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Advancing Timber for the Future Built Environment

ANALYZING COMPLEX TIMBER DIAPHRAGMS USING AN EQUIVALENT TRUSS METHOD

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ABSTRACT: This paper introduces an efficient yet accurate analysis method for modeling the real stiffness of timber diaphragms in a 3D model. Known as Equivalent Truss Method (ETM), this approach employs truss elements to simulate the sheathing shear stiffness and the nail slip, beam elements to model framing members (including chords and drags), and stiffness modification to represent splices. ETM enables accurate 3D analysis of timber buildings with irregular diaphragms, distributes lateral loads based on relative stiffness of diaphragms and shear walls (semi-rigid analysis), and considers torsional effects. To calibrate and validate the ETM, the data from previous diaphragm tests are used. The calibrated model is then utilized to analyze both simple and complex building examples, assuming rigid, semi-rigid, and flexible diaphragms. The lateral load distribution to vertical elements and the building deformation under each assumption are compared. Results show that analyzing timber buildings assuming fully flexible or rigid diaphragms can lead to underestimating overall building deflection and inaccurate load distribution. Semi-rigid analysis using ETM, on the other hand, can provide a more realistic load distribution, allowing for generating more optimized and cost-effective designs.

KEYWORDS: timber, diaphragm, semi-rigid analysis, modeling, lateral loads

1 – INTRODUCTION

This paper presents the Equivalent Truss Method (ETM) as a practical analytical process to perform analysis of light-framed timber diaphragms that explicitly considers the stiffness (i.e., semirigid modeling assumption). First, the different elements and methods making up the ETM are described. Then, to validate the method, several diaphragm tests from previous studies are modeled and results are discussed. Finally, a few case studies are presented where ETM is used as part of the full building analysis. The results are used to show the difference in the lateral load distribution to vertical elements and the building deformation when rigid, semirigid, and flexible diaphragm assumptions are used.

2 – BACKGROUND

In light frame timber construction, diaphragms are structural elements that are part of the lateral force resisting system in a building. They typically consist of wood structural panels (WSP) made of plywood or oriented strand board (OSB). Diaphragms are used to resist wind and seismic loads and transfer those loads to the vertical lateral force resisting elements of a building [6]. In timber construction, these vertical elements are typically wood structural panel shear walls.

American Wood Council's Special Design Provisions for Wind and Seismic (SDPWS) provides the design procedures for WSP diaphragms. For light-framed timber diaphragms, the capacity, stiffness, and deflection of the diaphragm depend on the sheathing thickness and material properties, panel nail slip, chord splice slip, and chord deformation. Calculations for diaphragm deflection are provided for simple cases in the SDPWS standards, including simply supported and cantilever conditions [2]. To distribute lateral forces to the vertical elements, diaphragms are commonly idealized as either a flexible beam or as a rigid body. The guideline for idealization is defined in ASCE's Minimum Design Loads for Buildings and Other Structures section 12.3.1.1 through 12.3.1.3 and is based on the relative stiffness of the diaphragm to the vertical lateral force resisting elements [1].

However, diaphragms in light-frame buildings, particularly in multi-family dwellings, may be neither

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fully flexible nor fully rigid. Also, the deflection equations provided by SPDWS are specific to overly simplistic conditions and may be inadequate for capturing the behavior of conditions such as complex diaphragm shapes, out-of-plane offsets of vertical lateral elements, large openings in the diaphragm, or variable nailing patterns [6]. These diaphragms should be analyzed as semirigid and the load should be distributed horizontally considering relative stiffnesses of diaphragms and vertical lateral elements. This analysis not only captures the distribution of the lateral load more accurately, but it also captures the internal elemental forces and diaphragm deformations more accurately, even for more complex irregular diaphragms. To reinforce this; although ASCE 7-22 has provisions for when diaphragms can be idealized as flexible or rigid, commentary section C12.3.1 states that semirigid modeling of the diaphragm is always permissible.

There are a number of tools available to analyze semirigid diaphragms, but they can be complex to use and require a large amount of computational resources and modelling expertise. In addition, these tools typically require inputs that are beyond basic parameters that are typically defined in design standards for timber diaphragms. This usually means additional efforts in order to determine those inputs. When shell elements are used in modeling, interpreting the results to meet design requirements can also be challenging.

