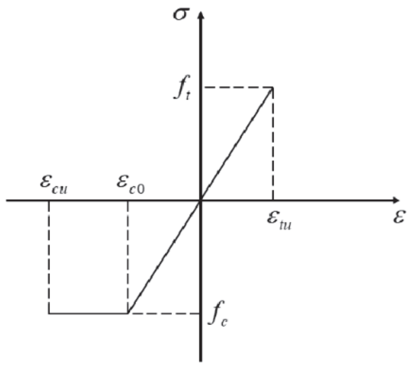


Fig.5 Constitutive curve of timber parallel to grain

2.3 SEMI-RIGID CONNECTION

In this study, the steel slotted-in plate joint widely used in actual engineerings are adopted and its stiffness range is estimated.

The bearing capacity and slip stiffness of the dowel-type fasteners are calculated according to Eurocode 5 [4]. The slip stiffness K_u of a single fastener in the steel slotted-in plate joint for the ultimate limit state is calculated as follows:

$$K_{\text{ser}} = \frac{2}{3} \times 4 \times \frac{\rho_{\text{m}}^{1.5} d}{23} \tag{4}$$

where ρ_m is the mean density in kg/m³, and d is the diameter of the fastener in mm. Equation (4) takes into account the adjustment of the stiffness by the number of shear planes of the fastener and the density of the steel.

Considering potential formula errors and initial gaps, an adjustment factor c_K not more than 1.0 is applied to the slip stiffness K_u:

$$K = c_K K_{\text{ser}} \tag{5}$$

where K is the slope of the first segment of the adopted dual-linear moment-rotation curve, while the slope of the second segment is set to zero. The stiffness adjustment factor c_K is assigned values of 1, 1/2, or 1/4.

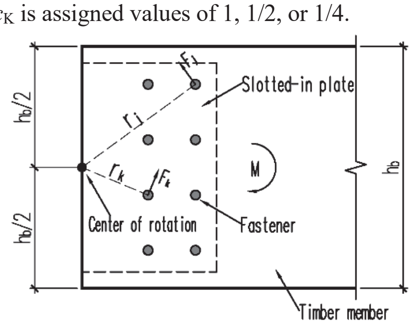


Fig.7 Joint forces under bending moment

Due to limitations of precision in manufacturing and installation, the timber member end may not be in full contact with the steel end plate. For safety considerations, the center of rotation is taken as the centroid of the cross-section height of the timber member end, as shown in Fig. 7. M is the moment at the member end, and h_b is the cross-section height of the timber member.

The force F_j on the fastener at a distance r_j from the center of rotation is the first fastener to reach its ultimate bearing capacity $F_{u,j}$. Noting that the corresponding joint rotation is θ_1 and the joint bending moment is M_1, there are:

$$\theta_1 = F_{\text{u,j}} \big/ \left(r_{\text{j}} K \right) \tag{6}$$

$$M_1 = \theta_1 \sum_{i=1}^{n} r_{\text{i}}^2 K \tag{7}$$

The force F_k on the fastener at a distance r_k from the center of rotation is the last fastener to reach its ultimate bearing capacity $F_{u,k}$. Noting that the corresponding joint rotation is θ_2 and the joint bending moment is M_2, there are:

$$\theta_2 = F_{\text{u,k}} \big/ \left(r_{\text{k}} K \right) \tag{8}$$

$$M_2 = \theta_2 \sum_{i=1}^{n} r_k^2 K \tag{9}$$

The moment-rotation curve of the joint rotation spring is shown in Fig. 8.

