Adaptive Neighborhood-Constrained Q Learning for Offline Reinforcement Learning

Yixiu Mao[1], **Yun Qu**[1], **Qi Wang**[1], **Xiangyang Ji**[1]
[1]Department of Automation, Tsinghua University
myx21@mails.tsinghua.edu.cn, xyji@tsinghua.edu.cn

Abstract

Offline reinforcement learning (RL) suffers from extrapolation errors induced by out-of-distribution (OOD) actions. To address this, offline RL algorithms typically impose constraints on action selection, which can be systematically categorized into density, support, and sample constraints. However, we show that each category has inherent limitations: density and sample constraints tend to be overly conservative in many scenarios, while the support constraint, though least restrictive, faces challenges in accurately modeling the behavior policy. To overcome these limitations, we propose a new neighborhood constraint that restricts action selection in the Bellman target to the union of neighborhoods of dataset actions. Theoretically, the constraint not only bounds extrapolation errors and distribution shift under certain conditions, but also approximates the support constraint without requiring behavior policy modeling. Moreover, it retains substantial flexibility and enables pointwise conservatism by adapting the neighborhood radius for each data point. In practice, we employ data quality as the adaptation criterion and design an adaptive neighborhood constraint. Building on an efficient bilevel optimization framework, we develop a simple yet effective algorithm, Adaptive Neighborhood-constrained Q learning (ANQ), to perform Q learning with target actions satisfying this constraint. Empirically, ANQ achieves state-of-the-art performance on standard offline RL benchmarks and exhibits strong robustness in scenarios with noisy or limited data.

1 Introduction

Reinforcement learning (RL) tackles sequential decision-making problems and has gained considerable attention in recent years [58, 70, 67, 14]. Despite its promise, RL faces practical challenges, notably the high data collection costs [38] and exploration risks [24]. Offline RL offers a compelling alternative by learning from a static dataset collected by a behavior policy [43, 44]. It enables the use of existing large-scale datasets [33, 52, 63] and reduces the dangers of unsafe exploration. However, it also introduces a key challenge: evaluating out-of-distribution (OOD) actions leads to extrapolation errors [23], which further causes value overestimation and significant performance degradation [44].

To address this issue, offline RL approaches typically impose constraints on the action selection process. A common strategy is to align the probability densities of the trained and behavior policies [83, 21], enforcing a *density constraint*. This is usually achieved using divergence metrics such as reverse Kullback-Leibler (KL) [83, 32], forward KL (i.e., behavior cloning) [21, 61], Fisher [39], or the implicit CQL divergence [42]. While straightforward, these methods can be overly restrictive both theoretically and empirically [41], in cases where the overall quality of the behavior policy is low. To overcome this limitation, recent work has explored the most relaxed *support constraint*, which only keeps the selected actions within the support of the behavior policy [82, 53, 93]. Accomplishing this generally necessitates high-fidelity estimation of the behavior policy through advanced generative modeling techniques [26, 82, 93, 9]. However, such modeling is challenging due to the

39th Conference on Neural Information Processing Systems (NeurIPS 2025).

 https://doi.org/10.52202/085713-0877

Table 1: A brief summary of constraint types in offline RL research.

Constraint type	Description	Algorithms	Key characteristics
Density	Enforce density proximity between the trained and behavior policies	BRAC [83], TD3BC [21], CQL [42]	Straightforward but heavily limited by the overall quality of behavior policy
Sample	Restrict action selection to dataset actions	IQL[40], XQL[25], SQL[88]	Avoid extrapolation error but lack action generalization beyond the dataset
Support	Restrict action selection to behavior policy's support	BCQ[23], BEAR[41], SPOT [82]	Least restrictive but require accurate behavior policy modeling
Neighborhood	Restrict action selection to certain neighborhoods of dataset actions	ANQ (Ours)	Flexible and approximate support constraint without behavior modeling

high-dimensional and multi-modal nature of real-world data [93], subjecting these methods to heightened error susceptibility and increased computational overhead. Alternatively, the *sample constraint* has emerged, formulating the Bellman target exclusively using actions in the dataset [6, 40, 92, 88]. These methods are easy to implement and effectively avoid extrapolation errors [40]. However, their performance is inherently limited by a lack of action generalization beyond the offline dataset, often resulting in overly conservative policies when near-optimal actions are rare in the dataset.

This work aims to address the over-conservatism of the density and sample constraints while avoiding complex behavior modeling required by the support constraint. To this end, we introduce a new *neighborhood constraint* that restricts action selection in the Bellman target to the union of neighborhoods of dataset actions. Theoretically, the constraint not only bounds extrapolation errors and distribution shift under certain conditions, but also approximates the least restrictive support constraint without behavior modeling. Moreover, it maintains high flexibility and can achieve pointwise conservatism by adapting the neighborhood radius for each data point. In light of real-world data patterns, we adopt data quality as the adaptation criterion and develop an adaptive neighborhood constraint in practice. It assigns larger neighborhood radii to low-advantage dataset actions to promote a broader search, and smaller neighborhood radii to high-advantage dataset actions to limit the overall extrapolation error.

To enforce the proposed constraint, we introduce an efficient bilevel optimization framework and, based on it, develop a simple yet effective algorithm, Adaptive Neighborhood-constrained Q learning (ANQ), which performs Q learning with target actions constrained accordingly. Specifically, in the inner optimization, we maximize the Q function separately within each dataset action's neighborhood; in the outer optimization, we implicitly maximize the Q function over all available neighborhoods via expectile regression [40]. With the Q function being trained, the policy is independently extracted by weighted regression toward optimized actions within the neighborhoods, obtained in the inner maximization. Empirically, ANQ achieves state-of-the-art performance on standard offline RL benchmarks [20], including Gym locomotion tasks and challenging AntMaze tasks. Moreover, benefiting from the flexible constraint without behavior modeling errors, ANQ attains superior performance in both noisy and limited data scenarios compared to algorithms with other types of constraints. The code is available at https://github.com/thu-rllab/ANQ.

2 Preliminaries

RL. In RL, the environment is typically modeled as a Markov Decision Process (MDP) $\mathcal{M} = (\mathcal{S}, \mathcal{A}, P, R, \gamma, d_0)$, with state space $\mathcal{S}$, action space $\mathcal{A}$, transition dynamics $P : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$, reward function $R : \mathcal{S} \times \mathcal{A} \to [0, R_{\max}]$, discount factor $\gamma \in [0, 1)$, and initial state distribution d_0 [73]. The agent seeks a policy $\pi : \mathcal{S} \to \Delta(\mathcal{A})$ that maximizes the expected return:

$$\eta(\pi) = \mathbb{E}_{s_0 \sim d_0, a_t \sim \pi(\cdot|s_t), s_{t+1} \sim P(\cdot|s_t, a_t)} \left[\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t) \right]. \tag{1}$$

For a given policy π, the state value function is defined as $V^{\pi}(s) = \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t R(s_t, a_t)|s_0 = s\right]$, and the state-action value function is defined as $Q^{\pi}(s, a) = \mathbb{E}_{\pi}\left[\sum_{t=0}^{\infty}\gamma^t R(s_t, a_t)|s_0 = s, a_0 = a\right]$.

Offline RL. In offline RL, the agent is able to access a fixed dataset $\mathcal{D} = \{(s_i, a_i, r_i, s_i')\}_{i=0}^{n-1}$ collected by some behavior policy π_β, and aims to learn an optimal policy without further data collection [43, 44]. Standard Q learning methods seek to learn the optimal Q function by minimizing:

$$L_Q(\theta) = \mathbb{E}_{(s,a,s')\sim\mathcal{D}}\left[(Q_\theta(s,a) - R(s,a) - \gamma \max_{a'} Q_{\theta'}(s', a'))^2\right], \tag{2}$$

where $Q_\theta(s, a)$ denotes a parameterized Q function, and $Q_{\theta'}(s, a)$ represents a target Q function with parameters updated using Polyak averaging [58].

A central challenge in offline RL is the presence of out-of-distribution (OOD) actions that fall outside the support of the behavior policy. These OOD actions often lead to inaccurate Q value estimates due to extrapolation errors [23]. As a result, maximizing the estimated Q functions tends to favor OOD actions with overestimated values, resulting in significant performance degradation [44].

3 Adaptive Neighborhood-Constrained Q Learning for Offline RL

This section focuses on developing action-selection constraints to address the OOD issue in offline RL. First, we provide a systematic categorization of existing approaches and analyze the strengths and limitations of each category. To overcome the limitations, we propose a new neighborhood constraint, supported by theoretical analyses that elucidate its properties. Furthermore, we design an adaptive variant of this flexible constraint, achieving pointwise conservatism in practice. Finally, we develop a simple yet effective algorithm to facilitate Q learning and policy extraction under the constraint.

3.1 A Categorization and Analysis of Constraints in Offline RL

To address the OOD issue, offline RL algorithms impose various constraints to prevent either the learned policy (in actor-critic training) or the Bellman target (in Q learning) from selecting OOD actions. In this context, many approaches inherently align the probability density of the trained policy with that of the behavior policy, either explicitly, through divergence measures such as reverse KL [32, 83], forward KL (i.e., behavior cloning) [21, 61], and Fisher divergence [39], or implicitly, via value penalties that reduce the Q values of trained policy's actions and while increasing those of dataset actions [42, 12]. We formalize this concept as the density constraint in Definition 1.

Definition 1 (Density constraint). *The trained policy satisfies the density constraint $\mathrm{D}(\pi, \pi_\beta) \le \epsilon$, where D represents a divergence measure between the trained policy π and the behavior policy π_β, e.g., KL, total variation (TV), or Fisher divergence.*

While straightforward, this density constraint can be overly restrictive in many scenarios. Lemma 1 provides a theoretical upper bound on policy performance under various forms of density constraints.

Lemma 1 (Performance bound under density constraints). *If any of the conditions $\mathrm{D}_{\mathrm{KL}}(\pi\|\pi_\beta) \le 2\epsilon$, $\mathrm{D}_{\mathrm{KL}}(\pi_\beta\|\pi) \le 2\epsilon$, or $\mathrm{D}_{\mathrm{TV}}(\pi, \pi_\beta) \le \sqrt{\epsilon}$ holds, then the policy performance η is bounded as follows:*

$$\eta(\pi) \le \eta(\pi_\beta) + \frac{2R_{\max}}{(1-\gamma)^2}\sqrt{\epsilon}. \tag{3}$$

Lemma 1 demonstrates that policy performance under the density constraints is affected by the overall quality of the behavior policy. Consequently, even if the optimal behavior is present in the dataset, the learned policy can still remain highly suboptimal if the overall behavior policy is of low quality.

