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DISCRETE OPTIMIZATION OF TIMBER STRUCTURES WITH M I N L P

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ABSTRACT: The paper deals with the discrete optimization of timber structures. Mixed-integer nonlinear programming is applied. For each structure, a superstructure of various structural alternatives and an optimization model are developed. The cost or mass objective function of the structure is subjected to the constraints of inner forces, stresses and dimensioning. The defined problem is solved with a modified outer-approximation/equality-relaxation algorithm. Cost optimizations of a timber-concrete composite floor system, a timber floor joist and a timber-steel hall structure are briefly presented here. For given input data and unit prices, the minimal self-manufacturing costs of the structures are determined together with the optimal number of structural elements and their sizes. MINLP proves to be a valuable method for the optimization of timber structures. Structural optimization is suitable for teaching at universities as well as for use in research and engineering practice.

KEYWORDS: timber structures, cost optimization, mixed-integer nonlinear programming, MINLP

1 – INTRODUCTION

The optimization of timber structures is an ever-increasing challenge, as it enables the calculation of the most favourable (economical) structural design in terms of material, energy and labour consumption. Minimizing the production costs of structures reduces the purchase price for the customer and increases the manufacturer`s competitiveness on the market, while also reducing wood consumption and logging, which is important for sustainable development.

Recent advances in the optimization of timber structures have focused on improving structural efficiency, material usage and sustainability. Over the past five years, various optimization techniques have been applied to different types of timber structures. For example, Mayencourt and Mueller [1] have researched the optimization of the structure of cross-laminated timber panels, focusing on the reduction of material consumption through a nonlinear programming approach using the fmincon solver in Matlab. Pech et al. [2] investigated the optimization of glued laminated timber beams employing metaheuristic algorithms to achieve efficiency. Garcia and Thompson [3] utilized parametric design in conjunction with gradient-based optimization to enhance the seismic performance of timber frame buildings. Lee and Kim [4] applied topology optimization to cross-laminated timber (CLT) panels and achieved significant weight reduction without compromising structural integrity. Santos et al. [5] aimed at cost optimization for cross-insulated timber panels and also used the fmincon solver in Matlab for their analysis. Zhang et al. [6] optimized glulam beam structures using genetic algorithms to improve load-bearing capacity while minimizing material consumption. Kravanja and Žula [7] optimized a single-story timber building structure using mixed-integer nonlinear programming, MINLP. Similarly, Jelušič and Kravanja [8] optimized timber floor joists employing a multi-parametric MINLP optimization. In another study, Nesheim et al. [9] investigated both cost and ECO2 optimization of timber floor components for adaptable buildings applying a mixed-integer sequential linearization method for the optimization process. In addition, Müller et al. [10] investigated multi-objective optimization for timber truss systems, using particle swarm optimization to balance cost efficiency and structural performance. These studies highlight the growing trend of integrating advanced computational methods into the

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design of timber structures, leading to more efficient, sustainable and resilient structural solutions.

This paper focuses on the discrete optimization of timber structures using MINLP. MINLP is a versatile method that can handle both continuous and discrete optimization variables simultaneously. Continuous variables are used to optimize continuous dimensions, stresses, masses, or costs of structures, while discrete variables determine the optimal number of structural elements, material grades and standard/discrete sizes. By using MINLP optimization, we thus obtain realistic optimal designs of structures that are directly applicable in practice. The optimization of three different timber structures is carried out and briefly explained.

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The optimization of three different timber structures is carried out and briefly explained: a timber-concrete composite floor system, a timber floor joist and a timber-steel hall structure, see Fig. 1.

2 – BACKGROUND – M I N L P

However, let us first provide some basics of the MINLP optimization approach, including mathematical formulation, algorithms, solvers, and handling of nonconvexities to obtain near-global solutions.