3 – EQUIVALENT TRUSS METHOD DESCRIPTION

The ETM is the use of matrix analysis on 2D or 3D models consisting of truss elements with varying properties representing the different components of diaphragm behavior. The method uses the equations given in SDPWS for diaphragm deflection as a basis for the types of stiffnesses and properties that are used in the model. The SDPWS 3-term equations for simply supported and cantilever diaphragms, include a term for each of the following: chord deformation, panel shear and nail slip, and chord splice slip (1).

$$\delta_{Dia} = \frac{5vL^3}{8EAH} + \frac{0.25vL}{1000G_a} + \frac{\sum(x\Delta_c)}{2H} \quad (1)$$

As shown in Fig. 1, the diaphragm is discretized into rectangular cells composed of horizontal and vertical elements bounding the cell and brace elements connecting the opposite corners of each cell. Each of these elements represents a different term in the SDPWS equations.

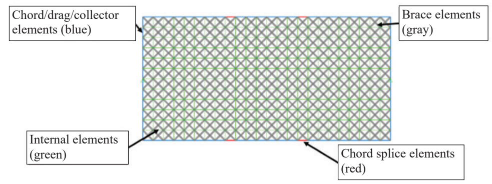

Figure 1. ETM Modeling

Term 1 of the SDPWS equation accounts for the deflection due to bending from chord deformation. This behavior is accounted for with the elements highlighted blue in Fig. 1. The elements represent the chord members of the diaphragm. They are modeled with the section and material properties of the actual chord member they are representing. These elements can also represent a drag or collector members since a chord may also serve as a drag/collector depending on the direction of the loading. The axial load in these elements can be used to design drags, chords, and/or collector members. Term 2 of the SDPWS equation accounts for the shear deflection including panel shear and nail slip. This deflection and the corresponding stiffness are modeled with brace elements as shown in Fig. 1 (colored gray). The material property and the geometry of these elements are selected based on the diaphragm specification and the apparent shear stiffness as defined in SDPWS. The axial forces in these elements are used to report the design shear forces for the diaphragm. Term 3 of the SDPWS equation accounts for the bending deflection due to chord splice slip. This behavior is accounted for in the model by modifying the chord members where splices exist. The modification depends on the type of splices. A nailed splice in an overlapped joint could have different stiffness than a bolted or a strap splice. The axial force in the modified elements can be used for verifying the design of the chord splices.

Fig. 1 also shows non-brace elements where there is no chord/drag/collector in green. These elements are designated as internal elements and are included to prevent unstable behavior in the model. These elements are assigned properties that provide minimal stiffness so that they do not contribute to the bending behavior of the diaphragm. These elements can also be used to model the floor joist contribution to the stiffness. Note that this contribution is ignored in the SDPWS equations, as such the initial ETM models would not include the floor joist properties.

Applied loads are distributed depending on the type of the loading. Seismic loads are distributed to each node based on the tributary area of the node in relation to adjacent nodes. Wind loads are distributed to each node at the edges of the diaphragm based on tributary width.

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Since the properties of all elements in this method are meant to represent the actual behavior of the horizontal and vertical lateral elements, the default analysis of the model represents a semirigid diaphragm analysis. However, this method can also be used to analyze a model with flexible or rigid diaphragm assumptions. The properties of the elements can be modified to have a minimal stiffness to deliver results similar to a hand calculated flexible diaphragm analysis. Similarly, modifying the element properties to have a sufficiently large stiffness will have results similar to a rigid diaphragm analysis.

4 – METHOD VALIDATION

To validate ETM, in the first step, the example provided in the SDPWS commentary for a simply supported diaphragm is used (Fig. 2).