To address this limitation inherent in the density constraint, recent studies have explored the more relaxed support constraint, which only requires selected actions to be within the support of the behavior policy [82, 26, 53, 93]. Prior to the formal definition, we introduce a general loss function for constrained Q learning in Eq. (4), where $\mathcal{C}(s)$ denotes a conditional set of actions for a given state.

$$L_{\mathcal{C}}(\theta) = \mathbb{E}_{(s,a,s')\sim\mathcal{D}}\left[(Q_\theta(s,a) - R(s,a) - \gamma \max_{a'\in\mathcal{C}(s')} Q_{\theta'}(s', a'))^2\right]. \tag{4}$$

https://doi.org/10.52202/085713-0877

Definition 2 (Support constraint)**.** *The selected action in the Bellman target is restricted to the support of the behavior policy, which is defined as $\mathcal{C}_{\mathrm{Supp}}(s) := \{a \in \mathcal{A} \mid \pi_\beta(a|s) > \epsilon\}$, where π_β is the behavior policy and ϵ is a threshold that determines the support.*

This support constraint is generally considered the least restrictive for offline RL [82], as the quality of actions outside the behavior policy's support cannot be reliably assessed. To enforce this constraint, existing approaches typically rely on behavior policy modeling, using techniques such as conditional variational autoencoders (CVAEs) [23, 41, 94, 82], autoregressive models [26], flow-GANs [93], and diffusion models [9]. Specifically, these methods either use pre-trained behavior density estimators to explicitly constrain the policy within the behavior support [41, 82, 93], or employ pre-trained behavior policy samplers to generate in-support actions and select the one with the highest Q value [23, 26, 94, 9]. However, their effectiveness is fundamentally limited by the accuracy of behavior policy modeling [26], which is well-known to be challenging due to the high-dimensional and multi-modal nature of real-world data [93]. Moreover, these methods incur extra computational costs due to behavior model training and, in some cases, extensive action sampling per state.

A parallel research direction has introduced the sample constraint, which constructs the Bellman target exclusively using actions in the dataset [40, 85, 92, 88], and derives policies via weighted behavior cloning [61, 81], thereby avoiding querying any out-of-dataset actions.

Definition 3 (Sample constraint)**.** *The selected action in the Bellman target is restricted to the sample set $\mathcal{C}_{\mathrm{Samp}}(s) := \{a \in \mathcal{A} \mid (s,a) \in \mathcal{D}\}$, consisting of actions in the dataset for a given state $s \in \mathcal{D}$.*

Sample constraint methods are computationally efficient, easy to implement, and effective in avoiding extrapolation errors [40]. However, their performance is inherently constrained by the inability to generalize beyond the offline dataset. This over-conservatism becomes especially problematic when the dataset lacks coverage of near-optimal actions, a common issue in environments with large or continuous action spaces, or when the dataset exhibits low quality or limited diversity. Moreover, to ensure computational stability, these methods often struggle to adequately suppress the impact of suboptimal dataset actions [88], diminishing their efficacy when such actions dominate the dataset.

For extended discussions on related work, we refer the reader to Appendix A.

3.2 Neighborhood Constraint for Offline RL

This work aims to mitigate the over-conservatism inherent in the density and sample constraints, while circumventing the behavior modeling requirement posed by the support constraint. To this end, we introduce a flexible neighborhood constraint for offline RL, which restricts action selection in the Bellman target to the union of neighborhoods of dataset actions on a given state.

Definition 4 (Neighborhood constraint)**.** *The selected action in the Bellman target is restricted to the neighborhood set $\mathcal{C}_{\mathrm{N}}(s) := \{\tilde{a} \in \mathcal{A} \mid \|\tilde{a} - a\| \leq \epsilon, (s,a) \in \mathcal{D}\}$, which comprises actions located within the ϵ-neighborhoods of all dataset actions on a given state $s \in \mathcal{D}$.*

In contrast to the sample constraint, this neighborhood constraint offers greater freedom, as it allows for seeking better actions beyond the dataset, within a flexible range. As shown in the following Theorem 1, the neighborhood constraint can serve as a viable approximation to the least restrictive support constraint, with the benefit of being achievable without behavior policy modeling. To establish the theorem, we introduce the standardness assumption commonly used in geometric measure theory [13, 8], which ensures that the measure does not exhibit "holes" at small scales.

Assumption 1 (Standardness)**.** *Let $S \subseteq \mathbb{R}^d$ be the support of a probability distribution ν, and $B(x,r)$ be the closed ball of radius r centered at $x \in \mathbb{R}^d$. There exist constants $r_0 > 0$ and $C_0 > 0$ such that:*

$$\forall x \in S, \;\; \forall r \leq r_0, \;\; \nu(B(x,r)) \geq C_0 \cdot r^d. \tag{5}$$

Theorem 1 (Support approximation via neighborhoods)**.** *Let $S \subseteq \mathbb{R}^d$ be the compact support of a distribution ν, and let $X_1, \ldots, X_n$ be independent and identically distributed samples from ν. Define $U_{n,\epsilon} = \bigcup_{i=1}^n B(X_i, \epsilon)$ as the union of closed balls of radius ϵ centered at the samples. Let $\mathcal{N}(S, \epsilon/2)$ denote the covering number of S, i.e., the minimal number of $\epsilon/2$-balls required to cover S. Under the standardness Assumption 1 with constants $r_0, C_0 > 0$, for any $\delta \in (0,1)$ and $\epsilon \leq 2r_0$, if*

$$n \geq \frac{1}{C_0(\epsilon/2)^d} \left(\log \mathcal{N}(S, \epsilon/2) + \log(1/\delta)\right), \tag{6}$$

then with probability at least $1-\delta$*, the Hausdorff distance between* S *and* $U_{n,\epsilon}$ *satisfies*

$$d_H(S, U_{n,\epsilon}) := \max\left(\sup_{x\in S}\inf_{u\in U_{n,\epsilon}} d(x,u), \sup_{u\in U_{n,\epsilon}}\inf_{x\in S} d(x,u)\right) \le \epsilon. \tag{7}$$

Since both the support and neighborhood constraint sets are defined over actions conditioned on a given state, Theorem 1 analyzes their relationship at a fixed state, focusing on their difference in the action space. In Theorem 1, ν represents the behavior policy distribution at a state, and S is defined as its support. $X_1, \dots, X_n$ are i.i.d. samples from ν, corresponding to dataset actions at that state. This theorem ensures that the union of sample neighborhoods $U_{n,\epsilon}$ approximates the support S within a controlled Hausdorff distance, capturing the trade-off between sample size n, neighborhood radius ϵ, and the geometric complexity of support S (as reflected by $\mathcal{N}(S, \epsilon/2)$). Note that this Hausdorff distance is sensitive to outliers [29], making it well-suited for evaluating approximation quality in our setting, where outlier actions can significantly affect Q function optimization.

In the following, we further investigate several properties of the proposed neighborhood constraint in the context of controlling extrapolation and distribution shift. The definition of this constraint is closely related to the concept of extrapolation, and Lemma 2 provides a theoretical characterization of the extrapolation behavior of deep Q functions under this constraint.

Lemma 2 (Extrapolation behavior)**.** *Under the neural tangent kernel (NTK) regime [30], for any in-sample state-action pair* $(s,a) \in \mathcal{D}$ *and in-neighborhood state-action pair* $(s,\tilde{a})$ *such that* $\|\tilde{a}-a\| \le \epsilon$*, the value difference of the deep Q function can be bounded as:*

$$\|Q_\theta(s,\tilde{a}) - Q_\theta(s,a)\| \le C(\sqrt{\min(\|s\oplus a\|, \|s\oplus\tilde{a}\|)}\sqrt{\epsilon} + 2\epsilon), \tag{8}$$

where $\oplus$ *denotes the vector concatenation operation, and* C *is a finite constant.*

Lemma 2 is a direct corollary of Theorem 1 in [45], specialized to the case of action extrapolation. It demonstrates that, for any unseen action $\tilde{a}$, its Q value $Q_\theta(s,\tilde{a})$ can be effectively controlled by a dataset action's Q value $Q_\theta(s,a)$ and the distance $\|\tilde{a}-a\|$. Specifically, a smaller neighborhood radius yields tighter control over the output of deep Q functions.

Furthermore, Proposition 1 demonstrates that, under mild continuity conditions on the transition dynamics [15, 87], the neighborhood constraint also helps to bound the degree of distribution shift.

Proposition 1 (Distribution shift)**.** *Let* π_1 *be a deterministic policy that satisfies the neighborhood constraint with threshold* ϵ*. Assume that the transition dynamics* P *is* K_P*-Lipschitz continuous:* $\forall s\in\mathcal{S}$*,* $\forall a_1,a_2\in\mathcal{A}$*,* $\|P(s'|s,a_1) - P(s'|s,a_2)\| \le K_P\|a_1-a_2\|$*. Then, there exists a policy* π_2 *satisfying the sample constraint such that:*

$$\mathrm{D_{TV}}\left(d^{\pi_1}(\cdot), d^{\pi_2}(\cdot)\right) \le \frac{\gamma K_P \epsilon}{2(1-\gamma)}, \tag{9}$$

where $d^\pi(s) = (1-\gamma)\sum_{t=0}^{\infty}\gamma^t\mathbb{E}_\pi\left[\mathbb{I}\left[s_t=s\right]\right]$ *is the state occupancy induced by policy* π*.*

Adaptive neighborhoods. The neighborhood constraint is highly flexible and can, in practice, achieve pointwise conservatism by adapting the neighborhood radius for each data point, enabling the design of an adaptive neighborhood constraint. Considering real-world data patterns, expert data typically clusters within a narrow distribution [3, 18], necessitating tighter constraints to mitigate extrapolation errors, while suboptimal data tends to be more dispersed and thus benefits from looser constraints that facilitate policy improvement. Inspired by this idea, we propose a concrete instantiation of adaptive neighborhoods in Definition 5, where per-sample radius is set as $\epsilon\exp(-\alpha A(s,a))$.

Definition 5 (Adaptive neighborhood constraint)**.** *The selected action in the Bellman target is restricted to the adaptive neighborhood set* $\mathcal{C}_{\mathrm{AN}}(s) := \{\tilde{a}\in\mathcal{A} \mid \|\tilde{a}-a\| \le \epsilon\exp(-\alpha A(s,a)), (s,a)\in\mathcal{D}\}$*, where* A *denotes the advantage function and* α *is an inverse temperature parameter that modulates the sensitivity of the neighborhood radius to advantage values.*

This adaptive neighborhood constraint assigns larger neighborhood radii to dataset actions with low advantage, thereby promoting a broader search over the action space and further mitigating the impact of low-quality data. Conversely, dataset actions with high advantage are assigned smaller neighborhood radii to more effectively reduce the overall extrapolation error. In practice, advantage

https://doi.org/10.52202/085713-0877

estimation errors are typically not a concern for two reasons: (1) In-distribution estimation: the advantage is computed only on dataset points $(s, a) \in \mathcal{D}$, where estimates are relatively reliable; (2) Qualitative use: the purpose of using advantage is to distinguish actions qualitatively, and the exponential form is merely a soft heuristic to bias the radius, without requiring precise values.

3.3 Adaptive Neighborhood-Constrained Q Learning

In the following, we develop an efficient bilevel optimization framework to achieve Q learning under the adaptive neighborhood constraint. Specifically, we aim to minimize the following Q learning loss:

$$L_{\mathrm{ANQ}}(\theta) = \mathbb{E}_{(s,a,s')\sim\mathcal{D}}\left[\left(Q_\theta(s,a) - R(s,a) - \gamma \max_{a'\in\mathcal{C}_{\mathrm{AN}}(s')} Q_{\theta'}(s',a')\right)^2\right]. \tag{10}$$

Bilevel optimization. The primary challenge in constrained Q learning lies in enforcing $\max_{a'\in\mathcal{C}(s')}$ in the Bellman target. While the support constraint $\mathcal{C}_{\mathrm{Supp}}$ typically necessitates accurate modeling of the behavior policy, we demonstrate that the adaptive neighborhood constraint $\mathcal{C}_{\mathrm{AN}}$ can be effectively enforced by decomposing the objective into a bilevel optimization structure:

$$\max_{a\in\mathcal{C}_{\mathrm{AN}}(s)} Q(s,a),\ \forall s\in\mathcal{D} \iff \begin{array}{c} \max\limits_{a\in\mathcal{D}(s)} Q(s, a+\delta_{sa}),\ \forall s \in \mathcal{D} \\ \text{s.t. } \delta_{sa} = \operatorname*{argmax}\limits_{\|\delta\|\le\epsilon\exp(-\alpha A(s,a))} Q(s,a+\delta),\ \forall (s,a)\in\mathcal{D}, \end{array} \tag{11}$$

where we use $\mathcal{D}(s)$ to denote the empirical action set observed in the dataset for a given state $s \in \mathcal{D}$.