Since the optimization of technical systems, including timber structures, involves continuous and discrete decisions, it mathematically gives rise to a mixed-integer description involving continuous variables for continuous decisions, denoted by vector $\boldsymbol{x}$, defined over the set X of real numbers of dimensionality n (R^n), each given by the corresponding lower and upper bounds, and integer variables for discrete decisions, usually represented by binary variables and denoted by vector $\boldsymbol{y}$, from a set Y of 0-1 numbers of dimensionality m ($\{0,1\}^m$), which take values of 1 for a realized decision and 0 otherwise. The objective of optimization is usually to maximize or minimize a single selected technical performance measure (weight, energy consumption, etc.) or economic measure (profit, cost). The objective is represented by an objective function, typically consisting of fixed charges associated to the selection of alternatives $\boldsymbol{c}^{\mathrm{T}}$ and (non)linear continuous charges $f(\boldsymbol{x})$ and subject to mixed-integer nonlinear equality and inequality constraints, denoted by $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y})$ and $\mathbf{g}(\boldsymbol{x}, \boldsymbol{y})$, respectively, as well as mixed-integer linear logical constraints, which together determine the (MINLP) problem. It represents a mathematical model defined over a given technical superstructure of topological alternatives.

$$
\begin{aligned}
&\min \ Z = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{y} + f(\boldsymbol{x}) \\
&\text{s.t} \\
&\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}) = \mathbf{0} \\
&\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \leq \mathbf{0} \qquad \text{(MINLP)} \\
&\boldsymbol{C}\boldsymbol{y} + \boldsymbol{D}\boldsymbol{x} \leq \boldsymbol{b} \\
&\boldsymbol{x} \in X = \{\boldsymbol{x} \mid \boldsymbol{x} \in R^{\mathrm{n}};\ \boldsymbol{x}^{\mathrm{lo}} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\mathrm{up}}\} \\
&\boldsymbol{y} \in Y = \{0,\ 1\}^{\mathrm{m}}
\end{aligned}
$$

Note that a problem should possess positive degrees of freedom to perform its optimization. The role of optimization is then to identify an optimal solution within the feasible space by selecting the best set of constitutive alternatives (y**) and their continuous parameters (x**), at which the given optimization criterion obtains its best realization. In this way, simultaneous topology (structure) and parameter (e.g., dimensions) optimization is performed. As the number of possible combinations increases exponentially with the number of alternatives, the combinatorial complexity of MINLP problems can be very high, requiring the use of efficient algorithms. Most of these algorithms rely on decomposition, where discrete decisions are performed using mixed-integer algorithms and continuous decisions by Newtonian algorithms. Typical algorithms include Generalized Benders Decomposition by Geoffrion [11], Outer-Approximation (OA) algorithm by Duran and Grossmann [12], Sequential Linear Discrete Programming method by Olsen and Vanderplaats [13] and Bremicker et al. [14], Extended Cutting-Plane algorithm by Westerlund and Petterson [15], and the LP/NLP Branch-and-Bound algorithm by Quesada and Grossmann [16], with OA being the most

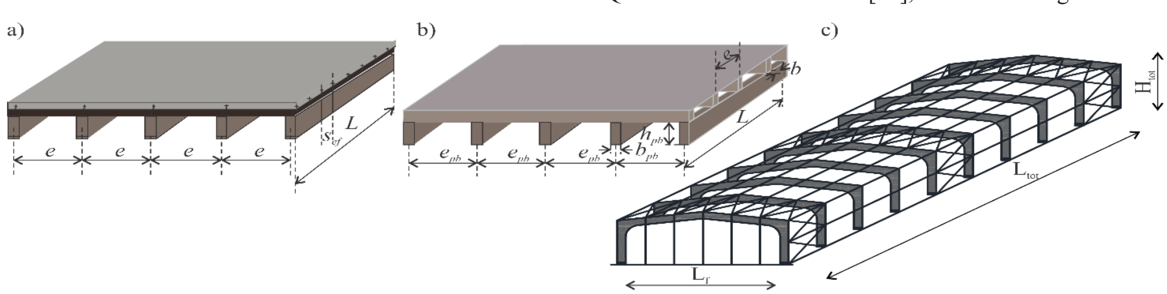

Figure 1. Three timber structures: a) a timber-concrete composite floor, b) a timber floor joist, and c) a timber-steel hall structure.