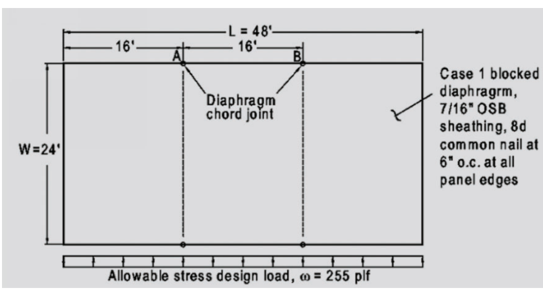

Figure 2. Simply Suppported Diaphragm Example

Fig. 3 shows three variations of the ETM model and the respective model deflections, and the comparison to the SDPWS calculated deflections for the different terms in the Equation 1. Model 1 was modeled with rigid properties for the chord elements and no chord splice elements, which isolates the deformation to just shear deformation. Model 2 used the actual chord properties and no chord splice elements to include both shear and bending due to chord deformation. Model 3 used the actual chord properties and includes chord splice elements to represent all of the contributing deformations. As can be seen from the comparison, the ETM model prediction of the diaphragm deflection is almost identical to calculated values in SDPWS.

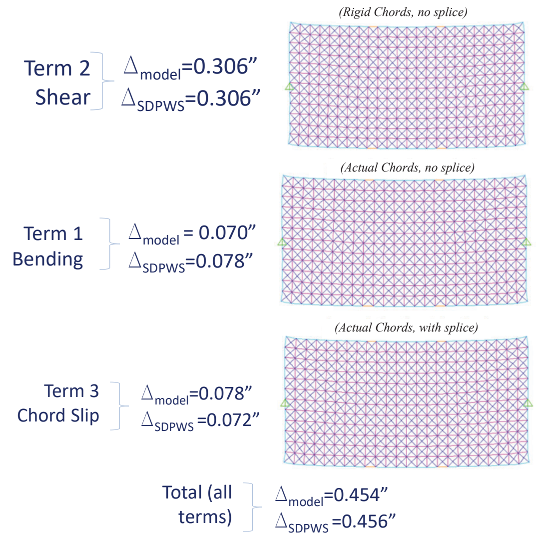

Figure 3. ETM Deflection vs. SDPWS Deflection

To further validate the ETM, the experimental data published in CUREE Publication No. W-27 and Bott 2005 was used next. In this study, multiple diaphragms were tested using cyclic displacements to determine the effect of various construction details on lateral stiffness of diaphragms. The diaphragm dimensions were 16 x 20 ft and 10 x 40 ft. Two loading cases were investigated, Case 1 with the load oriented parallel to the joists and perpendicular to the continuous the continuous joint in the sheathing, and Case 2, with the load oriented perpendicular to the joists and parallel to the sheathing continuous joints in the sheathing (Fig. 4). Construction details investigated included blocking, presence of designated chord members, and openings (center and corner) (Fig. 5 and Fig. 6) (CUREE, 2003). For each diaphragm, the stiffness was calculated and compared to a "benchmark" diaphragm (a diaphragm with designated chords, blocking and no openings) to find the reduction in stiffness if any of those construction details are omitted.

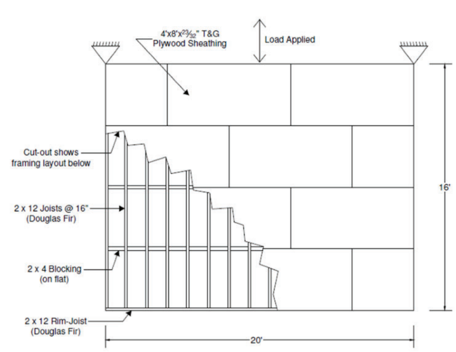

Figure 4. CUREE W-27 Test Details, Specimen 1 and 2 [3]

Figure 5. CUREE W-27 Test Details, Diaphragm with corner opening [3]

Figure 6. CUREE W-27 Test Details, Diaphragm with center opening [3]

A similar investigation study was performed through modeling CUREE diaphragm configurations using the ETM. Fig. 7 shows the ETM model of one of the "benchmark" diaphragms and a variation with an opening at the center. The reduction in stiffness due to the addition of the center opening was estimated to be about 40% by ETM which was close to the calculated tested value of 44% [3].