The inner maximization in Eq. (11) optimizes the Q function separately within each dataset action's neighborhood. To this end, we introduce an auxiliary policy μ_ω that takes state-action pairs (s, a) from the dataset as input and outputs action variations δ. This formulation enables straightforward enforcement of the adaptive neighborhood constraint by restricting the norm of $\mu_\omega(s, a)$ to stay within the bound $\epsilon \exp(-\alpha A(s, a))$. Practically, we multiply both sides of the constraint inequality by $\exp(\alpha A(s, a))$ to maintain a constant constraint threshold: $\exp(\alpha A(s, a))\|\mu_\omega(s, a)\| \le \epsilon$. Consequently, we optimize the Q function with respect to μ_ω to seek the optimal action within the adaptive neighborhood of each dataset action, according to the following objective:

$$\max_{\mu_\omega} Q_\theta(s, a+\mu_\omega(s,a)) \ \text{ s.t. } \exp(\alpha A(s,a))\|\mu_\omega(s,a)\| \le \epsilon,\ \forall (s,a)\in\mathcal{D}. \tag{12}$$

We reformulate the constrained optimization problem into an unconstrained one using a Lagrange multiplier $\lambda \in \mathbb{R}^+$. In addition, we introduce a state value function $V_\psi(s)$, whose training objective will be specified later, and use the difference $Q_{\theta'} - V_\psi$ to compute the advantage function. Accordingly, we optimize the following objective for the inner maximization:

$$\max_{\mu_\omega} \mathbb{E}_{(s,a)\sim\mathcal{D}}\left[Q_\theta(s,a+\mu_\omega(s,a)) - \lambda\exp(\alpha(Q_{\theta'}(s,a) - V_\psi(s)))\|\mu_\omega(s,a)\|\right]. \tag{13}$$

The outer maximization in Eq. (11) searches over all dataset actions on a given state and seeks the one whose corresponding neighborhood yields the highest Q value. To achieve this objective, we first sample state-action pairs from the dataset and refine the actions by adding the outputs of the trained auxiliary policy, thereby simulating the sampling of the optimized actions across the neighborhoods. We then employ expectile regression [40] to implicitly maximize the Q function over these optimized actions. Specifically, we fit a V function with the following asymmetric squared error loss, treating the Q values of the optimized actions as regression targets:

$$\min_{V_\psi} \mathop{\mathbb{E}}_{(s,a)\sim\mathcal{D}}\left[L_2^\tau\left(Q_{\theta'}(s, a+\mu_{\omega'}(s,a)) - V_\psi(s)\right)\right], \tag{14}$$

where $L_2^\tau(x) = |\tau - \mathbb{1}(x<0)|x^2$, $\tau \in (0,1)$, $Q_{\theta'}$ and $\mu_{\omega'}$ are the target Q function and target auxiliary policy, whose parameters are updated via Polyak averaging [58].

For $\tau \approx 1$, $V_\psi(s)$ captures the maximum Q value within the adaptive neighborhood set $\mathcal{C}_{\mathrm{AN}}(s)$. By substituting $\max_{a'\in\mathcal{C}_{\mathrm{AN}}(s')} Q_{\theta'}(s', a')$ in Eq. (10) with $V_\psi(s')$, adaptive neighborhood-constrained Q learning is achieved based on the following loss:

$$\min_{Q_\theta} \mathbb{E}_{(s,a,s')\sim\mathcal{D}}\left[\left(Q_\theta(s,a) - R(s,a) - \gamma V_\psi(s')\right)^2\right]. \tag{15}$$

A radius-agnostic framework. Although our Q learning algorithm is presented specifically for the adaptive neighborhood in Definition 5, i.e., using the per-sample radius $\epsilon \exp(-\alpha A(s,a))$, the overall framework is general and can accommodate arbitrary radius schemes. Specifically, one can simply replace $\epsilon \exp(-\alpha A(s,a))$ in Definition 5 (and Eq. (11)) with $\epsilon f(s,a)$ to define a generic per-sample neighborhood radius, where $f : \mathcal{S} \times \mathcal{A} \to \mathbb{R}^+$ is an arbitrary function that modulates the radius. Correspondingly, in Eq. (12), $\exp(\alpha A(s,a))$ becomes $1/f(s,a)$, and Eq. (13) becomes:

$$\max_{\mu_\omega} \mathbb{E}_{(s,a)\sim\mathcal{D}}[Q_\theta(s, a + \mu_\omega(s,a)) - \lambda \|\mu_\omega(s,a)\|/f(s,a)]. \tag{16}$$

With all other equations unchanged, the resulting algorithm supports arbitrary neighborhood schemes.

3.4 Policy Extraction via Weighted Regression Toward Optimized Actions

While our algorithm enables Q learning under the adaptive neighborhood constraint, it does not explicitly derive the corresponding policy, thereby requiring a separate policy extraction step. To this end, we employ the weighed behavior cloning method [17, 11] and, rather than imitating the actions in the dataset, we instead imitate the actions that have been refined through the auxiliary policy, which represents the optimal actions within the adaptive neighborhoods. Moreover, we set the weights as the exponentiated advantage function [79, 61, 60, 81]. Consequently, the final policy $\pi_\phi : \mathcal{S} \to \mathcal{A}$ is extracted according to the following loss:

$$\min_{\pi_\phi} \mathbb{E}_{(s,a)\sim\mathcal{D}} \exp(\beta(Q_{\theta'}(s, a + \mu_\omega(s,a)) - V_\psi(s))) \|a + \mu_\omega(s,a) - \pi_\phi(s)\|_2^2, \tag{17}$$

where β is an inverse temperature and $Q_{\theta'} - V_\psi$ computes the advantage function.

Remark. This policy extraction step, which does not interfere with the Q learning process described in Section 3.3, also constitutes a key distinction from existing regression-based policy learning objectives, such as those employed in AWR [61], AWAC [60], CRR [81], 10% BC [10], IQL [40], and SQL [88], all of which perform weighted regression toward the dataset actions. In contrast, our policy learning objective performs weighted regression toward the optimized actions within the adaptive neighborhoods. This enables the trained policy to select actions superior to those in the dataset, while also significantly mitigating the adverse effects of suboptimal dataset actions.

Algorithm 1 ANQ

1: Initialize policy π_ϕ, auxiliary policy μ_ω, target auxiliary policy $\mu_{\omega'}$, Q-network Q_θ, target Q-network $Q_{\theta'}$, and V-network V_ψ.
2: **for** each gradient step **do**
3: Update ψ by minimizing Eq. (14)
4: Update θ by minimizing Eq. (15)
5: Update ω by maximizing Eq. (13)
6: Update ϕ by maximizing Eq. (17)
7: Update target networks: $\theta' \leftarrow (1-\xi)\theta' + \xi\theta$, $\omega' \leftarrow (1-\xi)\omega' + \xi\omega$
8: **end for**

Integrating all components, we present our final algorithm in Algorithm 1.

4 Experiments

We conduct experiments to evaluate the performance and properties of the proposed approach ANQ. Experimental details and extended results are provided in Appendices C and D, respectively.

4.1 Benchmark Results

Tasks. We assess ANQ on two distinct task suites from D4RL [20]: the Gym-MuJoCo locomotion domains and the challenging AntMaze domains. The AntMaze tasks involve sparse rewards and require the ant agent to combine segments of suboptimal trajectories to reach the maze's goal.

Baselines. Our offline RL baselines span various constraint categories. For density constraints, we compare to TD3BC [21], CQL [42], and AWAC [60], where CQL and AWAC essentially enforce a density constraint as analyzed in Theorem 3.5 of [42] and Section 3 of [53], respectively. For support constraints, we include BCQ [23], BEAR [41], and SPOT [82]. For sample constraints, we compare against OneStep RL [6] and IQL [40]. We also include the sequence-modeling approach DT [10].

https://doi.org/10.52202/085713-0877

Table 2: Averaged normalized scores on Gym locomotion and Antmaze tasks over five random seeds. m = medium, m-r = medium-replay, m-e = medium-expert, e = expert, r = random; u = umaze, u-d = umaze-diverse, m-p = medium-play, m-d = medium-diverse, l-p= large-play, l-d = large-diverse.

Dataset-v2	BCQ	BEAR	DT	AWAC	OneStep	TD3BC	CQL	IQL	SPOT	ANQ (Ours)
halfcheetah-m	46.6	43.0	42.6	47.9	50.4	48.3	47.0	47.4	58.4	**61.8±1.4**
hopper-m	59.4	51.8	67.6	59.8	87.5	59.3	53.0	66.2	86.0	**100.9±0.6**
walker2d-m	71.8	-0.2	74.0	83.1	84.8	83.7	73.3	78.3	**86.4**	82.9±1.5
halfcheetah-m-r	42.2	36.3	36.6	44.8	42.7	44.6	45.5	44.2	52.2	**55.5±1.4**
hopper-m-r	60.9	52.2	82.7	69.8	98.5	60.9	88.7	94.7	**100.2**	**101.5±2.7**
walker2d-m-r	57.0	7.0	66.6	78.1	61.7	81.8	81.8	73.8	**91.6**	**92.7±3.8**
halfcheetah-m-e	**95.4**	46.0	86.8	64.9	75.1	90.7	75.6	86.7	86.9	**94.2±0.8**
hopper-m-e	**106.9**	50.6	**107.6**	100.1	**108.6**	98.0	105.6	91.5	99.3	**107.0±4.9**
walker2d-m-e	107.7	22.1	108.1	110.0	**111.3**	110.1	107.9	109.6	**112.0**	**111.7±0.2**
halfcheetah-e	89.9	92.7	87.7	81.7	88.2	**96.7**	**96.3**	**95.0**	94.8	**95.9±0.4**
hopper-e	109.0	54.6	94.2	109.5	106.9	107.8	96.5	109.4	**111.0**	**111.4±2.5**
walker2d-e	106.3	106.6	108.3	**110.1**	**110.7**	**110.2**	108.5	109.9	109.9	**111.8±0.1**
halfcheetah-r	2.2	2.3	2.2	6.1	2.3	11.0	17.5	13.1	**25.4**	**24.9±1.0**
hopper-r	7.8	3.9	5.4	9.2	5.6	8.5	7.9	7.9	23.4	**31.1±0.2**
walker2d-r	4.9	**12.8**	2.2	0.2	6.9	1.6	5.1	5.4	2.4	11.2±9.5
locomotion total	968.0	581.7	972.6	975.3	1041.2	1013.2	1010.2	1033.1	1139.9	**1194.5**
antmaze-u	78.9	73.0	54.2	80.0	54.0	73.0	82.6	89.6	**93.5**	**96.0±1.6**
antmaze-u-d	55.0	61.0	41.2	52.0	57.8	47.0	10.2	65.6	40.7	**80.2±1.8**
antmaze-m-p	0.0	0.0	0.0	0.0	0.0	0.0	59.0	**76.4**	**74.7**	**76.2±3.3**
antmaze-m-d	0.0	8.0	0.0	0.2	0.6	0.2	46.6	72.8	**79.1**	**77.2±6.1**
antmaze-l-p	6.7	0.0	0.0	0.0	0.0	0.0	16.4	42.0	35.3	**56.2±4.9**
antmaze-l-d	2.2	0.0	0.0	0.0	0.2	0.0	3.2	46.0	36.3	**55.8±4.0**
antmaze total	142.8	142.0	95.4	132.2	112.6	120.2	218.0	392.4	359.6	**441.6**