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efficient for solving highly nonlinear problems with dense matrices. These algorithms, however, are based on local methods, guaranteeing global optimality only for convex problems. On the other hand, global algorithms can only solve smaller and sometimes medium-sized MINLP problems to a global optimum. As alternatives can also be represented as disjunctions, an alternative way of problem formulation is Generalized Disjunctive Programming (GDP), which is increasingly gaining more and more attention [17]. Only the OA algorithm and its extension, the Modified Outer-Approximation/Equality Relaxation algorithm [18], are briefly explained, as the latter was used in this discrete optimization of timber structures.

The OA algorithm consists of a sequence of iterative executions of a mixed-integer linear programming (MILP) master problem and nonlinear programming (NLP) subproblems. The role of the MILP master problem, a linear approximation of the whole problem, is to perform discrete optimization, and the role of NLP subproblems is to perform continuous parameter optimization for a subsystem identified in the previous MILP master problem. At each main MINLP iteration, new linearizations derived from the NLP subproblem's solution point are added to the MILP master problem, which thus becomes more and more exact. When the direction of optimization is maximization, the MILP master problem provides an upper bound, and the NLP subproblems provide a lower bound to the MINLP problem. For convex problems, the search terminates when the bounds converge, while for nonconvex problems, it terminates when there is no improvement of the NLP solution over a predefined number of main MINLP iterations.

Since the presence of nonconvexities may cut off large portions of the feasible space, the globality of the solutions can be seriously compromised. As optimization models in mechanics are typically large scale, nonconvex, and with dense matrices, global optimization solvers cannot solve them. To circumvent the problem, the following non-structured convexifications, implemented in the Modified OA algorithm, were applied:

- Deactivation of linearizations to make them redundant when the corresponding alternatives are not selected.
- Decomposition of the objective function into groups of terms and deactivation of their linearization.
- Use of a penalty function to shift linearizations, thus opening the feasible space.
- Convexity testing.
- Either removal of linearizations that cut off the feasible space or performing some other validation of linearization.

The Modified OA algorithm was implemented in the MIPSYN synthesizer [19]. Additionally, for large-size problems with a large number of discrete variables associated with individual structural elements, the Linked Multilevel Hierarchical Strategy (LMHS) [20] can be applied to obtain a near-global solution for nonconvex problems.

3 – OPTIMIZATION MODELS

The MINLP optimization process is divided into three main steps. First, a timber MINLP superstructure is generated with a variety of different topological, material and dimensional discrete alternatives. The combination of these alternatives results in a variety of different structural design alternatives, one of which is the optimal one. A MINLP model is then formulated to encapsulate the optimization problem. Finally, an optimal solution is searched in the direction of the cost or mass objective function, taking into account the structural constraints (inner forces, stresses, deflections and dimensioning equations).

The effectiveness of the proposed MINLP approach is demonstrated by its application to various optimization tasks, such as the cost optimization of a timber-concrete composite floor system, a timber floor joist and a timber-steel hall structure. MINLP optimization models were developed for the optimization of these structures. The dimensioning constraints of the timber structures are