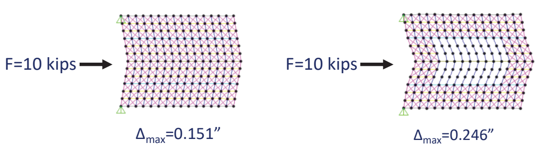

Figure 7. ETM Models for Specimen 3 and 4

The modeling and comparison were repeated for other test configurations. The results of these models and their corresponding tests are summarized in Fig. 8 with the blue bars representing the average test results and the black lines representing the range of test results. Note that several pairs of test specimens were used in the CUREE W-27 to calculate the effect of each parameter on the diaphragm stiffness. The red points in Fig. 8 display the estimated stiffness reduction from the ETM models. For the models investigating the effect of blocking, the unblocked model's shear stiffness properties were multiplied by 0.6 for Case 1 and 0.4 for other cases based on SDPWS provisions for unblocked diaphragms [2]. Overall, the model stiffness reductions fall within test ranges with several being similar to the average test value.

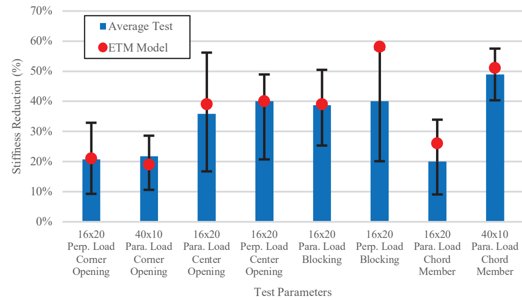

Figure 8. Stiffness Reduction

Another test data set that was used for the validation of ETM was the Report 138 published by APA – The Engineered Wood Association, a study for developing design recommendations for high load diaphragms was performed. In this study, eleven diaphragms were tested with varying details, such as opening and variable nail spacing. Diaphragm 1 was used as the "control" specimen in the report. Diaphragms 3 and 4 were tested with different openings. Diaphragms 9 and 10 were tested with multiple rows of nails and varying nail spacings through the span of the diaphragms (Fig. 9 and Fig. 10) [7].

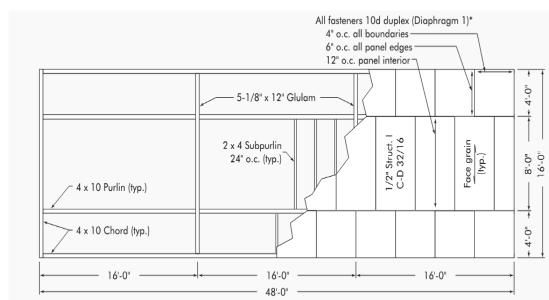

Figure 9. APA Report 138 Diaphragm Construction Details, Diaphragm 1 [7]

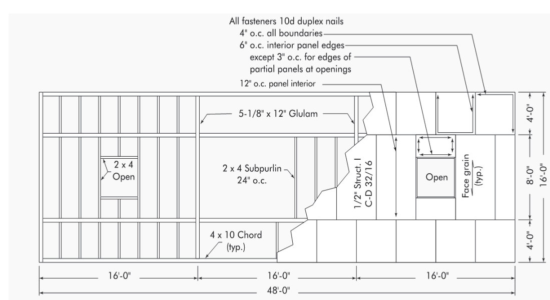

Figure 10. APA Report 138 Diaphragm Construction Details, Diaphragm 3 [7]

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An ETM model was generated for each diaphragm and the loads were distributed to each node such that the total load was equal to the applied load from the test. To accommodate the varying nail spacings in the ETM model for diaphragms 9 and 10, the brace properties were calibrated for the stiffness based on the nailing used in the different sections of the diaphragm.

As another point of comparison, the deflection was also hand calculated using equations provided in SDPWS 2021. It should be noted that the SDPWS deflections are not included for the diaphragms with openings because the equations are no longer valid when openings are present. Also, for diaphragms with variable nailing, the deflection is calculated assuming the support nailing for the whole diaphragm. Fig. 11 shows the resulting deflections from the tests, ETM models, and hand calculations. Overall, the ETM models were able to estimate the deflection more accurately than the SDPWS equations.