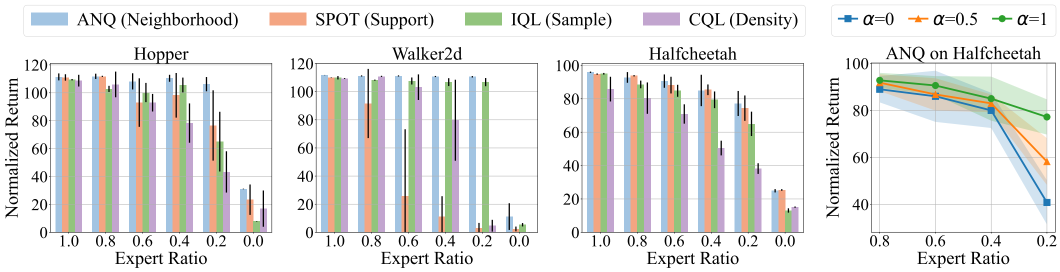
Comparisons. Aggregated results are reported in Table 2. On the Gym locomotion tasks, ANQ outperforms existing methods on most tasks and achieves the highest overall score. On the challenging AntMaze tasks, ANQ surpasses the baselines by a considerable margin, particularly in the most complex large maze settings. The learning curves are provided in Appendix D.5. We also extend our evaluation in Appendix D.2 by comparing ANQ with additional recent SOTA algorithms.

Runtime. We test the runtime of ANQ and some baseline methods on a GeForce RTX 3090. As shown in Appendix D.1, ANQ is among the fastest tier of offline RL algorithms, on par with efficient baselines such as AWAC, IQL, and TD3BC, with a detailed analysis provided in the same section.

4.2 Noisy Data Results

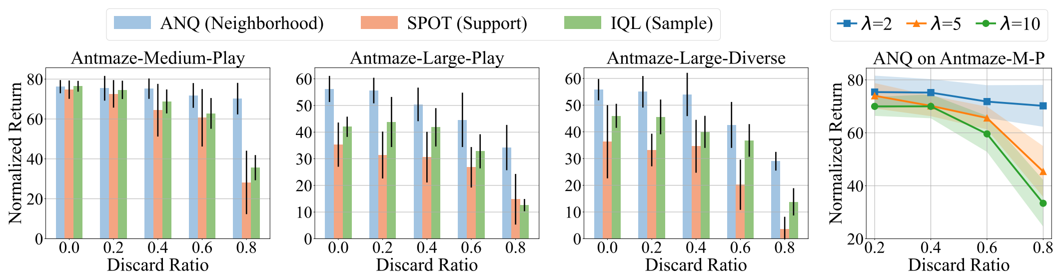
We examine the robustness of various constraint types under noisy data conditions. Specifically, we construct noisy datasets by mixing the random and expert datasets at varying ratios, thereby simulating real-world scenarios such as imperfect demonstrations in robotics or suboptimal data collection in autonomous systems. We then evaluate the performance of representative algorithms, including CQL (density), IQL (sample), SPOT (support), and ANQ (neighborhood).

As presented in Figure 1(a), ANQ generally outperforms the other algorithms across expert ratios, and its performance advantage becomes more pronounced as the proportion of expert data decreases. As analyzed in Section 3.1, the density constraint is sensitive to the overall quality of the behavior policy, which tends to be low in noisy datasets, while the support constraint often struggles with modeling the multi-modal behavior policy distribution inherent in such datasets. In contrast, the adaptive neighborhood constraint employed by ANQ exhibits greater robustness to noisy data. Moreover, we evaluate ANQ on such noisy datasets with varying inverse temperature α that controls the adaptiveness of neighborhood radius. The results in Figure 1(b) demonstrate that, compared with the uniform neighborhood constraint ($\alpha = 0$), the adaptive neighborhood constraint further mitigates the adverse effects of low-quality data in the datasets and exhibits greater robustness to such data.



(a) Evaluation of algorithms with various constraint types on noisy datasets (b) ANQ with varying α

Figure 1: (a) Evaluation on noisy datasets over five random seeds. (b) Evaluation of ANQ on noisy datasets with varying inverse temperature α that determines the adaptiveness of neighborhood radius.



(a) Evaluation of algorithms with various constraint types on reduced datasets (b) ANQ with varying λ

Figure 2: (a) Evaluation on reduced datasets over five random seeds. (b) Evaluation of ANQ on reduced datasets with varying Lagrange multiplier λ that controls the overall radius of neighborhoods.

4.3 Limited Data Results

We also investigate the robustness of these constraint types in limited data settings. To this end, we create reduced datasets by randomly discarding some portion of transitions from the AntMaze datasets. This setup mimics practical scenarios in which data is rare or partially missing, such as in healthcare applications. Again, we evaluate the performance of IQL, SPOT, and ANQ, omitting CQL due to its consistently inferior performance on Antmaze tasks as reported in Table 2.

As shown in Figure 2(a), ANQ demonstrates superior performance across nearly all discard ratios, with the performance gap widening as the amount of available data decreases. In such limited data settings, support constraint methods face even more difficulties in modeling the behavior policy due to sample scarcity, whereas sample constraint methods risk being overly conservative because of reduced coverage of near-optimal actions. In contrast, ANQ bypasses the need to model the behavior policy and leverages generalization to attain superior performance beyond the offline dataset. Furthermore, we evaluate ANQ on reduced datasets with varying Lagrange multiplier λ, which is inversely proportional to the overall radius of the neighborhoods. The results in Figure 1(b) show that an appropriately large neighborhood is crucial for achieving good performance in such limited data scenarios, further showcasing the benefit and flexibility of ANQ over sample constraint methods.

4.4 Ablation Study

Lagrange multiplier λ. The Lagrange multiplier λ controls the overall neighborhood radius in ANQ. We vary λ and present the learned Q values and performance across various tasks in Figure 3. As λ decreases from a sufficiently large value, ANQ enables larger neighborhoods, resulting in higher and possibly divergent Q values, and performance also exhibits a rise-then-fall trend. Note that ANQ with $\lambda = 0$ and $\lambda = \infty$ approximately corresponds to Q learning without any constraint and with the sample constraint, respectively. Therefore, the results provide evidence that ANQ not only effectively suppresses extrapolation error, but also mitigates the over-conservatism of the sample constraint.

https://doi.org/10.52202/085713-0877

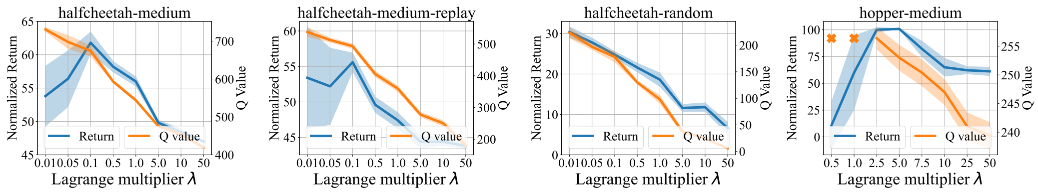


Figure 3: Performance and Q values of ANQ with varying Lagrange multiplier λ over five random seeds. The crosses $\times$ mean that the value functions diverge in some seeds. As λ decreases, ANQ enables larger overall neighborhood radii, resulting in higher and probably divergent learned Q values. A moderate λ (neighborhood constraint) is crucial for achieving superior performance.

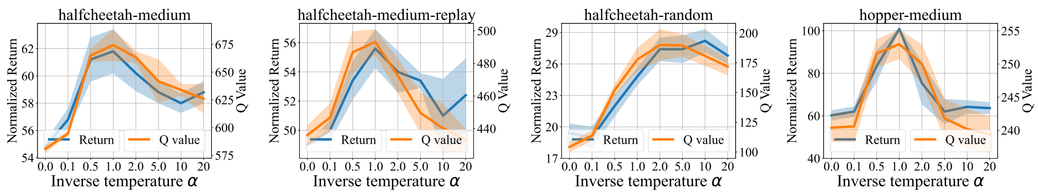


Figure 4: Performance and Q values of ANQ with varying inverse temperature α over five random seeds. An appropriately large α (adaptive neighborhoods) yields enhanced performance.

Inverse temperature α. The inverse temperature α determines how the neighborhood radius adapts to the action advantage, where $\alpha = 0$ corresponds to a fixed neighborhood radius. The results in Figure 4 demonstrate that an appropriately large α leads to enhanced performance, validating our design of advantage-based adaptive neighborhoods. However, an excessively large α ($\alpha = 20$) may degrade performance, likely due to the increased variance of the learning objective.

5 Conclusion and Limitations

This work focuses on developing action-selection constraints to address the OOD issue in offline RL. To overcome the identified limitations of existing approaches, we propose the flexible neighborhood constraint and the corresponding algorithm ANQ, which mitigates the over-conservatism inherent in the density and sample constraints, and approximates the least restrictive support constraint without challenging behavior modeling. Empirical results demonstrate that ANQ achieves SOTA performance on standard offline RL benchmarks and exhibits enhanced robustness to noisy or limited data.

At the algorithmic level, this work develops a general framework for achieving pointwise conservatism by adapting the neighborhood radius for each data point. In particular, the practical algorithm ANQ represents one instantiation of adaptive neighborhoods, using data point quality as the adaptation criterion. However, this design is not necessarily optimal; incorporating additional information, such as uncertainty quantification, could potentially lead to more effective neighborhood construction.

Acknowledgment

We thank the anonymous reviewers for feedback on an early version of this paper. This work was supported by the National Key R&D Program of China under Grant 2018AAA0102801.

References

[1] Joshua Achiam, David Held, Aviv Tamar, and Pieter Abbeel. Constrained policy optimization. In *International conference on machine learning*, pages 22–31. PMLR, 2017.

[2] Gaon An, Seungyong Moon, Jang-Hyun Kim, and Hyun Oh Song. Uncertainty-based offline reinforcement learning with diversified q-ensemble. *Advances in neural information processing systems*, 34:7436–7447, 2021.

[3] Brenna D Argall, Sonia Chernova, Manuela Veloso, and Brett Browning. A survey of robot learning from demonstration. *Robotics and autonomous systems*, 57(5):469–483, 2009.

[4] Sanjeev Arora, Simon Du, Wei Hu, Zhiyuan Li, and Ruosong Wang. Fine-grained analysis of optimization and generalization for overparameterized two-layer neural networks. In *International Conference on Machine Learning*, pages 322–332. PMLR, 2019.

[5] Chenjia Bai, Lingxiao Wang, Zhuoran Yang, Zhi-Hong Deng, Animesh Garg, Peng Liu, and Zhaoran Wang. Pessimistic bootstrapping for uncertainty-driven offline reinforcement learning. In *International Conference on Learning Representations*, 2022. URL https://openreview.net/forum?id=Y4cs1Z3HnqL.

[6] David Brandfonbrener, Will Whitney, Rajesh Ranganath, and Joan Bruna. Offline rl without off-policy evaluation. *Advances in Neural Information Processing Systems*, 34:4933–4946, 2021.

[7] Qi Cai, Zhuoran Yang, Jason D Lee, and Zhaoran Wang. Neural temporal-difference learning converges to global optima. *Advances in Neural Information Processing Systems*, 32, 2019.

