

A Bayesian model updating framework for seismic damage identification in mortise tenon jointed timber frame

Jingliang Dong¹, Xiaobin Song²

ABSTRACT: The performance degradation of traditional timber structures caused by man-induced or natural hazards is a pressing issue for the preservation and protection of historical timber structures. A reliable numerical model capable of identifying structural damages is key for seismic assessment of the existing traditional timber structures. However, damage identification based on Finite Element (FE) models usually requires a great deal of computation effort, and updated models can hardly be applied for performance evaluation. This paper proposes damage parameters for mortise and tenon connections, considering the seismic damage characteristics of the wooden tower. The Bayesian method is embedded into a parameterized model to automate the updating process, and the bottom-level timber frames of a Tang-style wooden pagoda in Shanghai is considered for modeling and damage identification. The predicted results agree well with actual damage results, confirming the suitability and stability of the proposed Bayesian framework in identifying damage of joints.

KEYWORDS: traditional timber structures, mortise-tenon connections, seismic damage, Bayesian model updating

1 – INTRODUCTION

Traditional Chinese timber pagodas are historical and cultural symbols in East Asian countries [1-3]. During the long service life, most pagodas exist aging and seismic damage [4, 5]. Detecting and Evaluating safety becomes more and more urgent, for which a damage identification framework is essential.

At present, the safety assessment of traditional timber pagodas is primarily based on on-site inspections [6]. The decay and cracking conditions of wooden columns and beams are manually measured and then analyzed [7]. However, this method is time-consuming and labor-intensive, and it cannot provide information on the damage to the mortise and tenon joints, as the condition of these joints is not visible. As a result, dynamic testing has become an alternative method to assess damage. For long-term monitoring, acceleration signals, obtained from environmental excitations, can be used to assess the performance degradation of the structure over time [8]. Signal processing techniques are then applied to identify the location and extent of the damage [9]. However, this method requires an intact structure as a baseline for comparison. Given that the timber pagoda has already suffered damage, this approach cannot be directly applied for post-damage performance evaluation.

Currently, finite element model updating technology has attracted much attention from researchers [10-13].

Structural damages are identified through the model parameters adjustment. The damage identification method based on Bayesian model updating considers the uncertainty during the identification process, enabling the simultaneous detection of both the location and severity of the damage. This method has been successfully applied in practical engineering cases [14, 15]. The above method assumes that each element is elastic when constructing the model, and defines the extent of the damage by identifying the reduction in the elastic modulus and stiffness. However, a large number of timber components increase candidate parameter dimension, make damage identification challenging. Moreover, most proposed damage parameters only reflect the damage severity and cannot be used for subsequent performance analysis of damaged structures. Since actual mortise and tenon joints exhibit complex nonlinear characteristics under seismic loading, the assumption of elasticity alone does not accurately reflect the real behavior.

This paper proposes a damage index to assess performance degradation of mortise and tenon connections following earthquakes. Meanwhile, the timber frames of a pagoda are selected for modeling. Model damage parameters are identified based on Bayesian method. The hysteretic behavior of the damaged timber frames is then analyzed using the determined parameters and compared with the actual damage results.

¹ Jingliang Dong, Tongji University, 1239 Siping Road, Shanghai, China. Email: 2210033@tongji.edu.cn

² Xiaobin Song, Tongji University, 1239 Siping Road, Shanghai, China. Email: xiaobins@tongji.edu.cn

2 – Seismic damage characterization of mortise and tenon joints

2.1 Typical damage in traditional Chinese timber pagodas

To improve the efficiency and accuracy of damage identification in traditional timber pagodas, it is necessary to reduce the dimensionality of the damage parameters. Currently, the typical service-related damages to traditional timber pagodas include natural aging damage and seismic damage [8, 16, 17].

