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MACHINE LEARNING INFORMED SIMULATION OF UNCERTAINTIES FOR PROGRESSIVE DAMAGE EVALUATION IN WOOD VENEER LAMINATES

Johannes Reiner[1]

ABSTRACT: This study presents the key steps in developing a robust computational framework that can represent the inherent uncertainties of mechanical properties in wood veneer laminates subjected to progressive fracture tests. A large dataset derived from efficient finite element simulations of compact tension tests serves as the foundation for developing a machine learning surrogate model by means of Gaussian process regression. This fast, yet accurate surrogate model is then coupled with a Markov Chain Monte Carlo method and statistical measurements from experiments to estimate the uncertainty of each finite element input parameter that contributes to the measured uncertainty of the experiments. This framework combines various computational methods to account for uncertainties in the simulation of thin wood veneer laminates, hence paving the way for efficient and realistic finite element simulations of wooden materials that can guide the design of safe and reliable structures.

KEYWORDS: veneers, progressive damage, finite element simulation, machine learning, Markov Chain Monte Carlo

1 – INTRODUCTION

The transportation sector is directly or indirectly responsible for a large part of the world's greenhouse gas emissions. Similar to many other countries, Australia has committed to a net zero emission target by 2050. Therefore, the use of sustainable lightweight materials from renewable resources is crucial for reducing emissions in transport applications [6].

Thanks to its capability to absorb and store carbon, wood is regarded as an environmentally friendly material, in contrast to conventionally used metals such as steel and aluminium. Many analytical and empirical methods exist to estimate the mechanical behaviour of wood structures, mostly limited to static loading conditions and elastic properties.

With the goal of introducing thin wood veneer laminates into future transport applications, extreme loading scenarios, such as dynamic load-bearing and crash-loaded structures, become relevant [8]. In these cases, most analytical methods are not applicable. Hence, more sophisticated simulation tools, such as Finite Element Analysis (FEA), are needed to predict the mechanical behaviour of wood veneer laminates in these load cases, with an emphasis on damage resistance [13]. Furthermore, the conventional application of wood veneers is insufficient to satisfy the critical safety standards (e.g. stiffness, strength, crashworthiness) in the automotive industry, therefore simulations can aid in optimising wooden structures with respect to thickness, weight or layup.

To accurately simulate the mechanical behaviour of natural products such as wood veneers, it is essential to account for their natural variation in mechanical properties. These variations may result from differences in geographical location, age of the tree, temperature, and moisture content. While many FEA studies of wood focus on replicating average values measured from experiments, they often overlook these inherent uncertainties. Typically, a single set of FEA input parameters is proposed to replicate the mean of experimental measurements.

Markov Chain Monte Carlo (MCMC) methods can estimate distributions of input parameters rather than single deterministic values [12]. However, these methods require many model evaluations, making them computationally expensive. To address this limitation, Machine Learning (ML) surrogate models can be developed to achieve the required speed-up [15]. These ML surrogate models can be trained and validated using data obtained from physically meaningful FEA results [17, 20].

This study presents a combination of FEA, ML and MCMC methods to estimate the distribution of various FEA input parameters in order to represent the variation in measured load vs displacement data obtained from compact tension tests of thin Beech veneer laminates [16]. This study builds on a collaboration with the German Aerospace Center (DLR) to explore safe and reliable designs of thin wooden structures as a sustainable alternative in future transport applications [6].

2 – MATERIALS & EXPERIMENTS

The material of interest is rotary-cut European Beech (Fagus sylvatica) veneers, procured from Metz & Co, Germany. Quasi-isotropic $[90/45/0/-45]_s$ veneer laminates were manufactured by stacking up individual veneer plies with varying grain orientations. The adhesive PURBOND HB S109 [7] was applied to each ply with a

[1]School of Engineering, Faculty of Science Engineering and Built Environment, Deakin University, Geelong, Australia, johannes.reiner@deakin.edu.au

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glue spread of 90-100 g/m^2 before consolidating the laminates in a hydraulic press. The laminates were uniformly pressed at 1 MPa for 20 hours to ensure complete curing of the adhesive and to eliminate press time as a potential variable influencing the mechanical behaviour. The average thickness of these laminates after manufacturing was 4.37 mm. The moisture content of the laminates is approximately 8% – 12% [10].