[8] Frédéric Chazal, Marc Glisse, Catherine Labruère, and Bertrand Michel. Convergence rates for persistence diagram estimation in topological data analysis. In *International Conference on Machine Learning*, pages 163–171. PMLR, 2014.

[9] Huayu Chen, Cheng Lu, Chengyang Ying, Hang Su, and Jun Zhu. Offline reinforcement learning via high-fidelity generative behavior modeling. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=42zs3qa2kpy.

[10] Lili Chen, Kevin Lu, Aravind Rajeswaran, Kimin Lee, Aditya Grover, Misha Laskin, Pieter Abbeel, Aravind Srinivas, and Igor Mordatch. Decision transformer: Reinforcement learning via sequence modeling. *Advances in neural information processing systems*, 34:15084–15097, 2021.

[11] Xinyue Chen, Zijian Zhou, Zheng Wang, Che Wang, Yanqiu Wu, and Keith Ross. Bail: Best-action imitation learning for batch deep reinforcement learning. *Advances in Neural Information Processing Systems*, 33:18353–18363, 2020.

[12] Ching-An Cheng, Tengyang Xie, Nan Jiang, and Alekh Agarwal. Adversarially trained actor critic for offline reinforcement learning. In *International Conference on Machine Learning*, pages 3852–3878. PMLR, 2022.

[13] Antonio Cuevas. Set estimation: Another bridge between statistics and geometry. *Bol. Estad. Investig. Oper*, 25(2):71–85, 2009.

[14] Jonas Degrave, Federico Felici, Jonas Buchli, Michael Neunert, Brendan Tracey, Francesco Carpanese, Timo Ewalds, Roland Hafner, Abbas Abdolmaleki, Diego de Las Casas, et al. Magnetic control of tokamak plasmas through deep reinforcement learning. *Nature*, 602(7897): 414–419, 2022.

[15] Francois Dufour and Tomas Prieto-Rumeau. Finite linear programming approximations of constrained discounted markov decision processes. *SIAM Journal on Control and Optimization*, 51(2):1298–1324, 2013.

[16] Francois Dufour and Tomas Prieto-Rumeau. Approximation of average cost markov decision processes using empirical distributions and concentration inequalities. *Stochastics An International Journal of Probability and Stochastic Processes*, 87(2):273–307, 2015.

[17] Scott Emmons, Benjamin Eysenbach, Ilya Kostrikov, and Sergey Levine. Rvs: What is essential for offline rl via supervised learning? *arXiv preprint arXiv:2112.10751*, 2021.

[18] K Anders Ericsson, Ralf T Krampe, and Clemens Tesch-Römer. The role of deliberate practice in the acquisition of expert performance. *Psychological review*, 100(3):363, 1993.

 https://doi.org/10.52202/085713-0877

[19] Jianqing Fan, Zhaoran Wang, Yuchen Xie, and Zhuoran Yang. A theoretical analysis of deep q-learning. In *Learning for dynamics and control*, pages 486–489. PMLR, 2020.

[20] Justin Fu, Aviral Kumar, Ofir Nachum, George Tucker, and Sergey Levine. D4rl: Datasets for deep data-driven reinforcement learning. *arXiv preprint arXiv:2004.07219*, 2020.

[21] Scott Fujimoto and Shixiang Shane Gu. A minimalist approach to offline reinforcement learning. *Advances in neural information processing systems*, 34:20132–20145, 2021.

[22] Scott Fujimoto, Herke Hoof, and David Meger. Addressing function approximation error in actor-critic methods. In *International conference on machine learning*, pages 1587–1596. PMLR, 2018.

[23] Scott Fujimoto, David Meger, and Doina Precup. Off-policy deep reinforcement learning without exploration. In *International conference on machine learning*, pages 2052–2062. PMLR, 2019.

[24] Javier Garcıa and Fernando Fernández. A comprehensive survey on safe reinforcement learning. *Journal of Machine Learning Research*, 16(1):1437–1480, 2015.

[25] Divyansh Garg, Joey Hejna, Matthieu Geist, and Stefano Ermon. Extreme q-learning: Maxent RL without entropy. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=SJ0Lde3tRL.

[26] Seyed Kamyar Seyed Ghasemipour, Dale Schuurmans, and Shixiang Shane Gu. Emaq: Expected-max q-learning operator for simple yet effective offline and online rl. In *International Conference on Machine Learning*, pages 3682–3691. PMLR, 2021.

[27] Shangding Gu, Long Yang, Yali Du, Guang Chen, Florian Walter, Jun Wang, and Alois Knoll. A review of safe reinforcement learning: Methods, theory and applications. *arXiv preprint arXiv:2205.10330*, 2022.

[28] Shengchao Hu, Ziqing Fan, Chaoqin Huang, Li Shen, Ya Zhang, Yanfeng Wang, and Dacheng Tao. Q-value regularized transformer for offline reinforcement learning. In *International Conference on Machine Learning*, pages 19165–19181. PMLR, 2024.

[29] Daniel P Huttenlocher, Gregory A. Klanderman, and William J Rucklidge. Comparing images using the hausdorff distance. *IEEE Transactions on pattern analysis and machine intelligence*, 15(9):850–863, 1993.

[30] Arthur Jacot, Franck Gabriel, and Clément Hongler. Neural tangent kernel: Convergence and generalization in neural networks. *Advances in neural information processing systems*, 31, 2018.

[31] Michael Janner, Justin Fu, Marvin Zhang, and Sergey Levine. When to trust your model: Model-based policy optimization. *Advances in neural information processing systems*, 32, 2019.

[32] Natasha Jaques, Asma Ghandeharioun, Judy Hanwen Shen, Craig Ferguson, Agata Lapedriza, Noah Jones, Shixiang Gu, and Rosalind Picard. Way off-policy batch deep reinforcement learning of implicit human preferences in dialog. *arXiv preprint arXiv:1907.00456*, 2019.

[33] Alistair EW Johnson, Tom J Pollard, Lu Shen, Li-wei H Lehman, Mengling Feng, Mohammad Ghassemi, Benjamin Moody, Peter Szolovits, Leo Anthony Celi, and Roger G Mark. Mimic-iii, a freely accessible critical care database. *Scientific data*, 3(1):1–9, 2016.

[34] Lukasz Kaiser, Mohammad Babaeizadeh, Piotr Milos, Blazej Osinski, Roy H Campbell, Konrad Czechowski, Dumitru Erhan, Chelsea Finn, Piotr Kozakowski, Sergey Levine, et al. Model-based reinforcement learning for atari. *arXiv preprint arXiv:1903.00374*, 2019.

[35] Sham Kakade and John Langford. Approximately optimal approximate reinforcement learning. In *In Proc. 19th International Conference on Machine Learning*. Citeseer, 2002.

[36] Rahul Kidambi, Aravind Rajeswaran, Praneeth Netrapalli, and Thorsten Joachims. Morel: Model-based offline reinforcement learning. *Advances in neural information processing systems*, 33:21810–21823, 2020.

[37] Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. *arXiv preprint arXiv:1412.6980*, 2014.

[38] Jens Kober, J Andrew Bagnell, and Jan Peters. Reinforcement learning in robotics: A survey. *The International Journal of Robotics Research*, 32(11):1238–1274, 2013.

[39] Ilya Kostrikov, Rob Fergus, Jonathan Tompson, and Ofir Nachum. Offline reinforcement learning with fisher divergence critic regularization. In *International Conference on Machine Learning*, pages 5774–5783. PMLR, 2021.

[40] Ilya Kostrikov, Ashvin Nair, and Sergey Levine. Offline reinforcement learning with implicit q-learning. In *International Conference on Learning Representations*, 2022. URL https://openreview.net/forum?id=68n2s9ZJWF8.

[41] Aviral Kumar, Justin Fu, Matthew Soh, George Tucker, and Sergey Levine. Stabilizing off-policy q-learning via bootstrapping error reduction. *Advances in Neural Information Processing Systems*, 32, 2019.

[42] Aviral Kumar, Aurick Zhou, George Tucker, and Sergey Levine. Conservative q-learning for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 33: 1179–1191, 2020.

[43] Sascha Lange, Thomas Gabel, and Martin Riedmiller. Batch reinforcement learning. *Reinforcement learning: State-of-the-art*, pages 45–73, 2012.

[44] Sergey Levine, Aviral Kumar, George Tucker, and Justin Fu. Offline reinforcement learning: Tutorial, review, and perspectives on open problems. *arXiv preprint arXiv:2005.01643*, 2020.

[45] Jianxiong Li, Xianyuan Zhan, Haoran Xu, Xiangyu Zhu, Jingjing Liu, and Ya-Qin Zhang. When data geometry meets deep function: Generalizing offline reinforcement learning. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=lMO7TC7cuuh.

[46] Boyi Liu, Qi Cai, Zhuoran Yang, and Zhaoran Wang. Neural trust region/proximal policy optimization attains globally optimal policy. *Advances in neural information processing systems*, 32, 2019.

[47] Xiaoteng Ma, Yiqin Yang, Hao Hu, Qihan Liu, Jun Yang, Chongjie Zhang, Qianchuan Zhao, and Bin Liang. Offline reinforcement learning with value-based episodic memory. *arXiv preprint arXiv:2110.09796*, 2021.

[48] Yecheng Ma, Dinesh Jayaraman, and Osbert Bastani. Conservative offline distributional reinforcement learning. *Advances in Neural Information Processing Systems*, 34:19235–19247, 2021.

[49] Yi Ma, Hongyao Tang, Dong Li, and Zhaopeng Meng. Reining generalization in offline reinforcement learning via representation distinction. *Advances in Neural Information Processing Systems*, 36:40773–40785, 2023.

[50] Yi Ma, Jianye Hao, Xiaohan Hu, Yan Zheng, and Chenjun Xiao. Iteratively refined behavior regularization for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 37:56215–56243, 2024.

[51] Yi Ma, Jianye Hao, Hebin Liang, and Chenjun Xiao. Rethinking decision transformer via hierarchical reinforcement learning. In *Proceedings of the 41st International Conference on Machine Learning*, pages 33730–33745, 2024.

[52] Will Maddern, Geoffrey Pascoe, Chris Linegar, and Paul Newman. 1 year, 1000 km: The oxford robotcar dataset. *The International Journal of Robotics Research*, 36(1):3–15, 2017.

 https://doi.org/10.52202/085713-0877

[53] Yixiu Mao, Hongchang Zhang, Chen Chen, Yi Xu, and Xiangyang Ji. Supported trust region optimization for offline reinforcement learning. In *International Conference on Machine Learning*, pages 23829–23851. PMLR, 2023.

[54] Yixiu Mao, Hongchang Zhang, Chen Chen, Yi Xu, and Xiangyang Ji. Supported value regularization for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 36:40587–40609, 2023.

[55] Yixiu Mao, Qi Wang, Chen Chen, Yun Qu, and Xiangyang Ji. Offline reinforcement learning with ood state correction and ood action suppression. *Advances in Neural Information Processing Systems*, 37:93568–93601, 2024.

[56] Yixiu Mao, Qi Wang, Yun Qu, Yuhang Jiang, and Xiangyang Ji. Doubly mild generalization for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 37: 51436–51473, 2024.

[57] Tatsuya Matsushima, Hiroki Furuta, Yutaka Matsuo, Ofir Nachum, and Shixiang Gu. Deployment-efficient reinforcement learning via model-based offline optimization. In *International Conference on Learning Representations*, 2021. URL https://openreview.net/forum?id=3hGNqpI4WS.