The typical aging damage in traditional timber pagodas includes aging cracks in components, degradation of gaps in mortise and tenon joints, and decay damage at the base of wooden columns. For localized cracking in wooden components, existing studies have conducted experimental research on the failure modes of components and proposed theoretical degradation models for damaged components [18, 19]. For mortise and tenon joints with gaps, some researchers have carried out experimental and finite element simulation studies, summarizing the degradation patterns of joint performance under different damage states [20-23]. Regarding column foot damage, existing research has simulated column foot decay through cutting experiments and performed fitting analysis on the performance degradation of damaged column foot [5]. Overall, the research on performance degradation due to natural aging damage is relatively abundant. In actual structural damage assessment, the performance of naturally aged components can be characterized through material and morphological inspections.

The typical post-earthquake damage in traditional timber structures is mainly related to the degradation of joint performance [17]. The joints often experience significant deformations to achieve seismic isolation and energy dissipation, while components such as beams and columns tend to suffer less damage [24, 25]. However, due to the self-centering behavior of timber pagodas and the structural features of mortise and tenon joints, traditional methods are insufficient for evaluating the post-earthquake performance of these joints. Current studies lack effective methods for characterizing and identifying seismic damage to joints. This paper proposes a method for identifying the post-earthquake performance of mortise and tenon joints, based on an improved restoring force model and Bayesian updating approach.

2.2 Hysteretic model of mortise and tenon joints considering seismic damage

Reversed cyclic loading tests on mortise and tenon joints indicate that the plastic deformation of the joints progressively accumulates as the loading displacement increases [20, 23]. The reloading path corresponding to each displacement stage changes, as shown in Fig. 1. The main reasons for the change in reloading behavior are as follows: As the loading displacement increases, the gap between the timber components widens, resulting in a greater slip displacement during reloading of the mortise and tenon joints; the contact surfaces repeatedly experience frictional slip, causing the surfaces of the timber components to become smoother and reducing the stiffness of the loading and unloading phases of the slip region; the tenon undergoes plastic deformation, causing the reloading strength of the joint to degrade at the same loading displacement.

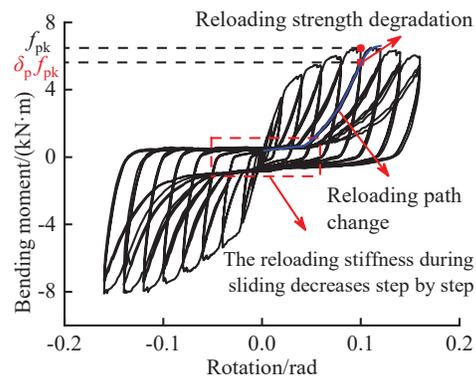


Figure 1. Hysteretic behavior of mortise-tenon joints [20]

Based on the simplified spring model of the bar system, the number of damage parameters to be identified in the timber pagoda can be significantly reduced. This paper utilizes the improved mortise and tenon joint hysteresis model proposed by Dong et al. [26] to characterize the hysteretic behavior of mortise and tenon joints considering seismic damage. The model definition takes into account the slip characteristics of the mortise and tenon joints and the strength degradation during reloading, thus enabling the characterization of seismic damage in these joints. As shown in Fig. 2, the backbone curve and reloading rules of the model are defined using linear segments and B-Spline curves. The backbone curve parameters are obtained through theoretical calculations, while the shape parameters are fitted from experimental data. The maximum historical displacement, d_{max} , is defined as the damage index, reflecting the characteristics of increased slip displacement during reloading, changes in stiffness during the reloading state of the slip segment, and strength degradation during reloading as the loading displacement increases.