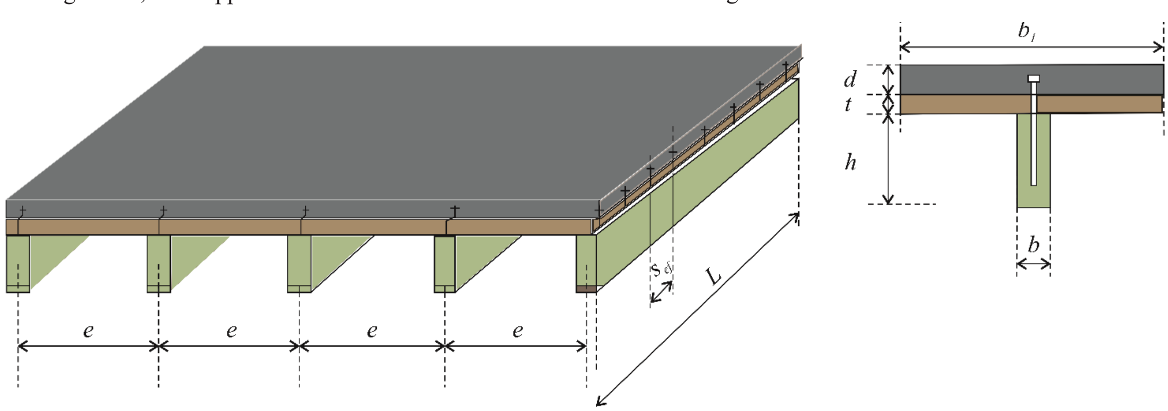

Figure 2. A timber-concrete composite floor system.

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developed for the optimization of these structures. The dimensioning constraints of the timber structures are defined according to the Eurocode specifications [21-23]. The models were modelled in GAMS (General algebraic Modelling System) [24].

3.1 TIMBER-CONCRETE COMPOSITE FLOOR SYSTEM

Timber-concrete composite floors are constructed from parallel timber beams in combination with a reinforced concrete slab, connected by shear connectors, see Fig. 2. To facilitate the construction of the concrete slab, permanent formwork panels are placed on top of the timber beams. To ensure a shear-resistant connection, various types of fasteners such as screws, anchor bolts or steel dowels can be used. In this structural system, the concrete slab primarily resists compressive forces, while the timber beams resist tensile and bending stresses. This flooring system is used to improve and strengthen existing timber floors in residential and office buildings as well as in new buildings.

To enhance the cost efficiency of timber-concrete composite floors, an optimization process was carried out. A mixed-integer nonlinear programming (MINLP) approach was implemented, which led to the development of the optimization model TCCF (timber-concrete composite floor). Since the model was designed in a general way, it allows optimization under various conditions, including different spans, applied loads and economic factors. The model includes input data (constants), variables and an economic objective function, which is subjected to equality and inequality constraints.

Input data: The input data include the span L of the floor beam, the vertical load q, the thickness of the formwork slab, the material density (steel and concrete), the properties of the fasteners (tensile strength, diameter and spacing), the amount of steel reinforcement, the materials and power unit prices, the labour cost, etc.

Variables: The variables within the model take into account the production costs of the composite structure, the total mass of the structure, the thickness of the concrete slab d, the timber beam width b, the beam height h, floor beam spacing e, the bending strength of the timber, the compressive strength of the concrete, etc.

Objective function: The objective function includes the self-manufacturing costs of the timber-concrete structure with detailed material, power and labor cost items. The objective function is subjected to the (in)equality constraints.

(In)equality constraints: These constraints limit the effective flange width, the design compressive and tensile stresses in the concrete, the design shear and bending stresses in the timber, the instantaneous and net deflections of the structure, while the design strength of the fasteners must be sufficient. Logical constraints used to calculate the standard values for dimensions and materials/strengths are also included. For more information on timber-concrete composite floors, see [25].

3.2 TIMBER FLOOR JOIST

Timber floor joists are composed of parallel timber beams combined with structural sheathing and connected by shear connectors. In order to achieve larger spans, the traditional sawn timber beams are here replaced by glulam beams. The aim is to identify the optimal costs for two different timber floor systems:

- Consisting solely of glulam beams in combination with sheathing, see Fig. 3.
- With glulam beams, secondary sawn timber beams and sheathing, see Fig. 4.

In order to achieve cost optimization of timber joist floors, a MINLP optimization model called TIMBFJ (timber floor joists) was created. This optimization model includes input data (constants), variables, a cost objective function of the structure, structural analysis/dimensioning constraints, and logical constraints.

