WORLD CONFERENCE ON TIMBER ENGINEERING 2025
BRISBANE, AUSTRALIA
Advancing Timber for the Future Built Environment

INTERNAL STRESS WITHIN TIMBER－STEEL BAR COMPOSITE BEAM DUE TO CHANGING OF TEMPERATURE AND MOISTURE CONTENT

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ABSTRACT: We have been developing a frame system formed by timber members reinforced by deformed steel bar(rebar) using epoxy resin adhesive. Two major problems of the composite timber member are that its timber and rebar have different coefficients of thermal expansion and the timber expands and contracts due to variations in moisture content. Due to these factors, there is a need for a generalized method that can estimate the variation of stress and strain in the wood and the rebar inside the composite member due to variations in wood temperature and moisture content. This paper proposes Equations to estimate the internal stress of the wood and rebar according the variation in those temperature and wood's moisture content.

KEYWORDS: Composite timber, Beam, Internal stress, Deformed steel bar, Temperature, Moisture content

1 – INTRODUCTION

S.Shioya, i.e., one of authors, proposed a structural system and construction for buildings adopting Hybrid Glulam Timber members using Steel bars (HGTSB, nicknamed "Samurai" in Japan), developed the structural design methodology, and constructed the first prototype building of two-way frame structure using the structural system [1]. And then a factory building with 25-m-span one-way frame was built by the system in 2018, and on the top floor of a new 11-storey high-rise building required two-hour fireproof, in Tokyo Japan, two beams was employed as a trial in 2020...

Characteristic of the composite beam under long-term loading have been being investigated. We have already reported the creep deformation of the composite beam under long-term loading in natural or high-temperature conditions, and a method for estimating the deformation [2-4]. However, methods for estimating the internal stress of the composite members under long-term environments due to variations in wood temperature and wood moisture content have not yet been studied. Wood is anisotropic, and its thermal expansibility and shrinkage with moisture content are extremely complex, and it is probably significantly difficult to derive an estimation equation that precisely takes those into account. It is realistic and advisable to first derive the basic equation by considering an essential and simplified mechanical model for them, and then to improve it by adjusting variables through experimental verification. In this paper, a mechanical model for the composite beam that expands and contracts in the axial direction of the member due to variations in temperature and moisture content is devised, the basic equation for estimating the amount of variation in the internal stress is derived as an expression for the general solution of a second-order linear differential equation, the validity of the equation is verified through numerical analysis, and trends in internal stress in the composite beam with the generally expected variation in temperature and moisture content is described. The constants in the derived evaluation Equations will be specified by our experimental data.

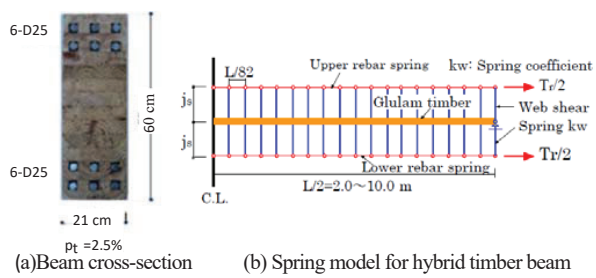

Figure 1: Composite timber beam and spring model for calculation

2 － SPRING MODEL OF STEEL BAR-TIMBER BEAM AND EQUATION FOR INCREASING STRESS INTENSITY

2.1 2.1 BASIC EQUATION

We will derive the basic equations using the model shown in Figure 1(b). The cross-section of the beam is a symmetrical cross-section, and it is also symmetrical in the

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https://doi.org/10.52202/080513-0713

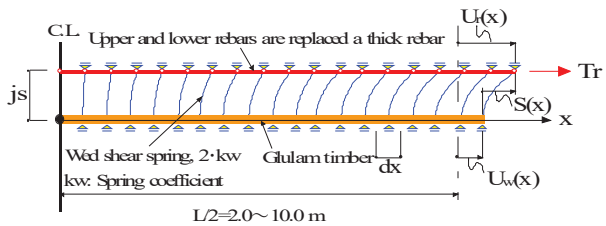

Figure 2: Spring model for derivation of differential equation

direction of the member length. We will consider only the right half of the beam from the centre of the span, making use of the symmetry of the shape and load.

Let L be the total span length and let ℓ be half of that length. The model is composed of axial elements of rebars and timber, and shear springs of the webs connecting both the components. The shear deformation function of the shear spring is defined as $S(x)$. The top and bottom rebars are assumed to be multiple, but each is integrated into one bar. In Figure 1(b), the origin is the center of the span, and the right direction is positive. The external force is assumed to cause only expansion and contraction due to temperature and moisture content. In the design, it is assumed that the internal stress caused by these factors is added to the stress caused by the design external force. Here, we will derive the basic equation for the case where the temperature rises. For the case where the temperature falls or the moisture content fluctuates, as described later, the derived basic equations can be applied.

Consider the state where the temperature rises and the rebar elongates relatively. Since the beam has a symmetrical cross-section, the rebar and glulam timber are assumed to elongate only in the direction of the timber axis. As shown in Figure 2, the rebar is concentrated into one at the top end, and no eccentricity is assumed to occur. The shear spring of the web is also doubled, with the top and bottom sides combined.

The function of the axial deformation of the top and bottom rebar are the same and are represented by $U_r(x)$. The function of the axial deformation of the timber is represented by $U_w(x)$. The function of the rebar $U_r(x)$ is expressed by Equation (1). When Equation (1) is differentiated twice with respect to x, Equation (2) is obtained.

$$U_r(x)=U_w(x)+S(x) \tag{1}$$

$$\frac{d^2}{dx^2}U_r(x)=\frac{d^2}{dx^2}U_w(x)+\frac{d^2}{dx^2}S(x) \tag{2}$$

where, $U_r(x)$: axial deformation of the rebar, $U_w(x)$: axial deformation of the timber, $S(x)$: shear deformation of the shear spring between the rebar and the timber, x, axial deformation, and shear deformation are positive in the direction on the right side in Figure 2