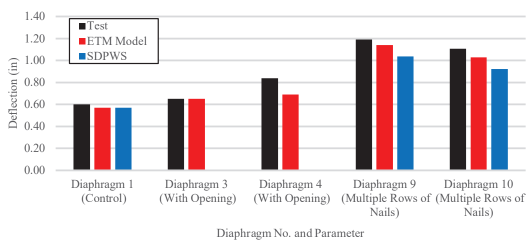

Figure 11. Diaphragm Deflection, ETM vs. APA Test vs. SDPWS

The previous studies provided validation for deflection and stiffness but there was no data available for validating the internal forces in diaphragm components. The Analysis of Irregular Shaped Structures [5] provides several examples of hand calculation methods for calculating diaphragm shears and chord and collector forces in complex diaphragms. Fig. 12 shows the example 5.1 from this book that represents a diaphragm with an opening.

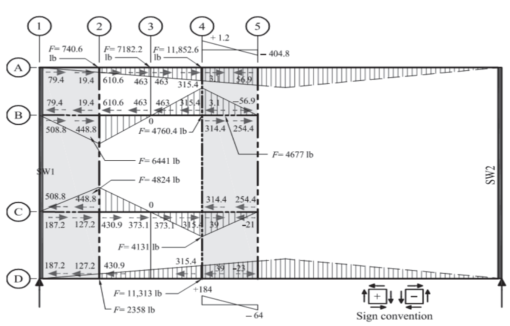

Figure 12. Chord and collector force diagram example [5]

The equivalent forces can be reported from the ETM analysis using the axial forces in the elements as shown in Fig. 13. The diaphragm shear forces near the opening and the chord/collector axial forces are shown in Fig. 13 and Fig. 14, respectively. Despite some differences, the overall load distribution within the diaphragm using the ETM was close to the hand-calculated distribution. For the values that showed greater differences, part of the discrepancy can be attributed to the variations between the model behavior and the assumptions used in the hand calculations. The hand calculation follows Diekmann method which assumes sections above and below the opening have points of contraflexure at their midlength; therefore, the collector forces in the middle of the members at the edge of the opening are zero [5]. In the ETM model, the load is distributed based on the actual relative stiffness of the diaphragm sections around the opening. Since the sections above, below and on the sides have different dimensions in this example, their stiffnesses are different and points of contraflexure would not occur at midpoint. In other words, the ETM will estimate the load distribution around the opening more accurately than the hand calculations through considering the actual stiffness of diaphragm segments.

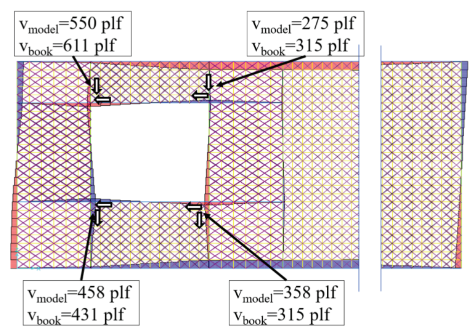

Figure 13. ETM axial force diagram with unit shear comparison

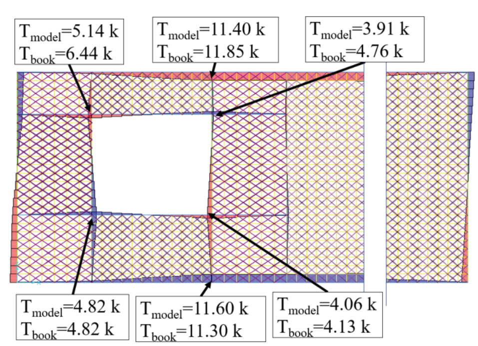

Figure 14. ETM axial force diagram with chord/collector force comparison

5 – CASE STUDY RESULTS

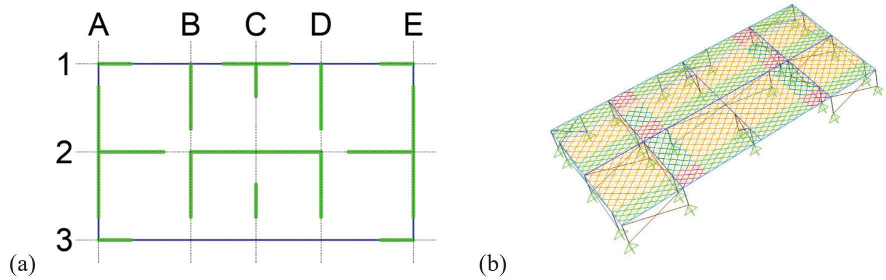

Figure 15. Case study 1, (a) Building plan and (b) ETM model

The first case study utilizes a simple rectangular building with walls of various lengths (Fig. 15). The green lines indicate wall locations. The diaphragm design was assumed to be 19/32" plywood with nailing at 6" o.c.