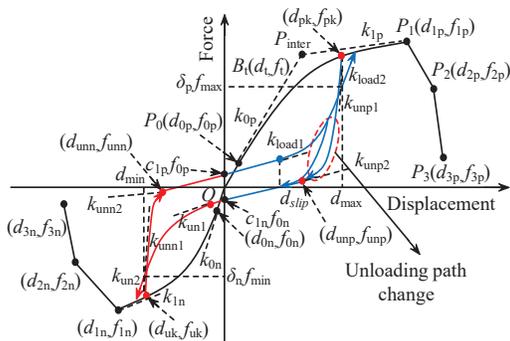


Figure 2. Improved hysteretic model^[26]

3 – Bayesian model updating framework

Seismic damage leads to a decrease in mortise and tenon joints stiffness, which in turn alters the modal parameters of the pagoda [24]. As shown in Fig. 1, the reloading stiffness of the joints gradually degrade with the accumulation of damage. By identifying the stiffness degradation $\theta = E_D/E_0$, the maximum historical loading displacement of the joints can be determined. Based on the improved hysteresis model, the post-earthquake damage state of the joints can be characterized.

This paper employs the Bayesian method to account for the uncertainty in the model updating process. The prior distribution of parameters is updated based on test data, and the posterior probability density distribution of the damage parameters is identified. In the context of damage identification for timber pagodas, the dimensionality of the parameters to be identified in the Bayesian equation is relatively high. This paper adopts the Markov Chain Monte Carlo method [27], avoiding the complexities of high-dimensional numerical integration calculations.

Based on the maximum entropy principle, it is assumed that the prior distribution of the damage parameters follows independent uniform distributions. The likelihood function is constructed based on the structural modal frequencies, and the posterior distribution of the damage parameters (Equation 1) is derived using the difference function (Equation 2).

$$\pi(\theta|D) = c \exp(-J(\theta)/2) \quad (1)$$

Where D represents the measured frequencies; c is the normalization constant; and $J(\theta)$ is the difference function (Equation 2).

$$J(\theta) = \sum_{j=1}^{N_m} \sum_{r=1}^m \left[\frac{(\hat{f}_{r,j} - f_r(\theta))^2}{\text{cov } f_r} \right] \quad (2)$$

Where $\hat{f}_{r,j}$ represents the j -th measured r -th order natural frequency; f_r is the r -th order theoretical natural frequency; r is the mode order; m is the maximum mode order; j is the mode testing number; N_m is the total number of tests; and $\text{cov } f_r$ is the covariance of the natural frequencies, with the r -th diagonal element.

This paper constructs a Markov chain for damage parameters using the Metropolis-Hastings sampling method. The specific steps are as follows: Initialize the state of the Markov chain $\theta_i = \theta_1$, set $i = 1$ and iterate N times; A candidate sample value θ^* is drawn from the proposal distribution q . The acceptance probability of the candidate sample is calculated by Equation 3. A random number u is generated from a uniform distribution $U(0,1)$. If $u \leq \alpha(\theta_i, \theta^*)$, the candidate sample is accepted, and $\theta_{i+1} = \theta^*$; otherwise, $\theta_{i+1} = \theta_i$. Then, set $i = i+1$ and repeat the above steps until a stationary distribution of the Markov chain is obtained.

$$\alpha(\theta_i, \theta^*) = \min \left[1, \frac{\pi(\theta^*|D)q(\theta_i|\theta^*)}{\pi(\theta_i|D)q(\theta^*|\theta_i)} \right] \quad (3)$$

4 – Numerical example of mortise and tenon timber frame

Earthquake disaster investigations have indicated that underlying timber frames are more susceptible to damage. This paper takes the bottom-layer frame of a seven-story Tang-style timber pagoda (Fig. 3) in Shanghai as research object and builds a numerical model to identify and evaluate the damage status of mortise and tenon joints.



Figure 3. Tang-style timber pagoda

Several timber structure models are built, as illustrated in Fig 4, which simulate seismic damage with cyclic loading. Models contain different boundary conditions, with the frame column base set in a floating configuration. The upper mortise and tenon joints are considered in two scenarios one with a cap beam and one without.