[58] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, Georg Ostrovski, et al. Human-level control through deep reinforcement learning. *nature*, 518(7540):529–533, 2015.

[59] Thomas M Moerland, Joost Broekens, Aske Plaat, Catholijn M Jonker, et al. Model-based reinforcement learning: A survey. *Foundations and Trends® in Machine Learning*, 16(1):1–118, 2023.

[60] Ashvin Nair, Abhishek Gupta, Murtaza Dalal, and Sergey Levine. Awac: Accelerating online reinforcement learning with offline datasets. *arXiv preprint arXiv:2006.09359*, 2020.

[61] Xue Bin Peng, Aviral Kumar, Grace Zhang, and Sergey Levine. Advantage-weighted regression: Simple and scalable off-policy reinforcement learning. *arXiv preprint arXiv:1910.00177*, 2019.

[62] Eduardo Pignatelli, Johan Ferret, Matthieu Geist, Thomas Mesnard, Hado van Hasselt, Olivier Pietquin, and Laura Toni. A survey of temporal credit assignment in deep reinforcement learning. *arXiv preprint arXiv:2312.01072*, 2023.

[63] Yun Qu, Boyuan Wang, Jianzhun Shao, Yuhang Jiang, Chen Chen, Zhenbin Ye, Linc Liu, Junfeng Yang, Lin Lai, Hongyang Qin, et al. Hokoff: Real game dataset from honor of kings and its offline reinforcement learning benchmarks. In *Thirty-seventh Conference on Neural Information Processing Systems Track on Datasets and Benchmarks*, 2023.

[64] Yun Qu, Yuhang Jiang, Boyuan Wang, Yixiu Mao, Cheems Wang, Chang Liu, and Xiangyang Ji. Latent reward: Llm-empowered credit assignment in episodic reinforcement learning. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, pages 20095–20103, 2025.

[65] Yun Qu, Cheems Wang, Yixiu Mao, Yiqin Lv, and Xiangyang Ji. Fast and robust: Task sampling with posterior and diversity synergies for adaptive decision-makers in randomized environments. In *Forty-second International Conference on Machine Learning*, 2025. URL https://openreview.net/forum?id=5UzT2VfPws.

[66] Yuhang Ran, Yi-Chen Li, Fuxiang Zhang, Zongzhang Zhang, and Yang Yu. Policy regularization with dataset constraint for offline reinforcement learning. In *International Conference on Machine Learning*, pages 28701–28717. PMLR, 2023.

[67] Julian Schrittwieser, Ioannis Antonoglou, Thomas Hubert, Karen Simonyan, Laurent Sifre, Simon Schmitt, Arthur Guez, Edward Lockhart, Demis Hassabis, Thore Graepel, et al. Mastering atari, go, chess and shogi by planning with a learned model. *Nature*, 588(7839):604–609, 2020.

[68] John Schulman, Sergey Levine, Pieter Abbeel, Michael Jordan, and Philipp Moritz. Trust region policy optimization. In *International conference on machine learning*, pages 1889–1897. PMLR, 2015.

[69] Jianzhun Shao, Yun Qu, Chen Chen, Hongchang Zhang, and Xiangyang Ji. Counterfactual conservative q learning for offline multi-agent reinforcement learning. In *Thirty-seventh Conference on Neural Information Processing Systems*, 2023. URL https://openreview.net/forum?id=62zm04mv8X.

[70] David Silver, Julian Schrittwieser, Karen Simonyan, Ioannis Antonoglou, Aja Huang, Arthur Guez, Thomas Hubert, Lucas Baker, Matthew Lai, Adrian Bolton, et al. Mastering the game of go without human knowledge. *nature*, 550(7676):354–359, 2017.

[71] Yihao Sun, Jiaji Zhang, Chengxing Jia, Haoxin Lin, Junyin Ye, and Yang Yu. Model-bellman inconsistency for model-based offline reinforcement learning. In *International Conference on Machine Learning*, pages 33177–33194. PMLR, 2023.

[72] Richard S Sutton. Dyna, an integrated architecture for learning, planning, and reacting. *ACM Sigart Bulletin*, 2(4):160–163, 1991.

[73] Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*. MIT press, 2018.

[74] Abdel Aziz Taha and Allan Hanbury. An efficient algorithm for calculating the exact hausdorff distance. *IEEE transactions on pattern analysis and machine intelligence*, 37(11):2153–2163, 2015.

[75] Denis Tarasov, Vladislav Kurenkov, Alexander Nikulin, and Sergey Kolesnikov. Revisiting the minimalist approach to offline reinforcement learning. *Advances in Neural Information Processing Systems*, 36, 2024.

[76] Emanuel Todorov, Tom Erez, and Yuval Tassa. Mujoco: A physics engine for model-based control. In *2012 IEEE/RSJ international conference on intelligent robots and systems*, pages 5026–5033. IEEE, 2012.

[77] Qi Wang and Herke Van Hoof. Learning expressive meta-representations with mixture of expert neural processes. *Advances in neural information processing systems*, 35:26242–26255, 2022.

[78] Qi Wang and Herke Van Hoof. Model-based meta reinforcement learning using graph structured surrogate models and amortized policy search. In *International Conference on Machine Learning*, pages 23055–23077. PMLR, 2022.

[79] Qing Wang, Jiechao Xiong, Lei Han, Han Liu, Tong Zhang, et al. Exponentially weighted imitation learning for batched historical data. *Advances in Neural Information Processing Systems*, 31, 2018.

[80] Zhendong Wang, Jonathan J Hunt, and Mingyuan Zhou. Diffusion policies as an expressive policy class for offline reinforcement learning. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=AHvFDPi-FA.

[81] Ziyu Wang, Alexander Novikov, Konrad Zolna, Josh S Merel, Jost Tobias Springenberg, Scott E Reed, Bobak Shahriari, Noah Siegel, Caglar Gulcehre, Nicolas Heess, et al. Critic regularized regression. *Advances in Neural Information Processing Systems*, 33:7768–7778, 2020.

[82] Jialong Wu, Haixu Wu, Zihan Qiu, Jianmin Wang, and Mingsheng Long. Supported policy optimization for offline reinforcement learning. In Alice H. Oh, Alekh Agarwal, Danielle Belgrave, and Kyunghyun Cho, editors, *Advances in Neural Information Processing Systems*, 2022. URL https://openreview.net/forum?id=KCXQ5HoM-fy.

[83] Yifan Wu, George Tucker, and Ofir Nachum. Behavior regularized offline reinforcement learning. *arXiv preprint arXiv:1911.11361*, 2019.

[84] Yueh-Hua Wu, Xiaolong Wang, and Masashi Hamaya. Elastic decision transformer. *Advances in neural information processing systems*, 36:18532–18550, 2023.

 https://doi.org/10.52202/085713-0877

[85] Chenjun Xiao, Han Wang, Yangchen Pan, Adam White, and Martha White. The in-sample softmax for offline reinforcement learning. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=u-RuvyDYqCM.

[86] Tengyang Xie, Ching-An Cheng, Nan Jiang, Paul Mineiro, and Alekh Agarwal. Bellman-consistent pessimism for offline reinforcement learning. *Advances in neural information processing systems*, 34:6683–6694, 2021.

[87] Huaqing Xiong, Tengyu Xu, Lin Zhao, Yingbin Liang, and Wei Zhang. Deterministic policy gradient: Convergence analysis. In *Uncertainty in Artificial Intelligence*, pages 2159–2169. PMLR, 2022.

[88] Haoran Xu, Li Jiang, Jianxiong Li, Zhuoran Yang, Zhaoran Wang, Victor Wai Kin Chan, and Xianyuan Zhan. Offline RL with no OOD actions: In-sample learning via implicit value regularization. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=ueYYgo2pSSU.

[89] Rui Yang, Chenjia Bai, Xiaoteng Ma, Zhaoran Wang, Chongjie Zhang, and Lei Han. RORL: Robust offline reinforcement learning via conservative smoothing. In Alice H. Oh, Alekh Agarwal, Danielle Belgrave, and Kyunghyun Cho, editors, *Advances in Neural Information Processing Systems*, 2022. URL https://openreview.net/forum?id=_QzJJGH_KE.

[90] Tianhe Yu, Garrett Thomas, Lantao Yu, Stefano Ermon, James Y Zou, Sergey Levine, Chelsea Finn, and Tengyu Ma. Mopo: Model-based offline policy optimization. *Advances in Neural Information Processing Systems*, 33:14129–14142, 2020.

[91] Tianhe Yu, Aviral Kumar, Rafael Rafailov, Aravind Rajeswaran, Sergey Levine, and Chelsea Finn. Combo: Conservative offline model-based policy optimization. *Advances in neural information processing systems*, 34:28954–28967, 2021.

[92] Hongchang Zhang, Yixiu Mao, Boyuan Wang, Shuncheng He, Yi Xu, and Xiangyang Ji. In-sample actor critic for offline reinforcement learning. In *The Eleventh International Conference on Learning Representations*, 2023. URL https://openreview.net/forum?id=dfDv0WU853R.

[93] Jing Zhang, Chi Zhang, Wenjia Wang, and Bingyi Jing. Constrained policy optimization with explicit behavior density for offline reinforcement learning. *Advances in Neural Information Processing Systems*, 36, 2024.

[94] Wenxuan Zhou, Sujay Bajracharya, and David Held. Plas: Latent action space for offline reinforcement learning. In *Conference on Robot Learning*, pages 1719–1735. PMLR, 2021.

[95] Heming Zou, Yunliang Zang, and Xiangyang Ji. Structural features of the fly olfactory circuit mitigate the stability-plasticity dilemma in continual learning. *arXiv preprint arXiv:2502.01427*, 2025.

NeurIPS Paper Checklist

1. **Claims**

   Question: Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope?

   Answer: [Yes]

   Justification: The main claims made in the abstract and introduction accurately reflect the paper's contributions and scope.

   Guidelines:

   - The answer NA means that the abstract and introduction do not include the claims made in the paper.
   - The abstract and/or introduction should clearly state the claims made, including the contributions made in the paper and important assumptions and limitations. A No or NA answer to this question will not be perceived well by the reviewers.
   - The claims made should match theoretical and experimental results, and reflect how much the results can be expected to generalize to other settings.
   - It is fine to include aspirational goals as motivation as long as it is clear that these goals are not attained by the paper.

2. **Limitations**

   Question: Does the paper discuss the limitations of the work performed by the authors?

   Answer: [Yes]

   Justification: Please refer to Section 5.

   Guidelines:

   - The answer NA means that the paper has no limitation while the answer No means that the paper has limitations, but those are not discussed in the paper.
   - The authors are encouraged to create a separate "Limitations" section in their paper.
   - The paper should point out any strong assumptions and how robust the results are to violations of these assumptions (e.g., independence assumptions, noiseless settings, model well-specification, asymptotic approximations only holding locally). The authors should reflect on how these assumptions might be violated in practice and what the implications would be.
   - The authors should reflect on the scope of the claims made, e.g., if the approach was only tested on a few datasets or with a few runs. In general, empirical results often depend on implicit assumptions, which should be articulated.
   - The authors should reflect on the factors that influence the performance of the approach. For example, a facial recognition algorithm may perform poorly when image resolution is low or images are taken in low lighting. Or a speech-to-text system might not be used reliably to provide closed captions for online lectures because it fails to handle technical jargon.
   - The authors should discuss the computational efficiency of the proposed algorithms and how they scale with dataset size.
   - If applicable, the authors should discuss possible limitations of their approach to address problems of privacy and fairness.
   - While the authors might fear that complete honesty about limitations might be used by reviewers as grounds for rejection, a worse outcome might be that reviewers discover limitations that aren't acknowledged in the paper. The authors should use their best judgment and recognize that individual actions in favor of transparency play an important role in developing norms that preserve the integrity of the community. Reviewers will be specifically instructed to not penalize honesty concerning limitations.