As the coefficient of thermal expansion of rebar is greater than that of timber, when the temperature rises, the timber restrains the expansion of the rebar, and the rebar expands the timber. This state is reproduced by applying tension Tr to the rebar due to the difference in the coefficients of thermal expansion of rebar and timber. This tension is applied to the rebar, which is the combined total of the top and bottom rebars. The axial force of the rebar is $N_r(x)$. Let the coefficient of the shear spring of the web per unit length of the interface between the rebar and the timber (N/mm/mm). When the rebar elongates, the balance of the increment of the axial force of the rebar, $dN_r(x)$, and the resultant of the adhesion stress intensity per fine length dx of the total surface of the rebar consolidated into one is expressed by Equation (3). When Equation (3) is expressed as a differential Equation, it becomes Equation (4).

$$dN_r(x)=k\,S(x)\,dx \tag{3}$$

$$\frac{d}{dx}N_r(x)=k\,S(x) \tag{4}$$

where, $N_r(x)$: axial force of the rebar, positive for tensile force, k: shear spring coefficient of the interface between the rebar and timber in Figure 2, k *in* N/mm/mm

$N_r(x)$ is expressed in Equation (5) as the product of the Young's modulus of rebar, E_r, the total cross-sectional area of the rebars aggregated into one bar, A_r, and the axial strain, $d/dx\ U_r(x)$

$$N_r(x)=E_r\,A_r\,\frac{d}{dx}U_r(x) \qquad 1 \tag{5}$$

where, E_r: Young's modulus of steel bar, A_r: Total cross-sectional area of steel bar, i.e. sum of cross-sectional areas of top and bottom steel bars

If both sides of Equation (5) are differentiated with respect to x, we get Equation (6).

$$\frac{d}{dx}N_r(x)=E_r\,A_r\,\frac{d^2}{dx^2}U_r(x) \tag{6}$$

If Equation (6) is rearranged by substituting Equation (4) into the left side of Equation (6), we get Equation (7).

$$\frac{d^2}{dx^2}U_r(x)=\frac{k}{E_r\cdot A_r}\,S(x) \tag{7}$$

Similarly, the axial force $N_w(x)$ of the timber can be expressed as shown in Equations (8) to (12) below. However, in Equation (8), as the axial force of the timber decreases when x moves in the positive (rightward) direction in the state shown in Figure 2, the increment needs to be negative. As $S(x)$ is positive in the state shown in the same figure, the right side of Equation (8) is made negative.

$$dN_w(x) = -k\,S(x)\,dx \tag{8}$$

$$\frac{d}{dx}N_w(x)=-k\,S(x) \tag{9}$$

$$N_w(x)=E_w\cdot A_w\cdot\frac{d}{dx}U_w(x) \tag{10}$$

$$\frac{d}{dx}N_w(x)=E_w\cdot A_w\cdot\frac{d^2}{dx^2}U_w(x) \tag{11}$$

https://doi.org/10.52202/080513-0713

$$\frac{d^2}{dx^2}U_w(x) = -\frac{k}{E_w \cdot A_w}S(x) \quad (12)$$

where $N_w(x)$: axial force of timber, E_w: bending Young's modulus of timber, A_w: axial cross-sectional area of timber

If we substitute Equations (7) and (12) into Equation (2), we get Equation (13), which we can rearrange to get Equation (14). This is a second-order homogeneous differential equation for S(x). Therefore, the general solution for S(x) is expressed by Equation (15).

$$\frac{k}{E_r \cdot A_r} \cdot S(x) = \frac{d^2}{dx^2} \cdot S(x) - \frac{k}{E_w \cdot A_w} \cdot S(x) \quad (13)$$

$$\frac{d^2}{dx^2} \cdot S(x) - \left(\frac{k}{E_r \cdot A_r} + \frac{k}{E_w \cdot A_w}\right) \cdot S(x) = 0 \quad (14)$$

$$S(x) = C_1 \cdot e^{\alpha x} + C_2 \cdot e^{-\alpha x} \quad (15)$$

At x=0, $S(x)$ in Equation (15) is zero, so Equation (16) is obtained, and $S(x)$ is expressed by Equation (17) or Equation (18).

$$C_2 = -C_1 \quad (16)$$

$$S(x) = C_1 \left(e^{\alpha x} - e^{-\alpha x} \right) \quad (17)$$

$$S(x) = 2\, C_1 \sinh(\alpha x) \quad (18)$$

where, $\alpha = \left(\frac{k}{E_r \cdot A_r} + \frac{k}{E_w \cdot A_w}\right)^{0.5}$

Equation (17) or (18) guarantees that the distribution shape of the shear deformation conforms to the difference in axial deformation between the rebar and the timber, as guaranteed by Equation (1). However, the continuity of the axial deformation of the rebar and the timber when tensile force Tr is applied to the rebar is not guaranteed. In order to guarantee this, it is necessary to satisfy the condition that the axial deformation of the rebar, the axial deformation of the timber, and the shear deformation of the shear spring are continuous at one point. In this case, the point of application of the tensile force Tr at the right end of the rebar, $x=\ell$, is taken as that point. In this case, Equation (19), which is obtained by substituting $x=\ell$ into Equation (1), is the equation of compatibility of deformation.

$$U_r(\ell) = U_w(\ell) + S(\ell) \quad (19)$$

The axial force $N_r(x)$ of the rebar is expressed as Equation (20), which is obtained by subtracting the shear force of the shear spring in the range from the point x to the rightmost point of the applied force ($x=\ell$) from the tensile force T_r. When Equation (18) is substituted into this and integrated with respect to x, it is expressed as Equation (21). These are expressed in a way that satisfies the boundary condition of the tensile force T_r.

$$N_r(x) = T_r - \int_x^{\ell} k\ S(x)dx \quad (20)$$

$$N_r(x) = T_r - 2k\frac{C_1}{\alpha}\{\cosh(\alpha l) - \cosh(\alpha x)\} \quad (21)$$

The axial deformation $U_r(x)$ is expressed by Equation (22), and when Equation (21) is substituted into this, the result is Equation (23).

$$U_r(x) = \int_0^x \frac{N_r(x)}{E_r \cdot A_r} dx \quad (22)$$

$$U_r(x) = \frac{1}{E_r \cdot A_r}\left[T_r\, x - 2k\frac{C_1}{\alpha^2}\{\alpha x \cosh(\alpha l) - \sinh(\alpha x)\}\right] \quad (23)$$

The deformation $U_r(\ell)$ at the point of loading is expressed by Equation (24) when Equation (23) is substituted for $x=\ell$.