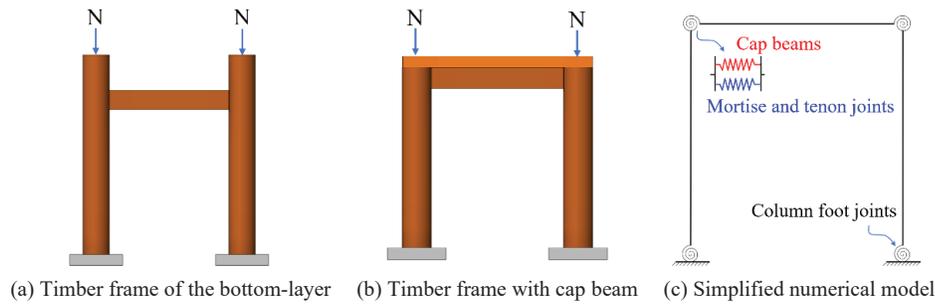


Figure 4. Mortise-tenon frames.

The dimensions of the frame are scaled to half of those of the prototype structure. For the model to be updated, both the beams, columns, and joint springs are selected as elastic elements to represent the dynamic characteristics of the frame. Based on existing material property tests, the parameters of each element in the model are determined. The rotational stiffness of the column base and the standard beam is determined based on the sway column lateral resistance theory [4, 28].

Based on the hysteresis model in Section 2.2, the damage stiffness at each level of the mortise and tenon joint can be determined, and performance analysis of the joint after damage can be conducted using this model. The key parameters of the backbone curve are calculated using the mortise and tenon bending theory [26, 29-31], and the shape parameters are determined based on experimental fitting results [26]. Three different mortise and tenon damage conditions are selected for identification analysis, corresponding to maximum loading displacements of 0.7Δ , 1.5Δ , 2Δ (reference displacement $\Delta = 60\text{mm}$). Under these three conditions, the reloading stiffness of the mortise and tenon joints degraded by 71%, 80%, 86%, and 90%, respectively.

Considering the influence of measurement noise, the sampling number is set to $N_m = 10$. The distribution of damage parameters is simulated using a normal distribution with the actual damage degree as the mean and a variance of 0.01. The corresponding first four modal frequencies are then calculated. The proposed sampling distribution is selected as a normal distribution with an initial value of 0 and a variance of 0.0009. The number of samples is set to $N = 6000$, and iterative calculations are performed.

The identified parameters fit well with the predefined severity of actual damage. The damage parameter identification error for each frame is less than 5%. The identification results for the case without cap beam are presented as an example. As shown in Fig. 5, the Markov chain for the mortise and tenon damage parameters has

converged, and the posterior parameters follow a normal distribution (Fig. 6).

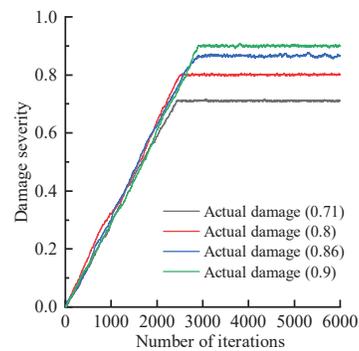


Figure 5. Markov chain of damage

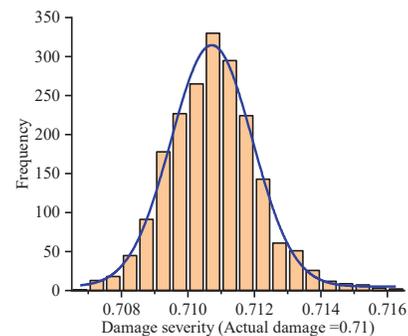


Figure 6. Posterior distribution of damage

The updated stiffness parameters are then converted into the historical maximum loading displacement, and the damage state of the mortise and tenon model is updated. The loading analysis corresponding to the maximum displacement of 2.5Δ is performed for each damaged model. The updated model can reflect the hysteretic performance degradation of the preset damaged joints, and demonstrate energy dissipation capabilities similar to real damaged conditions (Fig. 7).