Compact Tension (CT) tests are suitable to measure damage resistance. Here, double-tapered CT test samples (more details in Section 3.1), were used to evaluate progressive crack growth in the quasi-isotropic $[90/45/0/-45]_s$ Beech veneer laminates [16]. The tests were performed using an Instron 4505 testing machine with a 10 kN load cell and a cross-head displacement rate of 0.5 mm/min. Figure 1 shows the resulting force vs displacement graphs obtained from 10 CT tests. In addition, Open-Hole Tension (OHT) tests were conducted [16], consisting of 140 mm long and 20 mm wide test samples clamped at both ends over a length of 15 mm. The central hole has a diameter of 12 mm (more details about the geometry can be found in Section 4). The analysis of 20 OHT tests resulted in an average OHT strength of 17.3 MPa with a coefficient of variation of 6.9%.

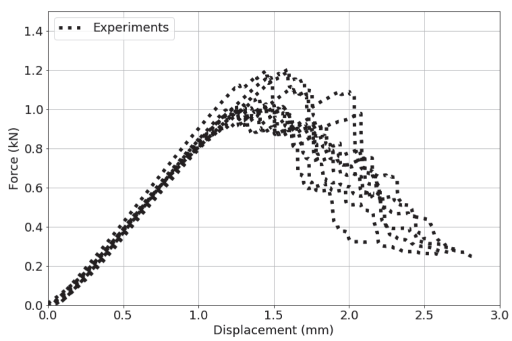

Figure 1: Force vs displacment graphs from 10 compact tension tests on quasi-isotropic $[90/45/0/-45]_s$ *Beech veneer laminates.*

During manufacturing and mechanical testing, care was taken to achieve high-quality test samples, and to minimise the spread of mechanical properties. Nevertheless, as shown in Figure 1, the mechanical behaviour of the natural material can vary significantly, necessitating the application of simulation methods that can represent these uncertainties.

3 – BAYESIAN PARAMETER ESTIMATION: SIMULATION OF COMPACT TENSION TESTS

The Bayesian method assumes parameters to be random variables whose unknown probability distributions, referred to as posterior distributions, quantify the probability of assuming any value in the considered parameter space. MCMC is one of the most popular Bayesian methods. It evaluates stochastic processes of ”walkers” to explore the full parameter space efficiently in order to find posterior distributions. To quantify the uncertainty measured in CT tests of the Beech veneer laminates using MCMC, fast FEA models and even faster ML surrogates are required.

3.1 FINITE ELEMENT ANALYSIS

The mechanical behaviour of the $[90/45/0/-45]_s$ Beech veneer laminates is described by the strain-based COMposite DAMage Model (CODAM2). This material model is incorporated as MAT219 in the commercial FEA software LS-DYNA [9]. A detailed description of the material model can be found in related literature [2, 3, 19]. Figure 2 illustrates the constitutive behaviour of CODAM2 in the principal directions of a veneer ply in the grain (longitudinal) direction and perpendicular (transverse) to it. In total, the material model requires eight input parameters: four related to the elastic behaviour (E_1, E_2, G_{12} and ν_{12}), and four strain-based inputs related to damage onset and evolution (ε_1^i, ε_2^i, ε_1^s, ε_2^s).

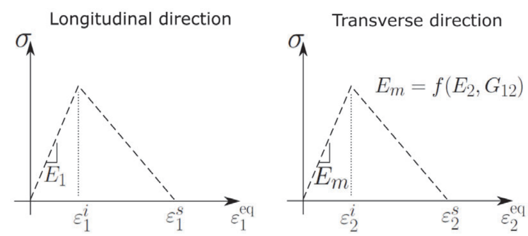

Figure 2: Illustration of CODAM2 stress-strain curves in (a) longitudinal (grain) and (b) transverse direction.