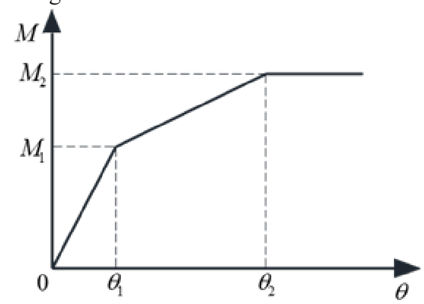
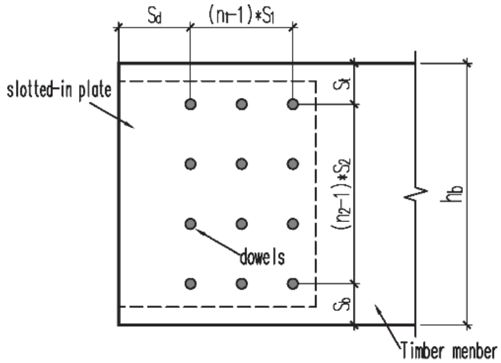
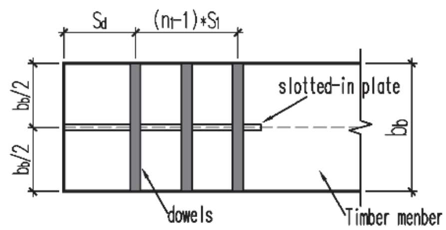


Fig.8 Moment-rotation curve of the joint

Referring to actual projects, the selected timber member cross-section and the corresponding slotted-in plate and dowels are shown in Table 1, and the dowels arrangement patterns is shown in Fig. 9 and Table 2, where b_b is the timber member cross-section width, t is the steel plate thickness, and d is the diameter of the dowels. A total of four timber member cross-sections were selected, labeled as SC1 to SC4. Each cross-section is paired with a fixed steel plate thickness and six dowels arrangement patterns. There are two dowel diameters for each set of sections, d_1 and d_2, respectively. The dowels arrangement patterns are labeled from I to VI. n_1 is the number of columns of dowels, n_2 is the number of rows of dowels. S_d is the edge distance of the columns, while S_t and S_b is the edge distances at the top and bottom of the rows. S_1 is the column spacing, and S_2 is the row spacing.



(a) Front view



(b) Top view

Fig.9 Arrangement of dowels in the joint

Table 1 Parameters of the joint

Labels	b_b/mm	h_b /mm	t/mm	d_1/mm	d_2/mm
SC1	100	300	10	12	16
SC2	130	390	14	16	20
SC3	170	510	18	20	24
SC4	220	660	22	24	30

Table 2 Arrangement patterns of dowels in the joint

Patterns	n_1	n_2	d	S_d	S_t	S_b	S_1	S_2
I	4	4	d_1	$7d_1$	$4d_1$	$4d_1$	$5d_1$	uniformly distributed in the remaining height
II	3	4						
III	3	3	d_2	$7d_2$	$4d_2$	$4d_2$	$5d_2$	
IV	2	3						
V	3	3	d_1	$7d_1$	$4d_1$	$4d_1$	$5d_1$	
VI	2	3						

3 –PLASTICITY REDUCTION FACTOR

JGJ 7-2010 "Technical specification for space frame structures" [1] specifies the stability safety factor K_s for steel reticulated shell. When analyzed by elasto-plastic full-process, K_s can be taken as 2.0. When analyzed by elastic full-process, K_s can be taken as 2.0 divided by plasticity reduction factor c_p. For the steel reticulated shell, based on numerous case studies from a statistical perspective, c_p is typically taken as 0.47, and the corresponding K_s is 4.2.

Referring to the design method of steel reticulated shells [1], the elasto-plastic stability safety factor $K_{s,p}$ and the elastic stability safety factor $K_{s,e}$ for timber reticulated shell can be expressed as:

$$K_{s,p} = \gamma_q \gamma_R \gamma_0 \tag{10}$$

$$K_{s,e} = K_{s,p} / c_p \tag{11}$$

where γ_q is the load partial factor; γ_R is the resistance partial factor; and γ_0 is the adjustment factor considering other uncertainties.

For common timber reticulated shells, the load partial factor γ_q can be determined based on the ratio of dead load to live load, generally taken as 1.35 for roof structures. The resistance partial factor γ_R, determined according to the ratio of strength characteristic value to design value, is typically taken as 1.45 according to GB50005-2017 "Code for Design of Timber Structures" [18]. Considering other uncertainties, the adjustment factor γ_0 can be taken as 1.20. Therefore, the elasto-plastic safety factor $K_{s,p}$, calculated using (10), can be approximately taken as 2.4, which is larger than the corresponding value of 2.0 for steel reticulated.