The building was modeled with actual diaphragm properties (semirigid) as well as rigid and flexible properties (Fig. 16). The plywood shear walls are represented in the 3D model with equivalent X braces (Fig. 15(b)). These braces are calibrated to have a lateral stiffness as defined in SDPWS (Equation 4.3-1). As expected, the flexible diaphragm deforms between the supports and the rigid diaphragm moves together as one body. The semirigid diaphragm shows a behavior that is somewhere in between the two.

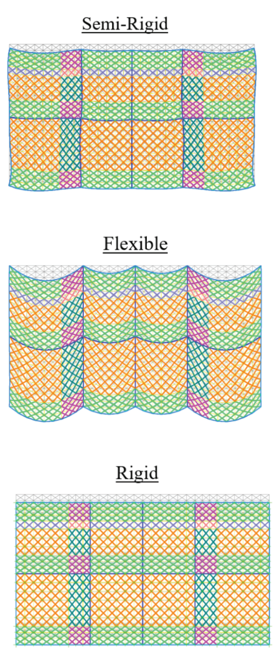

Figure 16. Case study 1, Deformed shapes

Fig. 17 and Fig. 18 show a normalized shear force distribution to each wall line for loading in the x and y directions. The results indicate a pattern similar to what was observed in the deformed shapes: using the actual properties of the diaphragm in analysis typical gives results between flexible and rigid behavior.

Due to the lack of a practical method for semirigid analysis of timber diaphragms, it is not uncommon for engineers to use an envelope method, where they use the maximum forces from both flexible and rigid diaphragm analysis for the design of the lateral systems. However, Fig. 18 suggests that an envelope approach can significantly overestimate the force in lines A, C and E. It can also potentially underestimate the force as seen with lines B and D.

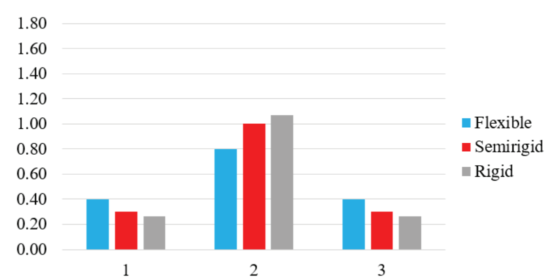

Figure 17. Case study 1 normalized shear force distribution at each wall line for x direction loading

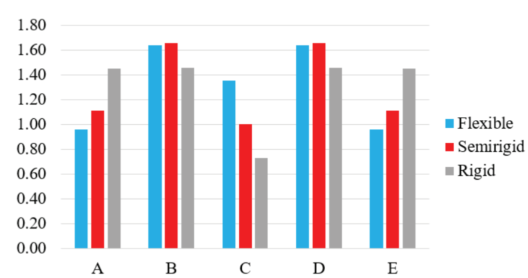

Figure 18. Case study 1 normalized shear force distribution at each wall line for y direction loading

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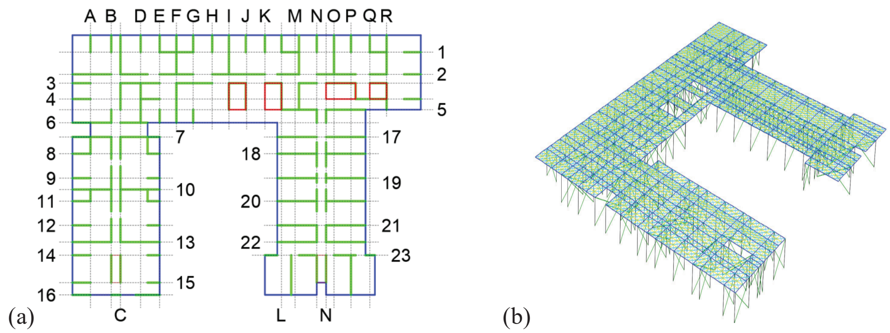