CODAM2 is capable of describing laminate behaviour using a single through-thickness integration point. This feature makes CODAM2 a significantly more efficient material model in comparison to other continuum damage models that require assigning ply-based material properties to multiple integration points through the thickness to describe laminates with various ply angles. The enhanced efficiency of CODAM2 further facilitates its integration with data-driven calibration methods, such as genetic algorithms [4] and ML [17, 20].

The CODAM2 material model is applied to simulate progressive damage in CT tests shown in Figure 3. The FEA model features a mesh that incorporates only one shell element through its thickness. Within the expected fracture process zone, the in-plane element size is 1 mm × 1 mm. A displacement is prescribed to the rigid loading pins (grey) in opposite vertical directions, as illustrated in Figure 3. The modelling of the quasi-isotropic $[90/45/0/-45]_s$ Beech veneer laminates consisting of only one through-thickness shell element leads to highly efficient computation times. A single CT simulation only takes 2–3 minutes on a conventional computer with 4-8 CPUs.

Such high efficiency is necessary to create large datasets for training and testing of ML surrogate models. Here, 6,000 FEA simulations with varying input parameters have been conducted. Table 1 lists the range of input parameters used to create the large dataset. Note that the

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Table 1: *Range of FEA input parameters to create large datasets for the generation of machine learning surrogate model.*

Description	Input parameter	Uniform range
Longitudinal modulus	E_1	7.5 - 20 GPa
Transverse modulus	E_2	0.5 - 4 GPa
Longitudinal damage initiation strain	ε_1^i	0.01 - 2 %
Transverse damage initiation strain	ε_2^i	0.01 - 2 %
Longitudinal damage saturation strain	ε_1^s	2 - 20 %
Transverse damage saturation strain	ε_2^s	2 - 20 %

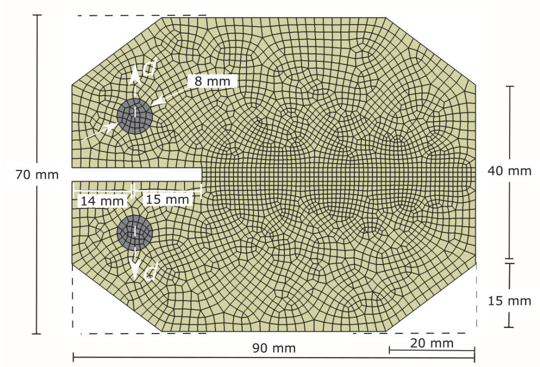

Figure 3: Geometry and dimensions of FEA model to simulate progressive damage evolution in compact tension tests.

in-plane Poisson's ratio $\nu_{12} = 0.073$ and shear modulus $G_{12} = 1.08$ GPa are held constant [13].

A global sensitivity analysis on this large dataset indicates that only three input parameters are critical for simulating the force vs displacement responses in CT tests [15]. These inputs relate to the grain (longitudinal) direction of a veneer ply, namely E_1, ε_1^i and ε_1^s.

3.2 MACHINE LEARNING

To create faster model evaluations, the large dataset from FEA simulations is divided into 80% for training and 20% for testing. Gaussian Process Regression (GPR) is used in Python to develop fast ML surrogate models of the CT tests, with the aim of reproducing the complete force vs displacement curve based on the three critical FEA input parameters. Hence, the GPR inputs are E_1, ε_1^i and ε_1^s, and the outputs are force vs displacement data. The GPR kernel selection was fine-tuned, with the optimal configuration determined based on mean squared error and R^2 scores from cross-validation. The Exp-Sine-Squared kernel, characterised by a length scale parameter l and a periodicity parameter p, was identified as the optimal kernel. Here, the optimal kernel parameters are $l = 1$ and $p = 3$. The application of the testing data revealed a root mean squared error of 0.0615 kN. Figure 4 shows selected comparisons of force vs dispalcement graphs obtained from actual FEA and the developed ML surrogate model. The results demonstrate that the trained GPR model is capable of replicating the simulated force vs displacement curves with reasonable accuracy. A notable advantage of using GPR simulations is the ability to rapidly evaluate multiple ML surrogates within seconds, thereby providing a convenient and efficient method for estimating force vs displacement graphs.