3. **Theory assumptions and proofs**

   Question: For each theoretical result, does the paper provide the full set of assumptions and a complete (and correct) proof?

   Answer: [Yes]

https://doi.org/10.52202/085713-0877

Justification: Please refer to Appendix B.

Guidelines:

- The answer NA means that the paper does not include theoretical results.
- All the theorems, formulas, and proofs in the paper should be numbered and cross-referenced.
- All assumptions should be clearly stated or referenced in the statement of any theorems.
- The proofs can either appear in the main paper or the supplemental material, but if they appear in the supplemental material, the authors are encouraged to provide a short proof sketch to provide intuition.
- Inversely, any informal proof provided in the core of the paper should be complemented by formal proofs provided in appendix or supplemental material.
- Theorems and Lemmas that the proof relies upon should be properly referenced.

4. **Experimental result reproducibility**

   Question: Does the paper fully disclose all the information needed to reproduce the main experimental results of the paper to the extent that it affects the main claims and/or conclusions of the paper (regardless of whether the code and data are provided or not)?

   Answer: [Yes]

   Justification: Please refer to Appendix C.

   Guidelines:

   - The answer NA means that the paper does not include experiments.
   - If the paper includes experiments, a No answer to this question will not be perceived well by the reviewers: Making the paper reproducible is important, regardless of whether the code and data are provided or not.
   - If the contribution is a dataset and/or model, the authors should describe the steps taken to make their results reproducible or verifiable.
   - Depending on the contribution, reproducibility can be accomplished in various ways. For example, if the contribution is a novel architecture, describing the architecture fully might suffice, or if the contribution is a specific model and empirical evaluation, it may be necessary to either make it possible for others to replicate the model with the same dataset, or provide access to the model. In general. releasing code and data is often one good way to accomplish this, but reproducibility can also be provided via detailed instructions for how to replicate the results, access to a hosted model (e.g., in the case of a large language model), releasing of a model checkpoint, or other means that are appropriate to the research performed.
   - While NeurIPS does not require releasing code, the conference does require all submissions to provide some reasonable avenue for reproducibility, which may depend on the nature of the contribution. For example
     (a) If the contribution is primarily a new algorithm, the paper should make it clear how to reproduce that algorithm.
     (b) If the contribution is primarily a new model architecture, the paper should describe the architecture clearly and fully.
     (c) If the contribution is a new model (e.g., a large language model), then there should either be a way to access this model for reproducing the results or a way to reproduce the model (e.g., with an open-source dataset or instructions for how to construct the dataset).
     (d) We recognize that reproducibility may be tricky in some cases, in which case authors are welcome to describe the particular way they provide for reproducibility. In the case of closed-source models, it may be that access to the model is limited in some way (e.g., to registered users), but it should be possible for other researchers to have some path to reproducing or verifying the results.

5. **Open access to data and code**

   Question: Does the paper provide open access to the data and code, with sufficient instructions to faithfully reproduce the main experimental results, as described in supplemental material?

https://doi.org/10.52202/085713-0877

Answer: [Yes]

Justification: Please refer to the code in the supplemental material.

Guidelines:

- The answer NA means that paper does not include experiments requiring code.
- Please see the NeurIPS code and data submission guidelines (https://nips.cc/public/guides/CodeSubmissionPolicy) for more details.
- While we encourage the release of code and data, we understand that this might not be possible, so "No" is an acceptable answer. Papers cannot be rejected simply for not including code, unless this is central to the contribution (e.g., for a new open-source benchmark).
- The instructions should contain the exact command and environment needed to run to reproduce the results. See the NeurIPS code and data submission guidelines (https://nips.cc/public/guides/CodeSubmissionPolicy) for more details.
- The authors should provide instructions on data access and preparation, including how to access the raw data, preprocessed data, intermediate data, and generated data, etc.
- The authors should provide scripts to reproduce all experimental results for the new proposed method and baselines. If only a subset of experiments are reproducible, they should state which ones are omitted from the script and why.
- At submission time, to preserve anonymity, the authors should release anonymized versions (if applicable).
- Providing as much information as possible in supplemental material (appended to the paper) is recommended, but including URLs to data and code is permitted.

6. **Experimental setting/details**

   Question: Does the paper specify all the training and test details (e.g., data splits, hyperparameters, how they were chosen, type of optimizer, etc.) necessary to understand the results?

   Answer: [Yes]

   Justification: Please refer to Appendix C.

   Guidelines:

   - The answer NA means that the paper does not include experiments.
   - The experimental setting should be presented in the core of the paper to a level of detail that is necessary to appreciate the results and make sense of them.
   - The full details can be provided either with the code, in appendix, or as supplemental material.

7. **Experiment statistical significance**

   Question: Does the paper report error bars suitably and correctly defined or other appropriate information about the statistical significance of the experiments?

   Answer: [Yes]

   Justification: The results in the paper are accompanied by standard deviations across multiple seeds.

   Guidelines:

   - The answer NA means that the paper does not include experiments.
   - The authors should answer "Yes" if the results are accompanied by error bars, confidence intervals, or statistical significance tests, at least for the experiments that support the main claims of the paper.
   - The factors of variability that the error bars are capturing should be clearly stated (for example, train/test split, initialization, random drawing of some parameter, or overall run with given experimental conditions).
   - The method for calculating the error bars should be explained (closed form formula, call to a library function, bootstrap, etc.)
   - The assumptions made should be given (e.g., Normally distributed errors).

https://doi.org/10.52202/085713-0877

- It should be clear whether the error bar is the standard deviation or the standard error of the mean.
- It is OK to report 1-sigma error bars, but one should state it. The authors should preferably report a 2-sigma error bar than state that they have a 96% CI, if the hypothesis of Normality of errors is not verified.
- For asymmetric distributions, the authors should be careful not to show in tables or figures symmetric error bars that would yield results that are out of range (e.g. negative error rates).
- If error bars are reported in tables or plots, The authors should explain in the text how they were calculated and reference the corresponding figures or tables in the text.

8. **Experiments compute resources**

   Question: For each experiment, does the paper provide sufficient information on the computer resources (type of compute workers, memory, time of execution) needed to reproduce the experiments?

   Answer: [Yes]

   Justification: Please refer to Appendix D.1.

   Guidelines:

   - The answer NA means that the paper does not include experiments.
   - The paper should indicate the type of compute workers CPU or GPU, internal cluster, or cloud provider, including relevant memory and storage.
   - The paper should provide the amount of compute required for each of the individual experimental runs as well as estimate the total compute.
   - The paper should disclose whether the full research project required more compute than the experiments reported in the paper (e.g., preliminary or failed experiments that didn't make it into the paper).

9. **Code of ethics**

   Question: Does the research conducted in the paper conform, in every respect, with the NeurIPS Code of Ethics https://neurips.cc/public/EthicsGuidelines?

   Answer: [Yes]

   Justification: The research conducted in the paper conforms, in every respect, with the NeurIPS Code of Ethics.

   Guidelines:

   - The answer NA means that the authors have not reviewed the NeurIPS Code of Ethics.
   - If the authors answer No, they should explain the special circumstances that require a deviation from the Code of Ethics.
   - The authors should make sure to preserve anonymity (e.g., if there is a special consideration due to laws or regulations in their jurisdiction).

10. **Broader impacts**

    Question: Does the paper discuss both potential positive societal impacts and negative societal impacts of the work performed?

    Answer: [Yes]

    Justification: Please refer to Appendix E.

    Guidelines:

    - The answer NA means that there is no societal impact of the work performed.
    - If the authors answer NA or No, they should explain why their work has no societal impact or why the paper does not address societal impact.
    - Examples of negative societal impacts include potential malicious or unintended uses (e.g., disinformation, generating fake profiles, surveillance), fairness considerations (e.g., deployment of technologies that could make decisions that unfairly impact specific groups), privacy considerations, and security considerations.

- The conference expects that many papers will be foundational research and not tied to particular applications, let alone deployments. However, if there is a direct path to any negative applications, the authors should point it out. For example, it is legitimate to point out that an improvement in the quality of generative models could be used to generate deepfakes for disinformation. On the other hand, it is not needed to point out that a generic algorithm for optimizing neural networks could enable people to train models that generate Deepfakes faster.
- The authors should consider possible harms that could arise when the technology is being used as intended and functioning correctly, harms that could arise when the technology is being used as intended but gives incorrect results, and harms following from (intentional or unintentional) misuse of the technology.
- If there are negative societal impacts, the authors could also discuss possible mitigation strategies (e.g., gated release of models, providing defenses in addition to attacks, mechanisms for monitoring misuse, mechanisms to monitor how a system learns from feedback over time, improving the efficiency and accessibility of ML).

11. **Safeguards**

    Question: Does the paper describe safeguards that have been put in place for responsible release of data or models that have a high risk for misuse (e.g., pretrained language models, image generators, or scraped datasets)?

    Answer: [NA]

    Justification: The paper poses no such risks.

    Guidelines:

    - The answer NA means that the paper poses no such risks.
    - Released models that have a high risk for misuse or dual-use should be released with necessary safeguards to allow for controlled use of the model, for example by requiring that users adhere to usage guidelines or restrictions to access the model or implementing safety filters.
    - Datasets that have been scraped from the Internet could pose safety risks. The authors should describe how they avoided releasing unsafe images.
    - We recognize that providing effective safeguards is challenging, and many papers do not require this, but we encourage authors to take this into account and make a best faith effort.

12. **Licenses for existing assets**

    Question: Are the creators or original owners of assets (e.g., code, data, models), used in the paper, properly credited and are the license and terms of use explicitly mentioned and properly respected?

    Answer: [Yes]

    Justification: The creators or original owners of assets used in the paper are properly credited and the license and terms of use are explicitly mentioned and properly respected.

    Guidelines:

    - The answer NA means that the paper does not use existing assets.
    - The authors should cite the original paper that produced the code package or dataset.
    - The authors should state which version of the asset is used and, if possible, include a URL.
    - The name of the license (e.g., CC-BY 4.0) should be included for each asset.
    - For scraped data from a particular source (e.g., website), the copyright and terms of service of that source should be provided.
    - If assets are released, the license, copyright information, and terms of use in the package should be provided. For popular datasets, `paperswithcode.com/datasets` has curated licenses for some datasets. Their licensing guide can help determine the license of a dataset.
    - For existing datasets that are re-packaged, both the original license and the license of the derived asset (if it has changed) should be provided.

https://doi.org/10.52202/085713-0877

- If this information is not available online, the authors are encouraged to reach out to the asset's creators.

13. **New assets**

    Question: Are new assets introduced in the paper well documented and is the documentation provided alongside the assets?

    Answer: [Yes]

    Justification: The code is well documented and anonymized.

    Guidelines:

    - The answer NA means that the paper does not release new assets.
    - Researchers should communicate the details of the dataset/code/model as part of their submissions via structured templates. This includes details about training, license, limitations, etc.
    - The paper should discuss whether and how consent was obtained from people whose asset is used.
    - At submission time, remember to anonymize your assets (if applicable). You can either create an anonymized URL or include an anonymized zip file.