Input data: Primary input data include the span of the structure L, the imposed load q, the material properties (moduli of elasticity, characteristic strengths, specific weights), various factors (amplification factor, deformation factor, loading

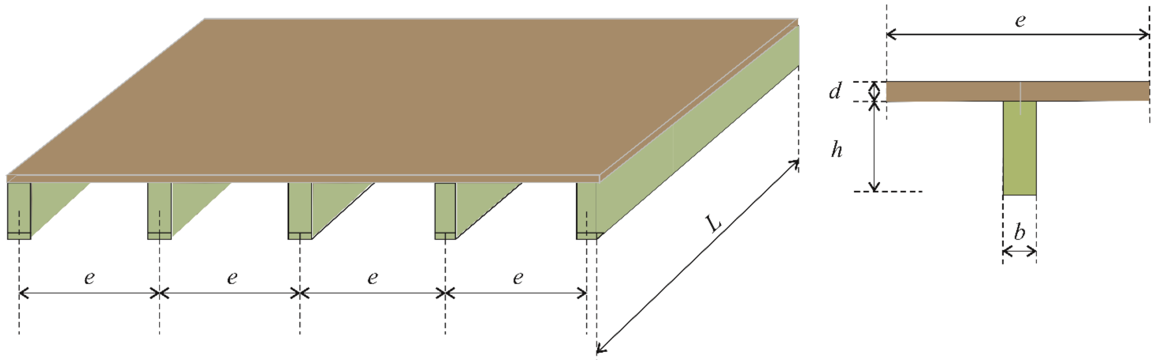

Figure 3. A timber floor joist with glulam beams and sheathing.

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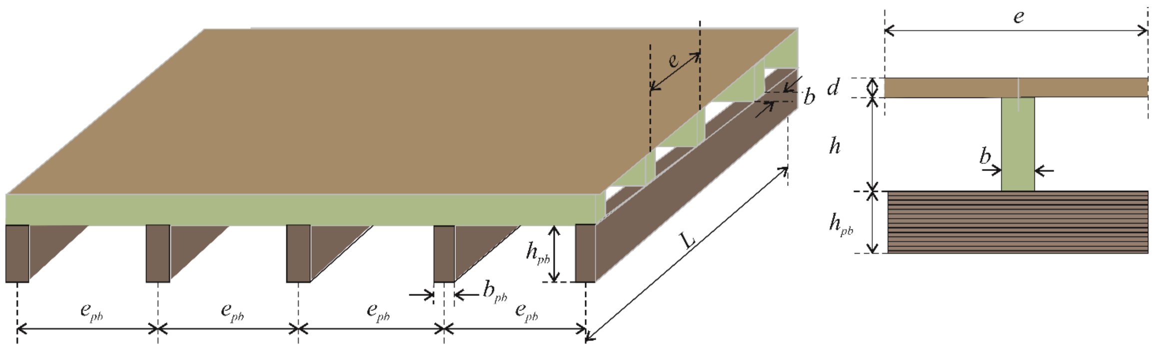

Figure 4. A timber floor joist with glulam beams, secondary sawn timber beams and sheathing.

sharing factor, partial factors for actions and material properties, composite actions, etc.) and cost information (unit prices for timber board, sawn timber, glulam, impregnation, etc.).

Variables: Continuous variables include aspects such as the total costs of production, geometric dimensions (widths and heights of main and secondary beams, spacings between beams, thickness of the timber board), cross-sectional properties (equivalent bending stiffness of the slab, equivalent bending stiffness of the joist, effective stiffness of the timber floor), stress levels (design bending and shear strengths, design bending and shear stresses), deflections (final deflections, instantaneous deflections, vertical deflections due to concentrated static force 1 kN) and masses. Discrete variables are used here for the calculation of standard sizes and bending strengths.

Objective function: The objective function of the self-manufacturing costs of the timber floor joists is determined, which includes the costs for the boards, the floorboard placing, the timber, the timber impregnation, the glued laminated timber beams and the glulam impregnation.

(In)equality constraints: The conditions for the ultimate limit state require that the maximum design bending stress in a timber beam does not exceed the design bending strength of the timber and that the design shear stress remains within the allowable shear strength of the timber. In addition, the design bending stress and shear stress in timber boards must be lower than their respective design strengths. For the serviceability limit state, both the instantaneous and final deflections of the timber beams and the deflection of the timber boards restrained between the beams must be checked. In addition, the vertical deflection due to concentrated static loads must be checked within certain limits. The fundamental vibration frequency of rectangular residential floors is also limited, and the unit impulse velocity of the floor must not exceed the allowable floor velocity. Logical constraints define the standard dimensions and strengths. For more information on the optimization model of the timber floor joist, see [26].