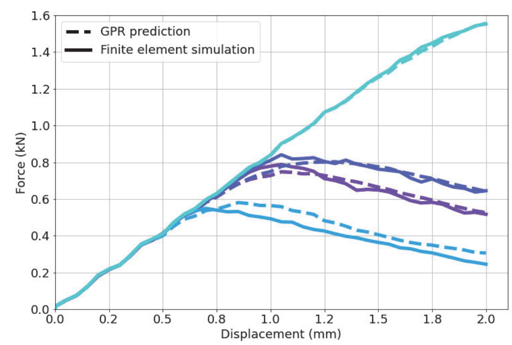

Figure 4: Examples of force vs displacement graphs in compact tension tests obtained from actual FEA simulations (solid lines) and machine learning surrogates (dashed lines).

3.3 MARKOV CHAIN MONTE CARLO

MCMC generates samples that estimate the input parameters by taking into account the prior distributions of the FEA input parameters, the experimental data shown in Figure 1, as well as the fast ML surrogate model outlined in previous section. Bayes' rule [5] in Equation (1) can estimate the posterior density $p(x_{\text{red}}|y_{\text{test}})$, where x_{red} denotes reduced FEA inputs E_1, ε_1^i and ε_1^s, and y_{test} the experimental data from CT tests shown in Figure 1.

$$\underbrace{p(x_{\text{red}}|y_{\text{test}})}_{\text{posterior}} \propto \underbrace{p(x_{\text{red}})}_{\text{prior}} \quad \underbrace{p(y_{\text{test}}|x_{\text{red}})}_{\text{likelihood}} \tag{1}$$

Bayes' rule states that the posterior density is proportional to the product of assumed prior densities $p(x_{\text{red}})$ of the FEA inputs and the likelihood $p(y_{\text{test}}|x_{\text{red}})$. The likelihood is evaluated by the GPR surrogate model $\mathcal{M}(x_{\text{red}})$, and is assumed to follow a distribution such that

$$p(y_{\text{test}}|x_{\text{red}}) = \mathcal{N}\left((y_{\text{test}} - \mathcal{M}(x_{\text{red}})), \Sigma_{\text{test}}\right), \tag{2}$$

where the experimental data follow a multivariate normal distribution $y_{\text{test}} \sim \mathcal{N}(\mu_{\text{test}}, \Sigma_{\text{test}})$, with mean vector μ_{test} and covariance matrix Σ_{test}, representing the average force vs displacement curve from experiments and their standard deviation, respectively.

To conduct MCMC, the affine invariant ensemble sampler implemented in the EMCEE Hammer [1], available in Python, is employed. Upon reaching convergence, Figure 5 shows the resulting MCMC simulations, which use

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the computed distributions of FEA input parameters. It can be seen that these simulations are capable of replicating the variation observed in experimental CT tests. The assumed prior distributions and the resulting posterior distributions of the three FEA input parameters E_1, ε_1^i and ε_1^s are shown in Figure 6. These results show that the posterior distributions for E_1 and ε_1^s differ significantly from their assumed uniform priors. The selection of the normal prior for ε_1^i was informed by a previous study on using genetic algorithms to estimate this distribution [13]. The obtained distributions enable the simulation of mechanical tests, including the consideration of uncertainty in $[90/45/0/-45]_s$ Beech veneer laminates.

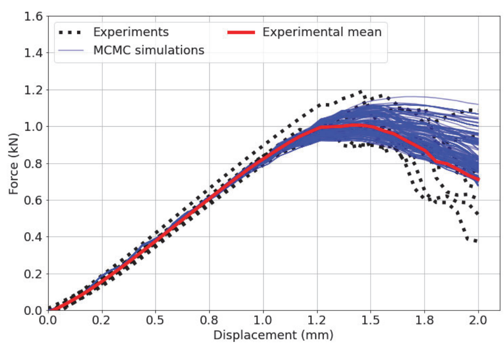

Figure 5: Comparison of force vs displacment graphs from experiments and FEA simulations using inputs sampled from posterior distributions shown in Figure 6.