$$U_r(\ell) = \frac{1}{E_r \cdot A_r}\left[T_r\, l - 2k\frac{C_1}{\alpha^2}\{\alpha l \cosh(\alpha l) - \sinh(\alpha l)\}\right] \quad (24)$$

The axial force $N_w(x)$ of the timber is expressed as Equation (25) by integrating the shear force of the shear spring in the range from the point x to the rightmost loading point ($x=\ell$), and by substituting Equation (18) into it and integrating and rearranging with respect to x, it is expressed as Equation (26).

$$N_w(x) = \int_x^{\ell} k\, S(x)dx \quad (25)$$

$$N_w(x) = 2k\frac{C_1}{\alpha}\{\cosh(\alpha l) - \cosh(\alpha x)\} \quad (26)$$

The axial deformation $U_w(x)$ of the timber is expressed by Equation (27), and when Equation (26) is substituted into this, the result is Equation (28).

$$U_w(x) = \int_0^x \frac{N_w(x)}{E_w\, A_w} dx \quad (27)$$

$$U_w(x) = 2k\frac{C_1}{E_w \cdot A_w \cdot \alpha^2}\{\alpha x \cosh(\alpha l) - \sinh(\alpha x)\} \quad (28)$$

The deformation $U_w(\ell)$ at the point of loading is expressed by Equation (29) when Equation (28) is substituted for $x=\ell$.

$$U_w(\ell) = 2k\frac{C_1}{E_w \cdot A_w \cdot \alpha^2}\{\alpha l \cdot \cosh(\alpha l) - \sinh(\alpha l)\} \quad (29)$$

The shear deformation $S(\ell)$ of the shear spring at the loading point is expressed by Equation (30) when Equation (18) is substituted for $x=\ell$.

$$S(\ell) = 2C_1 \sinh(\alpha \ell) \quad (30)$$

By substituting $U_r(\ell)$ from Equation (24), $U_w(\ell)$ from Equation (29) and $S(\ell)$ from Equation (30) into Equation (19) and rearranging, we can derive Equation (31) for C_1.

$$C_1 = \frac{T_r}{2\alpha\, E_r\, A_r \cosh(\alpha l)} \quad (31)$$

2.2 INCREASE IN STRESS OF REABER $\Delta\sigma_r(x)$

The axial stress of the rebar, $\sigma_r(x)$, is expressed by Equation (32) as the axial force of the rebar, $N_r(x)$, divided by the total cross-sectional area of the rebar, A_r.

Substituting Equation (21) into this gives Equation (33).

$$\sigma_r(x) = \frac{N_r(x)}{A_r} \tag{32}$$

$$\sigma_r(x) = \frac{T_r}{A_r} - 2k\frac{C_1}{\alpha}\frac{\cos h(\alpha l) - \cos h(\alpha x)}{A_r} \tag{33}$$

A tensile force T_r is applied to the most-right point of the rebar. If there were no shear springs connected, the tensile force of the rebar would be constant at T_r throughout its entire length L, over $\ell = L/2$. If there are shear springs connected, the tensile force will decrease from T_r within the section over the ℓ. The amount of decrease represents the restraining force that the rebar receives as the temperature rises. The stress intensity, which is the restraint force divided by the cross-sectional area of the rebar, is the compressive increase in the axial stress of the rebar, $\Delta\sigma_r(x)$. When only T_r is in effect, the axial stress of the rebar is T_r/A_r. If this is subtracted from the right-hand side of Equation (33), it becomes the increase/decrease in the axial stress of the rebar, $\Delta\sigma_r(x)$. $\Delta\sigma_r(x)$ is expressed by Equation (34).

$$\Delta\sigma_r(x) = -2k\frac{C_1}{\alpha}\frac{\cosh(\alpha l) - \cosh(\alpha x)}{A_r} \tag{34}$$

$\Delta\sigma_r(x)$ is expressed in the form of a negative number, which means that although the temperature rises and the rebar tries to expand, the wood around it, which has a relatively low thermal expansion coefficient, restrains that expansion, and so the rebar generates compressive stress when the temperature rises. The negative sign in Equation (34) indicates compressive stress.

2.3 INCREASE IN STRESS OF TIMBER $\Delta\sigma_w(x)$

The axial stress of the timber, $\sigma_w(x)$, is expressed in Equation (35) by dividing $N_w(x)$ in Equation (26) by the axial cross-sectional area of the timber, A_w. As no external force is directly applied to the timber, $\sigma_w(x)$ becomes the increase in axial stress of the timber, $\Delta\sigma_w(x)$.
Substituting this into Equation (26) and rearranging, we get Equation (36) for $\Delta\sigma_w(x)$.

$$\sigma_w(x) = \frac{N_w(x)}{A_w} \tag{35}$$

$$\Delta\sigma_w = 2k\frac{C_1}{\alpha}\frac{\cosh(\alpha l) - \cosh(\alpha x)}{A_w} \tag{36}$$

This means that when the temperature rises, the timber will produce tensile stress.

2.4 INCREASE IN BOND STRESS OF REBAR $\Delta\tau_b(x)$

In Figure 2, the shear force of the web in the length section divided in the direction of the axis is equal to the resultant of the bond stress of the rebar in the section. Let that section be dx. The shear force of the web is expressed as $k\,S(x)\,dx$. The balance between the resultant of the bond stress around the rebar in the section of dx, τ_b, and the shear force of the web is expressed in Equation (37).
Since the shear spring is also not directly affected by external forces, the bond stress increases as the temperature rises. Substituting Equation (18) into Equation (37), the increase in bond stress, $\Delta\tau_b(x)$, can be expressed as Equation (38).

$$\tau_b\,\phi\,dx = k\,S(x)\,dx \tag{37}$$

$$\Delta\tau_b(x) = 2k\frac{C_1}{\psi}\sinh(\alpha x) \tag{38}$$

where, ϕ: the total circumference of all rebar, including the top and bottom rebars

As for the direction of the bond stress, when the temperature rises, it becomes the direction that restrains the expansion of the rebar. This is positive. When the temperature decreases and the rebar contracts, it is negative.