3.3 TIMBER-STEEL HALL STRUCTURE

The design envisages a single-storey hall structure made of timber and steel, using equal timber portal frames connected to each other with steel purlins and rails. The primary frames are constructed from a glued laminated timber with rectangular cross-section, while the purlins and rails are made from hot-rolled IPE steel profiles. In addition, HEA steel sections are used for the façade columns on the front and rear façades, as can be seen in Fig. 5.

The structural analysis takes into account the swaying behaviour of the timber frame. The longitudinal stability and the stability of the roof structure are ensured by an integrated bracing system. The columns of the portal frame are supported on square concrete pad foundations. The structure is designed to withstand uniform vertical loads, such as snow, and concentrated horizontal loads from the wind. The optimization model, referred to as THS (timber hall structure), comprises various components: input data, design variables, a cost minimization objective function, and both equality and inequality constraints.

Input data: They include the overall dimensions of the building (length, height and span), material strengths and properties, cross-section characteristics, safety factors for loads and material properties, and unit prices for timber, steel and concrete.

Variables: The model includes both continuous and discrete (binary) variables. Key variables include load distributions, structural resistances, stress levels, deflections, dimensions of glulam cross-sections, steel I-sections, number of structural elements (frames, purlins and rails), and costs and masses for individual components and the overall structure.

Objective function: The main objective is to minimize material costs, i.e. the costs of glulam, structural and reinforcing steel and concrete.

(In)equality constraints: For glulam frames, the model evaluates axial compressive strength, bending moments, shear forces and combined conditions. Stability checks include resistance to compress buckling, lateral-torsional stability and the interaction between buckling and torsional effects. Critical stresses are evaluated in the areas of the apex and column-beam zones, focusing on bending stresses, tensile stresses perpendicular to the grain and combined shear-tensile stresses. The steel components are analysed for their axial, shear and bending resistances, as well as compression buckling, lateral-torsional buckling and the combined effects. The load-bearing capacity of the soil under concrete foundations is also verified. Both the vertical deflections of the timber and steel elements

and the horizontal deflections of the timber frames are limited. Logical constraints are implemented to determine the optimal number of frames, purlins and rails as well as the most efficient sizes for glulam sections, steel I-profiles, and foundation dimensions. For more information on the timber-steel hall structure, see [27].

4 – OPTIMIZATION

Given the complex, non-convex and highly non-linear nature of the optimization problems, the modified outer-approximation/equality-relaxation algorithm is applied. The user-friendly software MIPSYN [19] is used. MIPSYN integrates a variety of advanced optimization methods. Among these, the most prominent are the modified OA/ER algorithm [18] and the multilevel LMHS strategy [20]. GAMS/CONOPT [28], a tool based on the generalized reduced gradient method, was utilized to solve the NLP subproblems, while GAMS/CPLEX [29], which uses the branch-and-bound algorithm, dealt with the main MILP problems.

The multilevel LMHS strategy, which was applied to accelerate the convergence of the OA/ER algorithm, worked in three phases:

- The optimization process starts with a continuous NLP optimization of the structure, where all variables are treated as continuous in the initialization phase. This step provides a good initial solution and develops an efficient global linear approximation of the superstructure, which is then used for the subsequent optimization phases.
- In the second phase, the process moves on to discrete MINLP topology and material optimization. An iterative sequence of NLP and MILP solutions is performed until the suboptimal solution is reached. The global linear approximation is additionally enriched.
- In the third phase, a comprehensive discrete optimization is performed, focusing on topology, material selection, standard dimensions and rounded dimensions simultaneously. This process is continued iteratively until the optimal solution is achieved.

For the given input data, and the material, power and labour unit prices, the task of the optimizations is to find the minimal production costs and optimal designs for the considered three timber structures.