4 – VALIDATION: SIMULATION OF OPEN-HOLE TENSION TESTS

The CT tests outlined in Section 2 served as calibration of the distributions of the sensitive FEA input parameters shown in Figure 6. To validate these results, the OHT tests, described in Section 2, are simulated using the FEA input parameters sampled from these distributions. The remaining FEA input parameters use nominal values derived from a previous study [13] ($E_2 = 2.28\ GPa$, $\varepsilon_2^i = 0.65$ % and $\varepsilon_2^s = 8.1$ %).

As shown in Figure 7, the FEA simulation of the OHT test uses fully constrained nodes at one edge of the test sample, while a prescribed displacement is applied to the opposite edge. Here, 1,000 FEA simulations are evaluated with the sensitive FEA input parameters sampled from the posterior distributions obtained from MCMC and the nominal values for other input parameters.

Figure 8 compares the distributions of the OHT strength obtained from experiments and FEA simulations. It can be seen that the simulations not only predict the mean OHT value accurately but also estimate the variation of the measured OHT strength. Table 2 provides a quantitative comparison of these findings. The mean OHT strengths are closely aligned, with values of 17.3 MPa and 17.5 MPa for the experimental and simulated results, respectively. The coefficient of variation is 6.9% for the experimental data, compared to 10.5% for the simulated OHT tests. In addition, the results agree qualitatively. Figure 9 shows the result of a representative FEA simulation in comparison to an image of an OHT specimen after testing. The simulation shows the contour plot of the longitudinal damage variable associated with the vertical grain direction (0° ply) of the $[90/45/0/-45]_s$ Beech veneer laminate. Both the simulation and the experimental results indicate the presence of damage around the open hole, with the direction of damage propagation being perpendicular to the applied loading direction.

Table 2: Comparison of open-hole tensile strength in $[90/45/0/-45]_s$ *Beech veneer laminates.*

	Experiments*	Simulations**
Mean	17.3 MPa	17.5 MPa
Coefficient of variation	6.9%	10.5%

* based on 20 tests.
** based on 1,000 FEA simulations.

5 – DISCUSSION

The simulation results from the validation against OHT tests demonstrate that the presented framework can effectively predict the variations observed in mechanical tests of quasi-isotropic $[90/45/0/-45]_s$ Beech veneer laminates. This unique capability will enable the virtual development of design standards for crashworthiness of thin wood veneer laminates, which may be used in future automotive applications.

The mechanical tests conducted in this study were selected to be in the tensile direction, acknowledging that such tests have been successfully conducted in both experiments [16] and simulations [13]. It should be noted that other loading scenarios, such as compression or shear-dominated load cases, are equally important. Similarly, strain-rate sensitivities [14] should be taken into account when investigating crashworthiness involving highly dynamic loading conditions.

The presented framework integrates a range of computational techniques, each of which may be replaced with alternative methods. For example, FEA material models other than CODAM2 can be explored for the simulation of progressive damage in wood veneer laminates [11]. In particular, LS DYNA's MAT143 [18] is a promising candidate as it is specifically formulated for wood materials. Nevertheless, this material card is currently restricted to solid elements, which significantly increases the computational cost associated with simulating the mechanical behaviour of thin-walled wood veneers. Regarding ML surrogate models, alternative regression techniques can be considered. It has been demonstrated that Long Short-Term Memory (LSTM) architectures are capable of accurately representing load vs displacement curves obtained from carbon fibre reinfroced composites subjected to compact tension tests [17].