3 — ESTIMATION OF INTERNAL STRESS WHEN INCREASING MOSISTURE CONTENT

When the moisture content increases and the timber expands in the axial direction, the rebar will restrain the expansion of the timber. In other words, the restraining force in the axial direction that the timber is subjected to by the rebars can be estimated with the compressive force applied to the rebars.
In Figure 2, if Tr is set to Cr (negative value, i.e. leftward), the variation in the internal stress of each element can be calculated using Equations (34), (36) and (38). In this case, the incremental axial stress of the rebar is positive in tension, and the axial stress of the timber is negative in compression. The bond stress acts in the direction that restrains the axial elongation of the timber, and is negative.

4 — TEMPERATURE AND MOSISTURE CONTENT FLUCTUATION AND EXTERNAL FORCE

The temperature and moisture content fluctuations during the long-term use of the building will be determined by the building's usage environment, but for the purposes of this discussion, let's assume that they are as follows.

4.1 TEMPERATURE VARIATION

1) The temperature in the room when the composite member product is manufactured is assumed to be between 10℃ and 40℃.
2) The temperature range for the environment in which the timber is used is between -20℃ and 70℃. The temperature of 70℃ is assumed to be the case where there is no ventilation in the attic.
3) The maximum temperature difference from the time of manufacture to the time of use is between -60℃ and 60℃. On the basis of these, the maximum temperature change is assumed to be 60℃.

https://doi.org/10.52202/080513-0713

4.2 VARIATION IN MOISTURE CONTENT OF GLULAM TIMBER

The variation in moisture content of wood differs depending on the size of the timber, the region and environment in which it is used, the species of tree. Measurements were taken nationwide, in Japan, using thin plate test specimens measuring 20 x 7 x 2 cm. This data shows that the moisture content fluctuates by 3-7% per year, depending on the conditions. On the other hand, as Glulam timber in the composite members is a large-section timber, considering that the moisture penetration depth from the timber surface is about 17mm, such a fluctuation in moisture content of 3-7% cannot be considered for the entire timber.

The moisture content required here is for estimating the amount of expansion and contraction of the entire timber in the axial direction and is a change in moisture content based on the change in the total weight of the timber. As large-section glulam timber has a large weight and it is difficult to measure it on a regular basis, it is common to estimate the moisture content of the surface of the timber using electrical resistance or high frequency. There are almost no studies on the change in moisture content due to the weight of large section laminated timber.

In the measurements we are currently conducting on large-section laminated timber, the moisture content fluctuates by around 1.0% over a one-year period. In these measurements, the weight of test pieces with a cross-sectional dimension of 210 mm x 600 mm is measured to clarify the amount of fluctuation in moisture content. However, as the measurement period is still short at one year, we will consider the maximum fluctuation in moisture content from the time of manufacture to the time of use to be 3%. This value will also need to be examined in the future.

4.3 MITIGATING THE INCREASE IN STRESS

In Japan, annual temperature and humidity increase and decrease in tandem. Therefore, the temperature and moisture content of timber also increase and decrease in tandem. When the temperature increases and the rebars expand, the moisture content increases, and the timber wood also expands in the axial direction. This means that the increase in moisture content increases the expansion of the timber, mitigating the difference in expansion caused by the temperature, compared to the expansion of the rebar caused by the increase in temperature. It is also possible to model the correlation between temperature and moisture content and evaluate the increase in internal stress intensity by considering the effects of both at the same time, but for the time being, we will consider the increase in stress intensity of the more influential of the two, on the safe side.

4.4 EXTERNAL FORCE

The thermal expansion coefficients of the rebar and timber that make up the composite, and the shrinkage rate in the grain direction due to the moisture content of the Japanese ceder timber, are as follows.

Thermal expansion coefficient of steel bars: 1.0×10-5/℃
Thermal expansion coefficient of timber: 5.0×10-6/℃
Shrinkage rate in the grain direction of timber due to a 1% change in moisture content: 0.015%.

The environmental change that causes the greatest internal stress int the composite beam shall be the one that maximises the difference in the amount of axial expansion and contraction of the rebar and timber.

The difference between the above coefficient of thermal expansion and the expansion and contraction caused by the maximum temperature fluctuation (60°C), and the expansion and contraction rate of the timber caused by the above water content and the maximum water content fluctuation (3%) are as follows, and the fluctuation in water content (3%) has a much greater effect. Therefore, we here apply an external force due to the moisture content change.

Difference in expansion and contraction of rebar and timber when the temperature increases by 60℃ (per 1000mm of timber length) ΔT:

$$\Delta T=(1\times10\text{-}5\text{-}5\times10\text{-}6)\times60\times1000=0.30\text{mm}$$

The difference in expansion and contraction when the moisture content increases by 3%: ΔM

$$\Delta M=0.015/100\times3\times1000=0.45\text{mm}$$

The strain at the time is 0.45/1000×100=0.045

ΔT<ΔM, so the amount of expansion and contraction due to changes in the moisture content is greater.

As described in Chapter 3, in Figure 2, if an external force Cr (leftward in Figure 2) that causes a compressive strain of 0.045% in the rebar is applied, the increase in the axial stress and bond stress of the rebar and timber due to the increase in moisture content can be estimated. Cr is expressed by Equation (39) considering the rightward direction of Tr in Figure 2. If the Young's modulus of the rebar is 2.05×105N/mm^2, the equation becomes (40).

$$C_r=-E_r\cdot\frac{0.045}{100}\cdot A_r \tag{39}$$

$$=-2.05\times10^5\times\frac{0.045}{100}\cdot A_r$$

$$=-92.3\cdot A_r \tag{40}$$

where, A_r is the same as the term in Equation (5). The unit of C_r is N (Newton).

5 — ESTIMATION EQUATION FOR EXISTING STRESS WHEN MOISTURE CONTENT INCREASING BY 3%

The amount of change in stress that occurs when the moisture content increases by 3% can be expressed as follows by substituting Equation (40) into Tr in Equation (31).

$$C_1 = \frac{C_r}{2\alpha \cdot E_r \cdot A_r \cdot \cosh(\alpha l)} \quad (41)$$

$$C_1 = -\frac{92.3}{2\alpha \cdot E_r \cdot \cosh(\alpha l)} \quad (42)$$

where, α is the term in Equation (18), E_r is the term in Equation (5), and ℓ is the same in Equation (19).

If C_1 in Equation (42) is substituted into Equation (34) and Equations (36) and (38), the evaluation Equation for the amount of change in each stress ($\Delta\sigma_r$, $\Delta\sigma_w$, $\Delta\tau_b$) that occurs when the moisture content increases by 3% is expressed.