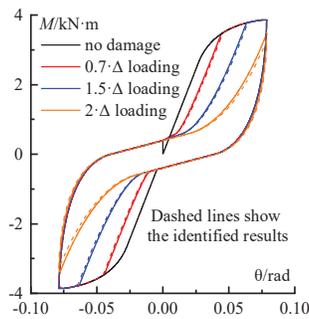


Figure 7. Comparison of energy dissipation

6 – CONCLUSION

The damage conditions of traditional timber pagodas are complex. Based on their damage characteristics, this paper proposes a seismic damage identification framework for mortise and tenon joints.

Using an improved hysteresis model for mortise and tenon joints, the number of damage parameters to be identified is effectively reduced. The damage state of the mortise and tenon joints can be defined by the historical maximum loading displacement, which is used for subsequent performance evaluation.

The Bayesian model updating method shows good identification accuracy for preset damage in the timber frame example. The hysteresis behavior of damaged timber frame in the updated model fixes well with actual damage. It shows the applicability and stability of the framework for identifying damage and assessing performance of mortise and tenon connections.

7 – REFERENCES

[1] He J, Liu K, Xie L, et al. Experimental and numerical studies on influences of wedge reinforcement on seismic performance of loose penetrated mortise-tenon joints[J]. *Journal of Building Engineering*. 2024, 91: 109610.

[2] Crayssac E, Song X, Wu Y, et al. Lateral performance of mortise-tenon jointed traditional timber frames with wood panel infill[J]. *Engineering Structures*. 2018, 161: 223-230.

[3] Mi X, Meng X, Yang Q, et al. Analysis of the Residual Deformation of Yingxian Wood Pagoda[J]. *Advances in Civil Engineering*. 2020, 2020(1): 2341375.

[4] Wu Y, Liu C, Wang L, et al. Seismic performance of stone supported traditional timber frames with or without cap beams[J]. *Journal of Building Engineering*. 2023, 70: 106395.

[5] Wu Y, Liu C, Xie Q, et al. Performance degradation by column foot damage: From column foot joints to frames[J]. *Engineering Structures*. 2023, 294: 116791.

[6] Ge X, Che A, Li Y. Use of non-destructive techniques in Chinese traditional timber structures[J]. *Proceedings of the ICE - Structures and Buildings*. 2013, 166: 307-315.

[7] Yuan J, Peng S, Shi Y, et al. Research on restoration scheme of Ying-Xian Timber Pagoda[M]. 2016.

[8] Li S, Zhang L, Tang J, et al. Damage Identification in Timber Structures Based on Wavelet Singular Spectrum Entropy[J]. *Mathematical Problems in Engineering*. 2018, 2018: 4898903.

[9] Jiang S, Wu M, Ma S, et al. Structural Stiffness Identification of Traditional Mortise-Tenon Joints Based on Statistical Process Control Chart[J]. *Journal of Aerospace Engineering*. 2018, 31(5): 4018066.

[10] Li X, Zhang C, Zheng Y, et al. A novel constrained UKF method for both updating structural parameters and identifying excitations for nonlinear structures[J]. *Soil Dynamics and Earthquake Engineering*. 2022, 158: 107291.

[11] Abdullah N, Sani M, Rahman M M, et al. A review on model updating in structural dynamics[C]. IOP Publishing, 2015.

[12] Rezaiee Pajand M, Entezami A, Sarmadi H. A sensitivity-based finite element model updating based on unconstrained optimization problem and regularized solution methods[J]. *Structural Control and Health Monitoring*. 2020, 27(5): e2481.

[13] Altunışık A C, Okur F Y, Genç A F, et al. Automated model updating of historical masonry structures based on ambient vibration measurements[J]. *Journal of Performance of Constructed Facilities*. 2018, 32(1): 4017126.