A timber-concrete composite floor with a span of 10 m, loaded with an imposed load of 2.0 kN/m^2. The thickness of the formwork panel is 20 mm and the fasteners are 10 mm in diameter at 150 mm spacings. The unit prices are as follows: timber (C24) 250 €/m^3, timber impregnation 125 €/m^3, concrete (C25/30) 85 €/m^3, steel reinforcement 0.7 €/kg, floor-slab panels 10 €/m^2 and shear fasteners 0.4 €/each;

A timber joist with a span of 10 m, also loaded with 2.0 kN/m^2. Two different systems are optimized: the system with only primary glulam beams and the system with primary glulam beams and secondary sawn timber beams. The unit prices are set as follows: timber (C24) 250 €/m^3, glulam 500 €/m^3, timber impregnation 125 €/m^3 and timber boards 21 €/m^2;

A 55 m long, 15.5 m wide and 5.0 m high timber-steel hall structure. The hall is loaded with a snow load of 0.8 kN/m^2 and a horizontal wind load of 0.6 kN/m^2. The unit prices

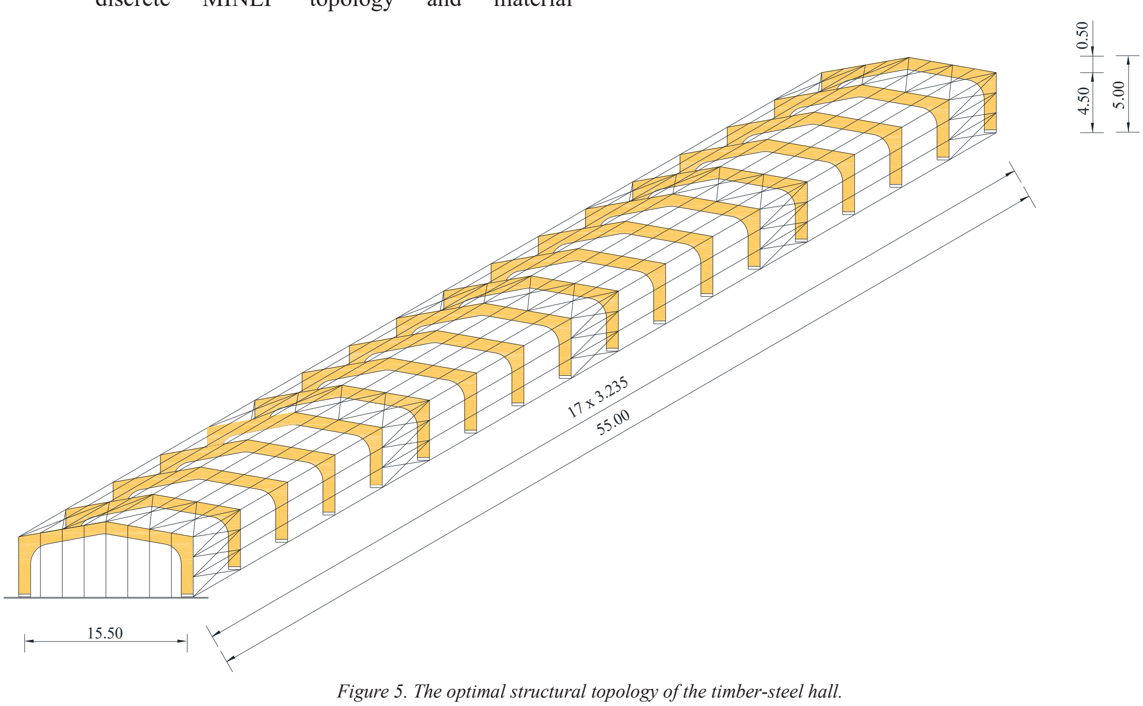

Figure 5. The optimal structural topology of the timber-steel hall.

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are €800/m^3 for glulam GL24h, €1.50/kg for steel S 355 and €110/m^3 for concrete C25/30.