6 — SPRING COEFFICIENT OF SHEAR SPRING OF WEB, k_w

k_w is the spring coefficient per unit length in the axial direction of the beam. It is expressed by the shear stiffness of the web of the timber in Figure 1(b). This shear stiffness is the shear stiffness of a shear bar when it undergoes shear deformation only in the axial direction of the beam, as shown in Figure 3.

To estimate this shear stiffness simply, the top and bottom rebars of multiple bars are aggregated into one, and it is considered as a single flat steel with the same cross-sectional area as the width of the beam width b.

The length of the shear bar is assumed to be j_s in Figure 1(b). The shear stress in the horizontal direction at the boundary between the rebar and shear spring in Figure 1(b) is assumed to be τ. The shear force Q of this shear spring is expressed by Eq. (43).

$$Q = \tau\ b\ 1 \quad (43)$$

where τ: shear stress intensity in the horizontal direction at the boundary between the rebar and shear spring, b: beam width

The shear deformation of the shear bar, δ_s, is expressed in Equation (44) as the product of the shear strain, γ, and the shear modulus, j_s. The shear strain is the value obtained by dividing the shear stress, τ, by the elastic shear modulus of the timber, G_w, so if we substitute this into Equation (44), the shear deformation, δ_s, is expressed in Equation (45).

If we rearrange this in terms of the shear stress, we get Equation (46). If this is substituted into Equation (43), we get Equation (47). The spring coefficient of the equation, or the slope, corresponds to the shear stiffness of the web, k_w, and k_w is expressed by Equation (48). In Figure 2, the differential equation is derived by aggregating the top and bottom rebars into one, so the spring coefficient of the shear spring, k , is expressed by Equation (49), which is twice the stiffness, k_w.

$$\delta_s = \gamma\ j_s \quad (44)$$

$$\delta_s = \frac{\tau}{G_w} j_s \quad (45)$$

$$\tau = \frac{\delta_s G_w}{j_s} \quad (46)$$

$$Q = \frac{b\ G_w}{j_s} \delta_s \quad (47)$$

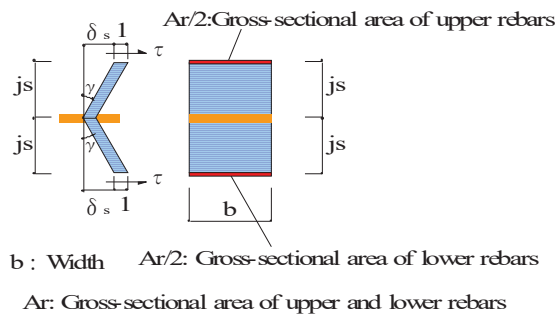

Figure 3: Deformation and modelling of web shear spring

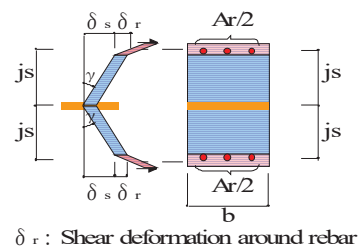

Figure 4: Ignored shear deformation around rebars

$$k_w = \frac{b\ G_w}{j_s} \quad (48)$$

$$k = 2\ k_w \quad (49)$$

where, b: beam width, G_w: shear modulus of timber, j_s: distance from the centroid of the timber to the top or bottom rebar

7 — ESTIMATION OF INCREMENTAL STRESS WHEN INCREASING MOISTURE CONTENT

In the previous chapter, the top and bottom reinforcing bars were each aggregated into a single flat steel bar. However, in reality, as shown in Figure 4, the rebars are deformed bars, and multiple bars are arranged in each case. The shear stress of the web and the shear stress of wood around a rebar are different. In the previous chapter, the shear deformation of the wood around the rebar was ignored, and the shear stiffness of the shear spring was estimated based on only the shear deformation of the web. If the shear deformation due to the bond stress is taken into account, the shear stiffness of the shear spring connecting the rebar and the timber will decrease. This deformation includes the shear deformation of the adhesive between the rebar and the timber. Considering the shear deformation caused by the bond stress will result in a smaller estimation of the axial stress of the timber and the bond stress of the rebar, which fluctuate with changes in moisture content or temperature. In design, it is safer to set the fluctuation of these stresses to a larger value. In the estimation of the variation in stress due to changes in moisture content and temperature in the design, it is also permissible to ignore the shear deformation of the wood around the rebar and to estimate the stiffness of the shear spring connecting the

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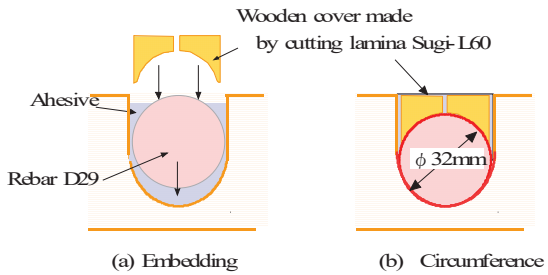


Figure 5: Embedding and circumference of rebar

rebar and timber using the Equation (48). In this case, we will not consider the shear deformation around the rebar.

8. CIRCUMFERENCE OF REBAR

Figure 5 shows the arrangement of the rebar into lamina and its circumference. As shown in Figure 5(a), the epoxy resin adhesive is first filled into the groove and the rebar is placed, and then two wood bars with an arc surface on the inside is dropped in and bonded to reduce the amount of adhesive used. After the adhesive has hardened, it is planed to finish the lamina to the specified thickness. The wood bar is assumed to be made from Japanese cedar lamina of grade L60 or higher. The rebar is surrounded by a cylindrical curved piece of wood. The radius of the curved surface of the groove is set by taking into account the maximum outer diameter of the rebar. The depth of the groove is also set by taking into account the relative position of the center of the rebar and the position in relation to the finished thickness of the lamina. Assuming that the rebar is D29, the radius of the U-groove and the radius of the curved surface of the wood bar are the same and are set at 16mm. From this, the circumference ψ_o per bar of rebar used to define the bond stress is the circumference with a diameter of 32mm, which is twice the radius of the U-groove. In the case of D29, this is 100.5mm (=2×16×3.14). The same approach is used to define the bond stress when using rebars other than D29.