[14] Mao J, Wang H, Li J. Bayesian Finite Element Model Updating of a Long-Span Suspension Bridge Utilizing Hybrid Monte Carlo Simulation and Kriging Predictor[J]. *KSCE Journal of Civil Engineering*. 2020, 24(2): 569-579.

[15] Hernández-Montes E, Jalón M L, Rodríguez-Romero R, et al. Bayesian structural parameter identification from ambient vibration in cultural heritage buildings: The case of the San Jerónimo monastery in Granada, Spain[J]. *Engineering Structures*. 2023, 284: 115924.

- [16] Lyu M, Zhu X, Yang Q. Connection stiffness identification of historic timber buildings using Temperature-based sensitivity analysis[J]. *Engineering Structures*. 2017, 131: 180-191.
- [17] Xie Q, Wang L, Zhang L, et al. Seismic behaviour of a traditional timber structure: shaking table tests, energy dissipation mechanism and damage assessment model[J]. *Bulletin of Earthquake Engineering*. 2019, 17(3): 1689-1714.
- [18] Li S, Xie Q, Zhang B, et al. Compressive behavior of damaged timber columns: Experimental tests, degradation model, and repair method[J]. *Engineering Structures*. 2023, 293: 116579.
- [19] Zhang C, Chun Q, Lin Y, et al. Quantitative assessment method of structural safety for complex timber structures with decay diseases[J]. *Journal of Building Engineering*. 2021, 44: 103355.
- [20] Zhang B, Xie Q, Li S, et al. Effects of gaps on the rotational performance of traditional straight mortise-tenon joints[J]. *Engineering Structures*. 2022, 260: 114231.
- [21] Yang Q, Yu P, Law S. Load resisting mechanism of the mortise-tenon connection with gaps under in-plane forces and moments[J]. *Engineering Structures*. 2020, 219: 110755.
- [22] Zhang B, Xie Q, Li S, et al. Effects of gaps on the rotational performance of traditional straight mortise-tenon joints[J]. *Engineering Structures*. 2022, 260: 114231.
- [23] Zhang B, Xie Q, Hu J, et al. Rotational Performance of Traditional Straight Mortise-Tenon Joints with Gap: Theoretical Model and Numerical Analyses[J]. *International Journal of Architectural Heritage*. 2023, 17(10): 1701-1718.
- [24] Wu Y, Song X, Gu X, et al. Dynamic performance of a multi-story traditional timber pagoda[J]. *Engineering Structures*. 2018, 159: 277-285.
- [25] Zhang X, Xue J, Zhao H, et al. Experimental study on Chinese ancient timber-frame building by shaking table test[J]. *Structural Engineering and Mechanics*. 2011, 40(4): 453-469.
- [26] Dong J, Song X. Hysteretic model of straight and semi-tenon joints considering hysteretic damage[J]. *Engineering Mechanics*. 2024: 1-10. (in chinese)
- [27] Luo J, Huang M, Xiang C, et al. Bayesian damage identification based on autoregressive model and MH-PSO hybrid MCMC sampling method[J]. *Journal of Civil Structural Health Monitoring*. 2022, 12(2): 361-390.
- [28] Wu Y, Lin H, Wang L, et al. Influence of support interface on the lateral performance of rocking columns in traditional Chinese timber structures[J]. *Structures*. 2023, 48: 2037-2047.
- [29] Wu Y, Wang L, Lin H, et al. Effect of shear force on the rotational performance of straight mortise-tenon joints[J]. *Structures*. 2022, 41: 501-510.
- [30] Chang W, Hsu M, Komatsu K. Rotational performance of traditional Nuki joints with gap I: theory and verification[J]. *Journal of Wood Science*. 2006, 52(1): 58-62.
- [31] Chang W, Hsu M. Rotational performance of traditional Nuki joints with gap II: the behavior of butted Nuki joint and its comparison with continuous Nuki joint[J]. *Journal of Wood Science*. 2007, 53(5): 401-407.