Within MCMC, a common challenge is the justified selection of prior distributions. It is known that Bayesian methods are sensitive to prior distributions. In instances where limited knowledge is available, uniform distributions, or flat priors, are typically used. Here, uniform pri-

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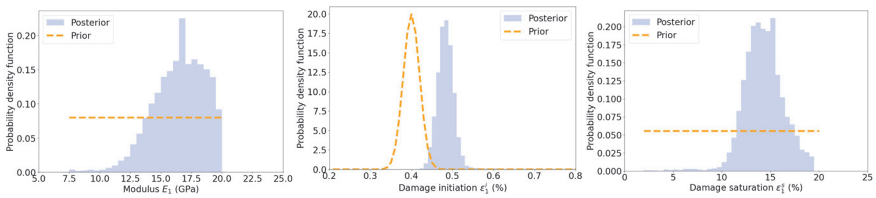

(a) Modulus E_1 *(b) Damage initiation ε_1^i* *(c) Damage saturation ε_1^s*

Figure 6: Prior and posterior distributions of sensitive FEA input parameters used in Markov Chain Monte Carlo method.

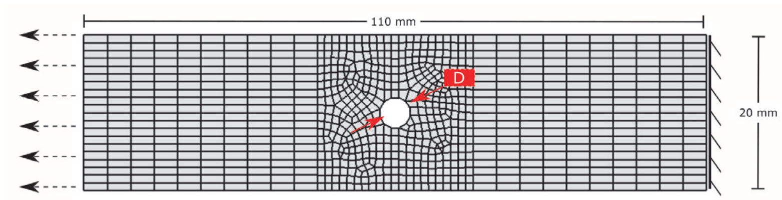

Figure 7: Geometry and dimensions to simulate open-hole tension test with hole diameter $D = 12$ *mm.*

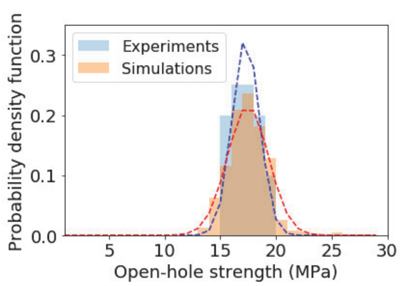

Figure 8: Experimental and simulated distributions of open-hole tensile strength in $[90/45/0/-45]_s$ *Beech veneer laminates.*

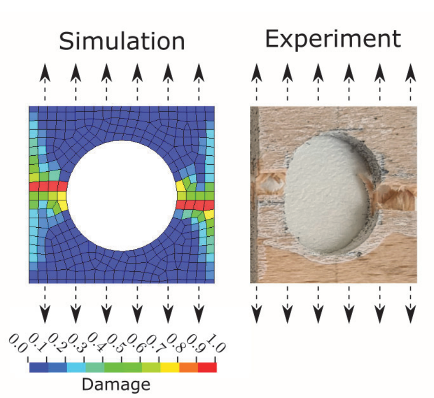

Figure 9: Qualitative damage comparison between representative FEA simulation and experiment after open-hole tensile tests in $[90/45/0/-45]_s$ *Beech veneer laminates.*

ors were selected for the modulus E_1 and the damage saturation strain ε_1^s, see Figure 6. Ideally, the priors should be informed by isolated experimental measurements. For example, the longitudinal modulus E_1 could be independently measured through tensile tests of uni-directional wood veneers, and this data could inform the prior selection.

Irrespective of replacing some of these computational techniques, the presented framework remains valid and represents a transition from empricial modelling or determinstic simulations to a series of computational methods that enable the consideration of uncertainties in the mechanical behavior of wood veneer laminates. Future work will focus on applying these techniques to large-scale automotive components, with the goal of designing sustainable wooden structural parts that could incrementally replace traditional metallic parts.

6 – CONCLUSION

This study presents a combined experimental-numerical framework to incorporate uncertainty into Finite Element Analysis (FEA) of progressive damage evolution in Beech veneer laminates subjected to compact tension tests. The key requirements include a sufficiently large experimental dataset and a fast prediction method in order to apply Markov Chain Monte Carlo. It is demonstrated that a machine learning surrogate model, based on FEA-simulated data and Gaussian Process Regression, provides the necessary speed-up to estimate the distribution of sensitive FEA input parameters. Validation against open-hole tension tests indicates that FEA simulations, using the obtained input distributions, are capable of predicting the mean values and standard deviations of open-hole tension strength.

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