9－VERIFICATION

The cross-section shown in Figure 6(a) is the largest cross-section assumed for the composite timber beam, and it has the largest rebar t ratio. The ratio of the total cross-sectional area of the rebar to the total cross-sectional area of beam, p_g, is 6.80%. The cross-section is symmetrical. The upper and lower rebars are of equal quantity, and their rebar ratio is 3.4%. As the bond strength of rebar in timber is extremely high compared to reinforced concrete members, it is a feature of the composite timber member that it can achieve high rebar.

Figure 6(b) shows the case where the amount of rebar is small. These cross-sections will be verified. In these cross-sections, the increase in stress intensity when the moisture content changes by 3% is calculated using numerical software, and the validity of Equations (34), (36) and (38) is verified using Equation (42).

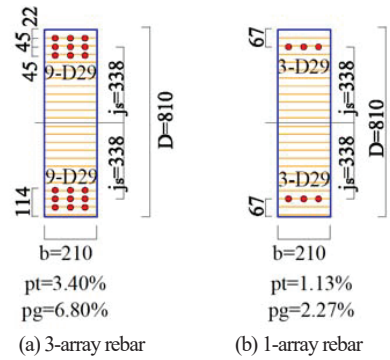


Figure 6: Beam sections for calculation

The model for verification is the model shown in Figure 1(b). The right half of the model is modelled from the center of the span of a beam using symmetry. The length of the timber is divided into 82 parts, and the timber is modelled as a beam element, the rebar is modelled as a truss element, and the shear spring of the web is modelled as a beam element stiffness connected to the timber beam element. The connection to the rebar is a pin connection. The element is a cantilever beam with a distributed bending moment, and the bending stiffness of the cantilever beam was set so that the stiffness of the shear spring in Equation (48) would be equivalent to the stiffness of the cantilever beam. The total length of the beam, L, was varied from 4.0m to 20.0m. The number of divisions in the direction of the length of the beam was set to 41, which is half the span length, regardless of the length of the beam. The values of various characteristics are as follows. The specific values and estimation equation are shown for the case of a three-array rebar section with a span length of L=4m, as shown in Figure 6(a).

i) Various values for the case of a three-array bar

Total cross-sectional area of the top and bottom rebar: a_t= 9×642.4= 5.782×10^3 mm^2

Gross cross-sectional area of the rebar:

A_r= 2×5.782×10^3 mm^2= 11.56×10^3 mm^2

ϕ o= 32×π= 100.5 mm^2

ϕ = 2×9×100.5= 1809 mm^2

A_w= 210×810-A_r

= 210×810-11.56×10^3= 158.54×10^3 mm^2

E_r=2.05×10^5 N/mm^2, E_w= 6500 N/mm^2, G_w= 433 N/mm^2

b=210mm, D=810mm

$$j_s=\frac{\left(\frac{810}{2}-22\right)+\left(\frac{810}{2}-67\right)+\left(\frac{810}{2}-112\right)}{3}=338\text{mm}$$

$$k=\frac{2bG_w}{j_s}=2\times210\times\frac{433}{338}==538.5\text{N/mm}$$

$$\alpha=\left(\frac{k}{E_r\,A_r}+\frac{k}{E_w\,A_w}\right)^{0.5}$$

$$=\left(\frac{538.5}{2.05\times10^5\times11.56\times10^3}+\frac{538.5}{6500\times158.54\times10^3}\right)^{0.5}$$

$$=8.658\times10^{-4}$$

ii) Calculation for external force and internal stress intensity when the total length of the beam is 4.0m

L=4000mm, ℓ=2000mm, $\alpha\ell$=8.65810^{-4} 2000=1.7317

$$C_1 = -\frac{92.3A_r}{2\alpha \cdot E_r \cdot A_r \cdot \cosh(\alpha l)} = -8.919\times10^{-2}$$

The increase in stress in the axial direction of the rebar is as follows.

$$\Delta\sigma_r(x) = -2k\frac{C_1}{\alpha}\cdot\frac{\cosh(\alpha l)-\cosh(\alpha x)}{A_r}$$
$$=9.597\{2.913-\cosh(8.65810^{-4}\ x)\} \qquad (50)$$

The maximum value occurs at the center of the span at a position where x=0 and is as follows.

$$\Delta\sigma_r(0) = 9.597\times(2.913-1.0) = 18.4\ \mathrm{N/mm^2}$$

This is a positive value and indicates the increase in stress in tension. The increase in stress in the axial direction of the timber is as follows.

$$\Delta\sigma_w(x) = 2k\frac{C_1}{\alpha}\cdot\frac{\cosh(\alpha l)-\cosh(\alpha x)}{A_w}$$
$$=0.6998\times\{2.913-\cosh(8.658\times10^{-4}x)\} \qquad (51)$$

The maximum value occurs at the center of the span at a position x =0 and is as follows.