5 – RESULTS

The superstructure of the timber-concrete composite floor system consists of 16800 different structural variants with different sizes and strengths (one of which is the optimal one). The calculation took about five minutes of working time. The calculated optimal self-manufacturing costs of the considered timber-concrete composite floor are 52.26 €/m^2 with the following optimal dimensions: the concrete slab thickness *d* is 40 mm, the width of the timber beam *b* is 120 mm, the height of the timber beam *h* is 260 mm and the spacing between the beams *e* is 600 mm (see Fig. 2).

The superstructure of the first timber joist system, with primary glulam beams only, comprises 6912 different structural alternatives. The calculated production costs reached 63.20 €/m^2. About five minutes were needed for the calculation. The calculated optimal dimensions are: the timber board thickness *d* is 20 mm, the width of the glulam beam *b* is 60 mm, the height *h* is 700 mm and the spacing between the beams *e* is 900 mm (see Fig. 3).

The superstructure of the second timber joist system, with primary glulam beams and secondary sawn timber beams, comprises $1.69917 \cdot 10^8$ different structural alternatives (one structure is the optimal one). The calculation took about five minutes. The calculated production costs reached 52.40 €/m^2. This joist system is cheaper than the system with primary beams only. The calculated timber board thickness *d* is 20 mm, the upper (secondary) sawn timber beams have the dimensions *b* 50 mm, *h* 140 mm and *e* 900 mm, while the lower (primary) glulam beams have the dimensions: b_{pb} 80 mm, h_{pb} 960 mm and e_{pb} 3.10 m (see Fig. 4).

The superstructure of the timber-steel hall structure is very extensive as it includes $1.42757 \cdot 10^{13}$ different structural alternatives (one of which is the optimal one). About fifteen minutes were needed for the optimization. The minimal material costs of the timber-steel hall structure resulted in 79.26 €/m^2. The optimal number of 18 main portal frames, 10 purlins and 8 façade rails are obtained, see Fig. 5. All cross-sections are also calculated: IPE 80 for purlins and rails, HEA 140 for façade columns, a rectangular glulam cross-section of 150/800 mm^2 for the main frames and a square cross-section of 147.5/147.5 cm^2 for the concrete pad foundations, see Fig. 6.

6 – CONCLUSION

This paper presents the MINLP optimization of timber structures. For this purpose, different optimization models for different timber structures are modelled and cost objective functions of structures are defined. In the paper, the optimizations of three different timber structures are presented to show the effectiveness of the proposed MINLP approach. The MINLP computer program MIPSYN is applied together with the modified OA/ER algorithm and the multilevel strategy to solve this task.

Based on this work, we will conduct future research on the development of global optimization algorithms and strategies and further develop the optimization models for arched timber hall structures and multi-storey timber buildings, including life cycle and carbon footprint concepts. MINLP optimization provides insight into mass or cost optimal solutions, optimal structural topologies, optimal material and standard cross-section choices, confirming MINLP as a valuable method for optimizing timber structures. Structural optimization is useful for teaching at universities and for use in research projects and daily engineering practice.

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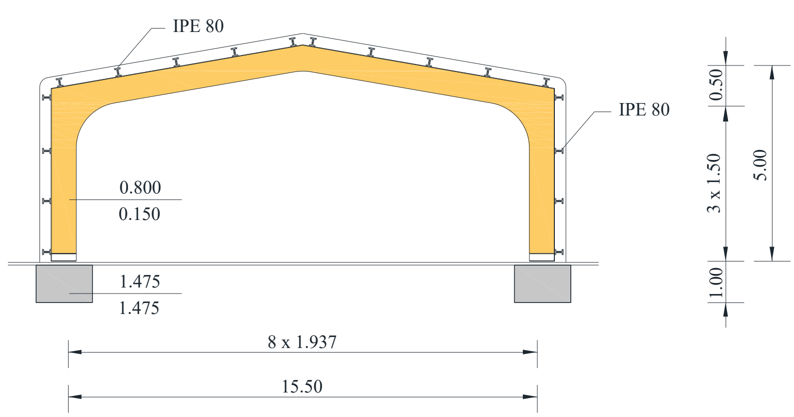

Figure 6. The optimal cross-sections of the timber-steel hall.

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