In order to obtain the elastic stability safety factor $K_{s,e}$ using (11), it is necessary to obtain the statistical value of the plasticity reduction factor c_p. Parametric calculations are performed on complete combination of parameters listed in Table 3, totaling 6,480 cases. In Table 3, the optimal cross-section refers to the one that achieves the highest utilization rate in load bearing capacity verification according to GB50005-2017 "Code for Design of Timber Structures" [18]. The elastic ultimate bearing capacity factor K_e and elasto-plastic ultimate bearing capacity coefficient K_p for each case are calculated to obtain its plasticity reduction factor c_p, which equals K_e divided by K_p.

The statistical values of c_p are determined using quintile values, as shown in Table 4. When the height-to-span ratio or joint stiffness adjustment factor falls between the parameters listed in the table, interpolation can be performed.

Table 3 Values of parametric calculations

Parameter	Value
Timber Strength Grade	TC_T28，Oven-day density is 460kg/m^3 TC_T32，Oven-day density is 500kg/m^3
Support	Fixed hinge
Initial defect ratio	1/300
Height-to-span ratio	1/6, 1/5, 1/4, 1/3, 1/2
Span	40m, 60m, 80m
Radial division number	5, 6, 7（for 40m span） 7, 9, 11（for 60m span） 9, 12, 15（for 80m span）
Dead load	1.0kN/m^2
Live load	0.5 kN/m^2, 1.0 kN/m^2
Live load pattern	Full-span uniform, Half-span uniform
Dowels layout pattern	I, II, III, IV, V, VI
The joint stiffness adjustment factor	1, 1/2, 1/4
Cross section	The optimal cross-section from： 100mm×300mm,130mm×390mm, 170mm×510mm,220mm×660mm

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Table 4 Statistical result of plasticity reduction factors

The joint stiffness adjustment factor	The plasticity reduction factor c_p				
	H/L=1/6	H/L=1/5	H/L=1/4	H/L=1/3	H/L=1/2
1	0.37	0.29	0.28	0.23	0.22
1/2	0.38	0.33	0.30	0.28	0.28
1/4	0.42	0.40	0.34	0.34	0.33

As shown in Table 4, the plasticity reduction factor c_p decreases as the height-to-span ratio increases and increases as the joint stiffness adjustment factor decreases. Considering that the fastener slip stiffness calculated by EC5 [4] may be overestimated, some studies [5-7] have suggested that when an adjustment factor of 1/4 is applied, the results align more closely with experiments or finite element results. Therefore, in the absence of joint test or numerical simulation data for practical engineering projects, it is recommended to take the adjustment factor as 1/4. The plasticity reduction factors c_p are 0.42, 0.40, 0.34, 0.34, and 0.33 for the height-to-span ratios of 1/6, 1/5, 1/4, 1/3, and 1/2, respectively, and the corresponding elastic stability safety factors $K_{s,e}$ are 5.7, 6.0, 7.1, 7.1, and 7.3, respectively, which are all greater than the corresponding value of 4.2 for steel reticulated shell.

4 –CONCLUSION

In this paper, a parametric finite element study is carried out on the overall stability safety factor and plasticity reduction factor of timber reticulated shells with reference to the Verification method of the overall stability of steel reticulated shells in the Chinese code. The main conclusions are as follows:

(1) The elasto-plastic stability safety factor $K_{s,p}$ of timber reticulated shells is taken to be 2.4, which is larger than the recommended value of 2.0 for steel reticulated shells.

(3) Statistical results of the plasticity reduction factor c_p are given by a parametric finite element analysis for total 6480 cases, so as to determine the value of the elastic stability safety factor $K_{s,e}$ of timber reticulated shells. The plasticity reduction factor c_p decreases as the height-to-span ratio increases and increases as the joint stiffness adjustment factor decreases, and it should be noted that its values are all smaller than the corresponding value of 0.47 for steel reticulated shells.

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