Figure 19. Case study 2 – (a) Building plan and (b) ETM model

The second case study used a larger building that had a more complex shape and included cantilevered diaphragms (Fig. 19). The green lines indicate wall locations and red lines indicate openings within the diaphragm. The diaphragm design was assumed to be 19/32" plywood with nailing at 6" o.c. This building was used to observe torsional effects, cantilevered diaphragm behavior, and the difference in force distribution in the chord elements between different analysis methods. Note that the shear walls are simulated using X braces as described before.

When assigning rigid properties for the diaphragm, Fig. 20(a) and (c) shows the rotational deformation due to torsion without any deformation of the diaphragm between supports. In the semirigid model in Fig. 20(b) and (d), there is some torsional deformation, but it is much less noticeable than the rigid model due to the deformation of the diaphragm between walls being significantly larger.

Due to the cantilever diaphragms, the model cannot be run as flexible. However, there are differences between fully flexible and semirigid behavior that can still be observed. In Fig. 20(b), it can be seen that the right wing has close to fully flexible behavior between the supporting wall lines. However, in the left wing there are some short walls on gridlines 8 and 9. Due to the lower stiffness at those lines, the diaphragm appears to act like a flexible diaphragm that is supported only at lines 7 and 10.

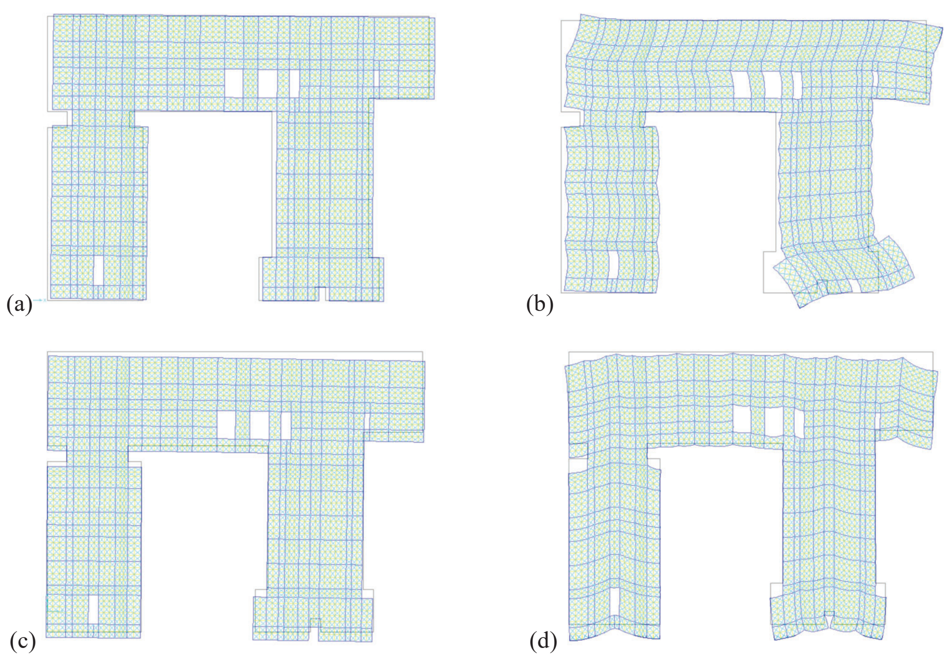

Figure 20. Case study 2 – Deformed shapes
(a) Rigid model, x direction load (b) Semirigid model, x direction load (c) Rigid model, y direction load (d) Semirigid model, y direction load