$$\Delta\sigma_w(0) = -0.6998\times(2.913-1.0) = -1.34\mathrm{N/mm^2}$$

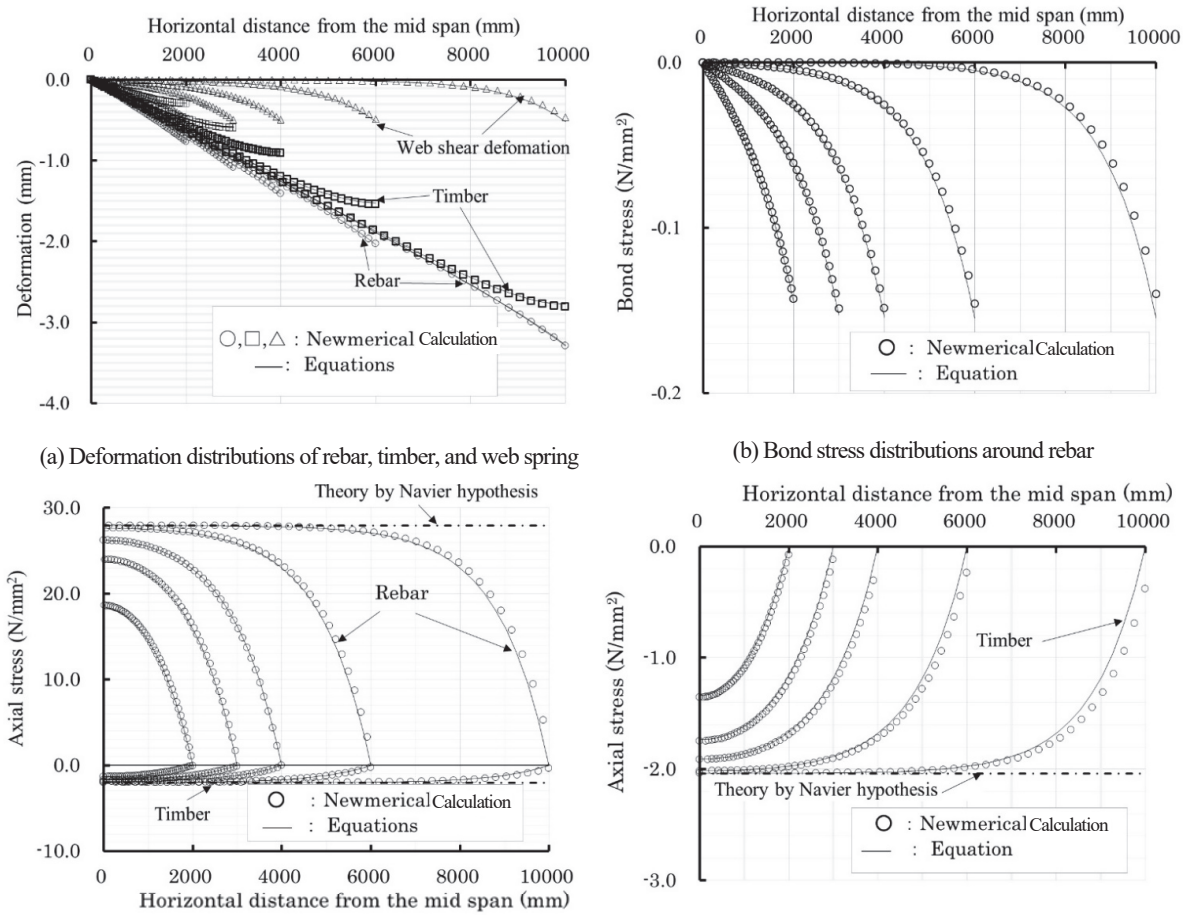
This is a negative value and indicates the increase in stress due to compression. The increase in stress of the bond stress of rebar is as follows.

$$\Delta\tau_b(x) = 2k\frac{C_1}{\psi}\sinh(\alpha x)$$
$$= -5.31\times10^{-2}\times\sinh(8.658\times10^{-4}x) \qquad (52)$$

The maximum value occurs at the beam end at a position x = 2000 and is as follows.

$$\Delta\tau_b(2000) = -0.0531\times\sinh(8.658\times10^{-4}\times2000)$$
$$= -0.145\ \mathrm{N/mm^2}$$

This is a negative value and means deformation in the opposite direction to that of the shear spring in Figure 2. Figure 7(a) compares the numerical calculation results for the distribution of the axial deformation of the rebar $U_r(x)$, the axial deformation of the laminated timber $U_w(x)$, and



(a) Deformation distributions of rebar, timber, and web spring

(b) Bond stress distributions around rebar

(c) Axial stress distributions of rebar and timber

(d) Axial stress distributions of timber

Figure 7: Calculation results for Beam section in Figure6(a)

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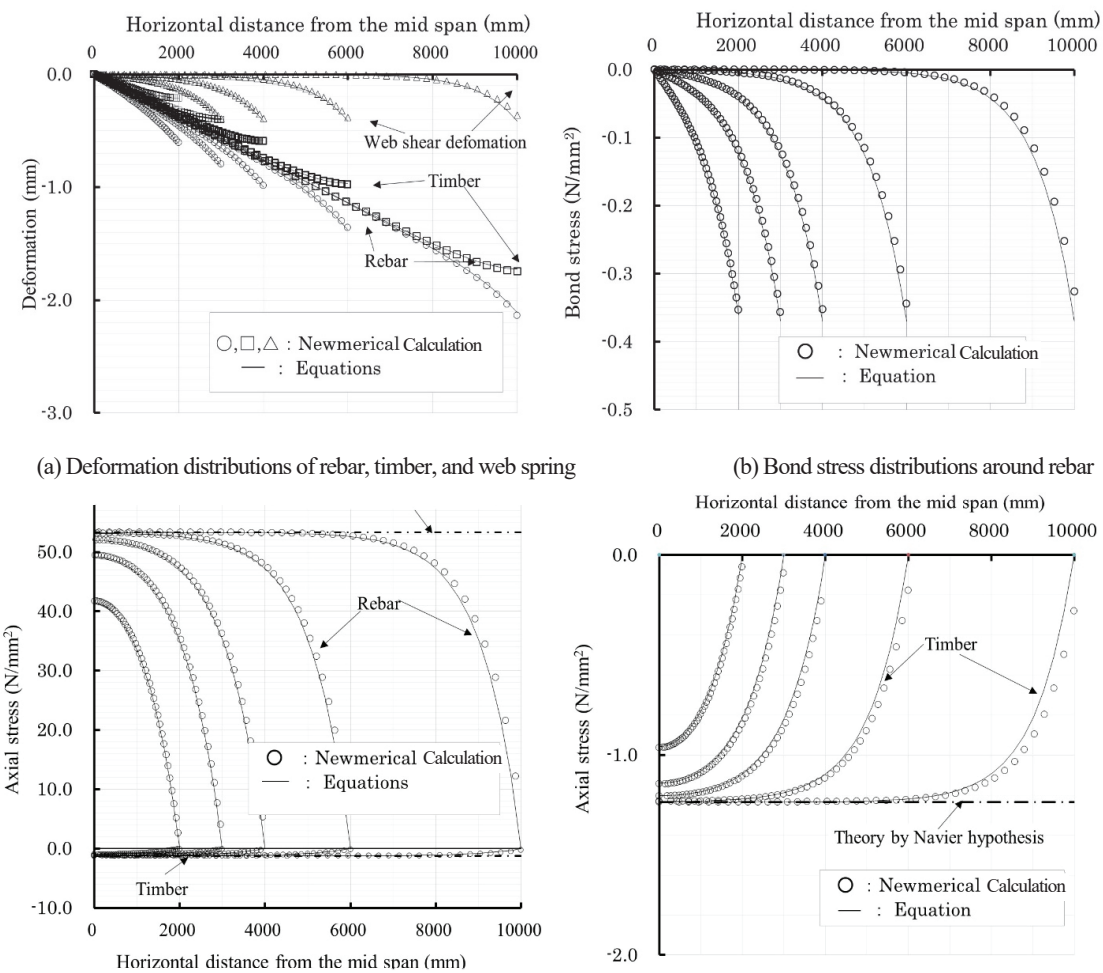
the shear deformation of the shear spring $S(x)$, which are calculated from Equations (24), (29), and (30) of the general solution of the differential equation for the three-array beam (general equation) in Figure 7(a). The ○, △, and □ symbols indicate the results by the numerical calculation, and the solid curves indicate the calculated values of the general Equation. Figure7(b) shows a comparison of the distribution of the bond stress of the re bar calculated from Equation (52) and the results of the numerical calculation. Figure7(c) shows the distributions of the axial stress of the rebar and timber calculated from Equations (50) and (51). Figure7(d) shows an enlarged version of the distribution of the axial stress of the timber only. Figure 8 shows the same results for the single-array rebar beam cross-section in Figure 6(b). In the numerical calculation, the model in Figure 1(b) was used, and even though the length of the beam was long, the number of divisions was set to 41, so there is a difference between the calculated values of the general equation and the numerical calculation as the length of the beam increases. If we exclude this cause of the difference, we can confirm that the curve by the general equation is consistent with the results of the numerical calculation.

If we compare Figure 7(b) and 8(b), and Figure 7(c) and 8(c), respectively, we can see that in the cross-section with a larger amount of rebars in Figure 7, the increase in the axial stress and bond stress of the rebar is smaller, and the increase in the axial stress of the laminated timber is larger. Conversely, in the cross-section with a smaller amount of rebars in Figure 8, the trend is the opposite. The composite timber beam is designed to have a large amount of rebars, so the trend is similar to that shown in Figure 7. If the long-term allowable stress of rebar is assumed to be 200 N/mm^2, the maximum increase in stress of the rebar in Figure 7(c) is 28 N/mm^2, which is 14% of the allowable stress. On the other hand, if the allowable stress for long-term bending of the cedar timber is set to 8.25 N/mm2 (= 1.1/3 x 22.5), the maximum increase in stress for the timber in Figure 7(d) is -2.0 N/mm^2, which is 24% of the allowable stress. When, in design, checking the allowable stress under long-term load, it is necessary to be careful as there is less margin for the timber.

The axial stress of the rebar and timber at the mid span x=0 converges to a constant value as the length of the beam



(a) Deformation distributions of rebar, timber, and web spring

(b) Bond stress distributions around rebar

(c) Axial stress distributions of rebar and timber

(d) Axial stress distributions of timber

Figure 8 Calculation results for beam section in Figure6(b)

increases. The value of the horizontal dotted line is a calculated value based on the concept of equivalent cross-section if the cross-section of the composite timber beam maintains in a plane after deformed. As the length of the beam increases, the axial stress of the rebar and timber converges to the value of the dotted line. This is because the effect of the concentrated load at the right end of Figure 1(b) is mitigated at the center of the span.

On the other hand, the bond stress reaches its maximum at the right-hand end, the point of the concentrated load, and its value is hardly affected by the span length L. When L is 6.0 m or more, it is almost a constant value. However, as can be seen by comparing Figure 7(b) and Figure 8(b), it should be noted that the maximum value of the bond stress increases when the amount of rebars decreases.

In Reference 5, an experiment was conducted to investigate the decrease in the bending stiffness, yield moment and moment strength of the composite beam under high-temperature environments, and a method for estimating these values was reported. It was reported that the bending stiffness and moment strength could be estimated by ignoring the internal stress caused by changes in temperature and, but that the yield moment needed to take the internal stress into account.

Figure 9 shows the results of comparing the calculated and experimental values of the yield moment using the method proposed in this report and the formula (5) in Reference 5. The moment M_{yc} when the upper rebar yields in compression and the moment M_{yt} when the lower rebar yields in tension are shown. After that, we also conducted an experiment in which, mainly, the moisture content of the wood was further reduced. This was reported in Reference 6. The data for temperatures below 20°C are for the additional specimens [6]. The change in the calculated yield moment, which takes into account the change in internal stress due to temperature change and the moisture content of the wood, is shown by the black solid line. The calculated values are almost the same as the experimental values. The results demonstrate that the internal stress can be roughly estimated using the proposed method. For details of the experiment, please refer to References 5 and 6.

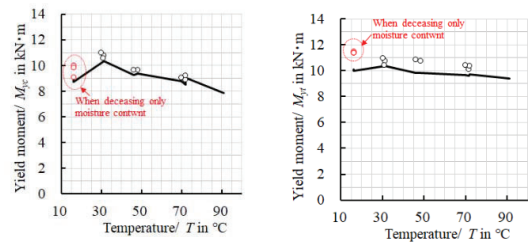
10－SUMMARY

We proposed equations for estimating the amount of change in the stress of timber and rebar of the steel bar-timber composite beam, which occurs when the composite beam is subjected to fluctuations in temperature and moisture content. The results have been summarized as the follows.

i: Using the general solution of the differential equation, we have shown specific equations that can be used to estimate the change in axial stress in the rebar and laminated timber, and the change in bond stress around the rebar, because of fluctuations in temperature and moisture content. We have carried out numerical calculations on specific cross-sections and confirmed that the increase and decrease in each stress can be estimated using the specific equations.

ii: In the case of the composite beam, a large amount of rebars is used, so when, in design, checking the allowable stress under long-term loading, it is necessary to be careful of effects of temperature and moisture content, as these have a greater impact on the composite beam.

iii: In this study, the temperature was set at 60℃ and the moisture content was set at 3%, and it was found that the change in internal stress was greater when the moisture content changed by 3% than when the temperature of the composite beam changed by 60℃. However, in natural environments, when temperature and humidity increase in tandem in the summer, it is possible that the increase in stress due to differences in the thermal expansion coefficient and moisture content could be mitigated.



(a) Yielding of compression rebar (b) Yielding of tension rebar

Figure 9: Yield moments by rebar yielding of HW [5,6] with temperature rising

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https://doi.org/10.52202/080513-0713