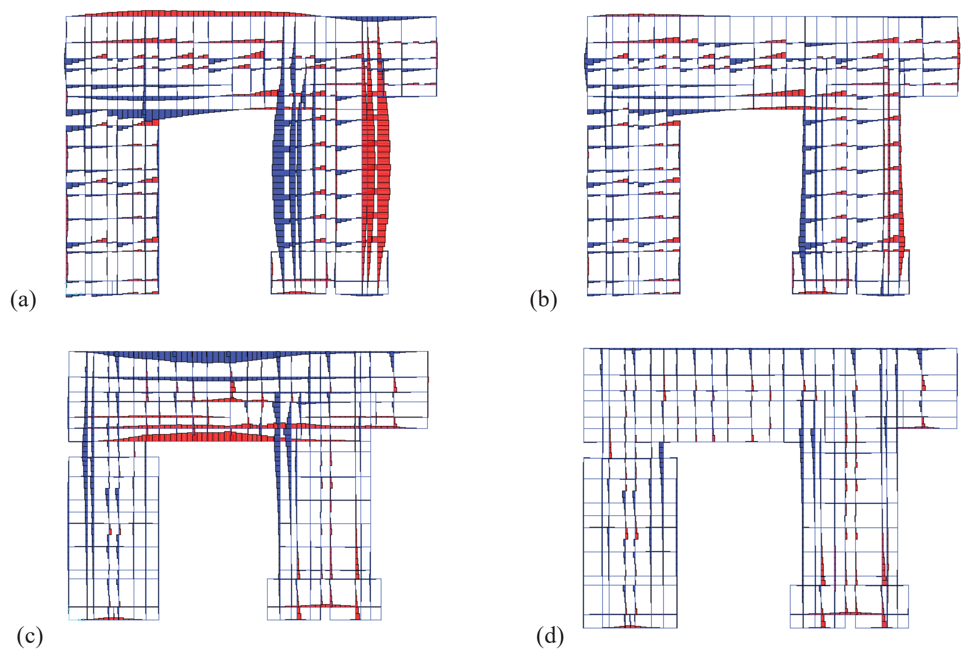

Figure 21. Case study 2 - Axial force diagrams
(a) Rigid model, x direction load (b) Semirigid model, x direction load (c) Rigid model, y direction load (d) Semirigid model, y direction load

The axial force in chords and collectors are shown in Fig. 21. This figure indicates that there is a large increase in axial forces when assuming a rigid diaphragm, not only in the chord members, but also in other collector members that are normally not considered as chords. This is most apparent in the right wing for x direction loading (wall lines L, M, O, P, and Q) and the top wing for y direction loading (wall lines 1, 4, and 5 and the cantilevered edge of the diaphragm). However, performing semirigid analysis would provide a more realistic load distribution to chords and collectors. This can be followed by a more cost effective design of these elements.

6 – CONCLUSION

This paper provides a description, validation and application of an alternate diaphragm analysis method, the Equivalent Truss Method (ETM). The ETM uses a discretized model consisting of truss elements with various stiffness properties to more accurately represent the diaphragm behavior including deflections and internal elemental forces.

Several studies were conducted to validate the accuracy of the proposed ETM. A model representing an example from SDPWS verified that the method achieves comparable results to current practice for simple diaphragms. The models based on tests performed by CUREE and APA confirmed that the method is capable of accurately estimating stiffness and deflection for diaphragms beyond the scope of the standard equations (diaphragms with openings, variable nail spacings, etc.). The method was also proven to give force distributions that are comparable to that of current practice.

Finally, the ETM was used to analyze real building layouts assuming rigid, semirigid, and flexible diaphragms. The lateral load distribution within the diaphragm, as well as the lateral load distribution to vertical elements, and the building deformation are compared. The results showed that using a flexible or rigid diaphragm assumption can result in significant error in force estimation in diaphragm components (axial force in chords and collectors, and shear in diaphragm sheathing). These assumptions may also lead to underestimation of diaphragms' local and torsional deflection. Semirigid analysis using the proposed ETM on the other hand, can provide a more realistic load distribution to chords/collectors and shear walls, and estimates the diaphragm deflection more accurately. This allows for generating more optimized and cost-effective designs that could lead to more affordable, yet reliable performing buildings.

In summary, this paper has shown that the proposed ETM addresses the limitations of the current procedures for diaphragm analysis and is well suited for analyzing diaphragms in modern light-frame timber construction, which typically fall outside of the scope of simplified equations provided in nationally recognized material standards.

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