14. **Crowdsourcing and research with human subjects**

    Question: For crowdsourcing experiments and research with human subjects, does the paper include the full text of instructions given to participants and screenshots, if applicable, as well as details about compensation (if any)?

    Answer: [NA]

    Justification: The paper does not involve crowdsourcing nor research with human subjects.

    Guidelines:

    - The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
    - Including this information in the supplemental material is fine, but if the main contribution of the paper involves human subjects, then as much detail as possible should be included in the main paper.
    - According to the NeurIPS Code of Ethics, workers involved in data collection, curation, or other labor should be paid at least the minimum wage in the country of the data collector.

15. **Institutional review board (IRB) approvals or equivalent for research with human subjects**

    Question: Does the paper describe potential risks incurred by study participants, whether such risks were disclosed to the subjects, and whether Institutional Review Board (IRB) approvals (or an equivalent approval/review based on the requirements of your country or institution) were obtained?

    Answer: [NA]

    Justification: The paper does not involve crowdsourcing nor research with human subjects.

    Guidelines:

    - The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
    - Depending on the country in which research is conducted, IRB approval (or equivalent) may be required for any human subjects research. If you obtained IRB approval, you should clearly state this in the paper.
    - We recognize that the procedures for this may vary significantly between institutions and locations, and we expect authors to adhere to the NeurIPS Code of Ethics and the guidelines for their institution.
    - For initial submissions, do not include any information that would break anonymity (if applicable), such as the institution conducting the review.

16. **Declaration of LLM usage**

Question: Does the paper describe the usage of LLMs if it is an important, original, or non-standard component of the core methods in this research? Note that if the LLM is used only for writing, editing, or formatting purposes and does not impact the core methodology, scientific rigorousness, or originality of the research, declaration is not required.

Answer: [NA]

Justification: The core method development in this research does not involve LLMs as any important, original, or non-standard components.

Guidelines:

- The answer NA means that the core method development in this research does not involve LLMs as any important, original, or non-standard components.
- Please refer to our LLM policy (https://neurips.cc/Conferences/2025/LLM) for what should or should not be described.

https://doi.org/10.52202/085713-0877

A Extended Related Work

Model-free offline RL. Offline RL aims to learn policies from a fixed dataset without any further interaction with the environment [43, 44]. Standard off-policy methods often struggle in this setting due to extrapolation errors induced by OOD actions [23]. To address this issue, a variety of algorithms have been developed, broadly falling into model-free and model-based categories. Within the model-free family, value regularization methods promote conservative value estimation, either by explicitly penalizing overestimated Q values [42, 39, 48, 86, 12, 69, 54], or by employing ensembles to capture epistemic uncertainty [2, 5, 89]. Policy constraint approaches, by contrast, aim to keep the learned policy close to the behavior policy, accomplished either explicitly through divergence penalties [83, 41, 32, 21, 82], implicitly via weighted behavior cloning [11, 61, 60, 81, 53], or through carefully designed policy parameterizations [23, 26, 94]. More recently, these methods have increasingly shifted toward learning the optimal policy within the support of the behavior policy, offering theoretical guarantees and reduced sensitivity to the overall quality of datasets [82, 53, 54, 50]. Another direction emphasizes in-sample learning, which avoids estimating values of unobserved actions by constructing Bellman targets solely from dataset samples [6, 47, 40, 85, 88, 25, 92]. For example, OneStep RL [6] employs SARSA-style value updates [73] and performs a single round of policy improvement. IQL [40] extends this idea using expectile regression within the SARSA update, enabling multi-step dynamic programming. In contrast to strict in-sample learning, some methods propose to exploit mild generalization beyond the dataset to further boost performance [56, 49]. Orthogonal to the above, another influential branch of work formulates offline RL as conditional sequence modeling [10]. These methods employ causal transformers conditioned on the desired return, past states, and actions to autoregressively generate future actions [10, 84, 51, 28], bypassing the need for bootstrapping in long-term credit assignment [62, 64].

Model-based offline RL. Model-based offline RL approaches construct a learned model of the environment dynamics, which is subsequently used to generate synthetic trajectories for policy improvement [72, 31, 34]. Given the challenges of distributional shift in offline settings, algorithms such as MOPO [90] and MOReL [36] incorporate mechanisms to quantify the model uncertainty and penalize high-uncertainty state-action pairs, thereby discouraging the exploitation of unreliable model predictions. MOBILE [71] further advances this line of work by proposing an uncertainty quantification approach based on inconsistencies in Bellman estimations within an ensemble of learned dynamics models. Some algorithms bridge the gap between model-based and model-free paradigms by incorporating similar regularization techniques. For instance, COMBO [91] integrates value penalization into model-based rollouts, while BREMEN [57] emphasizes behavior-regularized policy optimization. More recently, SCAS [55] proposes a generic model-based regularizer for model-free offline RL that unifies OOD state correction and OOD action suppression. Despite their promise, model-based methods often entail substantial computational costs [31], and their efficacy is highly contingent on the fidelity of the learned dynamics model [59].

Neighborhood in offline RL. This work formalizes the neighborhood constraint for offline RL and demonstrates its key properties through theoretical analyses. Some offline RL methods implicitly involve the concept of neighborhood by incorporating regularization terms that guide the trained policy toward dataset actions [21, 75, 66, 56]. However, due to the multi-modal nature of dataset action distributions [9] and the limited expressiveness of policy architectures [80], these policy iteration approaches cannot ensure that the trained policy's output actions for the Bellman target remain within the union of dataset action neighborhoods. In contrast, we adopt a value iteration framework and introduce an auxiliary policy to effectively enforce the proposed neighborhood constraint. BCQ [23] employs a perturbation model related to our auxiliary policy. Specifically, BCQ samples multiple candidate actions per state from a pre-trained generative behavior policy model, perturbs them using the perturbation model, and selects the one with the highest Q value. The perturbation model constructs neighborhoods around the generated actions and is intended to avoid "sampling from the generative model for a prohibitive number of times [23]". Subsequent works such as EMaQ [26] and SfBC [9] show that this perturbation model can be omitted without sacrificing performance. By contrast, our auxiliary policy constructs neighborhoods directly around dataset actions and is designed to support Q learning under the proposed constraint, without requiring complex behavior policy modeling. Based on the proposed neighborhood constraint, this work provides a general constrained Q learning framework for adaptive distribution-shift control, with the potential to extend to broader RL paradigms such as safe RL [1, 27, 24] and meta RL [78, 77, 65].

B Proofs

Lemma 1 in the main text characterizes the performance difference between two policies subject to divergence constraints, which is related to the results in TRPO [68] and CPO [1]. To prove Lemma 1, we begin with the performance difference lemma [35], which decomposes the difference in policy performance, $\eta(\pi') - \eta(\pi)$, into an expectation of advantages.

Lemma 3. *For any two policies π' and π, the following equality holds:*

$$\eta(\pi') - \eta(\pi) = \frac{1}{1-\gamma} \sum_s d_{\pi'}(s) \sum_a \left[\pi'(a|s) A^\pi(s,a)\right] \tag{18}$$

where $d^\pi(s) = (1-\gamma)\sum_{t=0}^{\infty} \gamma^t \mathbb{E}_\pi\left[\mathbb{I}\left[s_t = s\right]\right]$ is the state occupancy induced by the policy π.

Proof. Please refer to the proof of Lemma 6.1 in Kakade and Langford [35]. □

Lemma 4 (Performance bound under density constraint, Lemma 1)**.** *If any of the conditions $\mathrm{D_{KL}}(\pi\|\pi_\beta) \leq 2\epsilon$, $\mathrm{D_{KL}}(\pi_\beta\|\pi) \leq 2\epsilon$, or $\mathrm{D_{TV}}(\pi, \pi_\beta) \leq \sqrt{\epsilon}$ holds, then the policy performance η is bounded as follows:*

$$\eta(\pi) \leq \eta(\pi_\beta) + \frac{2R_{\max}}{(1-\gamma)^2}\sqrt{\epsilon}. \tag{19}$$

Proof. By Lemma 3, we have

$$\begin{aligned}
|\eta(\pi_\beta) - \eta(\pi)| &= \frac{1}{1-\gamma}\left|\sum_s d_{\pi_\beta}(s) \sum_a [\pi_\beta(a|s) A^\pi(s,a)]\right| && (20)\\
&= \frac{1}{1-\gamma}\left|\sum_s d_{\pi_\beta}(s) \sum_a \pi_\beta(a|s)\left(Q^\pi(s,a) - V^\pi(s)\right)\right| && (21)\\
&= \frac{1}{1-\gamma}\left|\sum_s d_{\pi_\beta}(s) \left(\sum_a \pi_\beta(a|s) Q^\pi(s,a) - V^\pi(s)\right)\right| && (22)\\
&= \frac{1}{1-\gamma}\left|\sum_s d_{\pi_\beta}(s) \left(\sum_a \pi_\beta(a|s) Q^\pi(s,a) - \sum_a \pi(a|s) Q^\pi(s,a)\right)\right| && (23)\\
&= \frac{1}{1-\gamma}\left|\sum_s d_{\pi_\beta}(s) \sum_a (\pi_\beta(a|s) - \pi(a|s)) Q^\pi(s,a)\right| && (24)\\
&\leq \frac{1}{1-\gamma}\sum_s d_{\pi_\beta}(s) \sum_a |\pi_\beta(a|s) - \pi(a|s)|\,|Q^\pi(s,a)| && (25)\\
&\leq \frac{R_{\max}}{(1-\gamma)^2}\sum_s d_{\pi_\beta}(s) \sum_a |\pi_\beta(a|s) - \pi(a|s)| && (26)\\
&= \frac{2R_{\max}}{(1-\gamma)^2}\sum_s d_{\pi_\beta}(s) \mathrm{D_{TV}}(\pi, \pi_\beta)[s] && (27)\\
&\leq \frac{\sqrt{2}R_{\max}}{(1-\gamma)^2}\sum_s d_{\pi_\beta}(s) \min\left\{\sqrt{\mathrm{D_{KL}}(\pi\|\pi_\beta)[s]}, \sqrt{\mathrm{D_{KL}}(\pi_\beta\|\pi)[s]}\right\} && (28)
\end{aligned}$$

where the last inequality holds by Pinsker's inequality.

By substituting the TV condition, $\mathrm{D_{TV}}(\pi, \pi_\beta) \leq \sqrt{\epsilon}$, into Eq. (27), or the KL condition, $\mathrm{D_{KL}}(\pi\|\pi_\beta) \leq 2\epsilon$ or $\mathrm{D_{KL}}(\pi_\beta\|\pi) \leq 2\epsilon$, into Eq. (28), we derive the final performance bound:

$$|\eta(\pi) - \eta(\pi_\beta)| \leq \frac{2R_{\max}}{(1-\gamma)^2}\sqrt{\epsilon}. \tag{29}$$

□

https://doi.org/10.52202/085713-0877

Lemma 2 in the main text analyzes the impact of neighborhood constraint on deep Q functions under the neural tangent kernel (NTK) regime [30]. The result is a direct corollary of Theorem 1 in [45], specialized to the case of action extrapolation. The NTK assumption is presented in Assumption 2.

Assumption 2 (NTK assumption)**.** *The Q function approximators are two-layer fully-connected ReLU neural networks with infinite width and are trained with an infinitesimally small learning rate.*

While not applicable to some practically advanced architectures [95], NTK remains one of the most influential theoretical frameworks for analyzing the generalization of deep neural networks. Assumption 2 is common in previous analyses on the generalization of deep neural networks [30, 4] and the convergence of deep RL [7, 46, 19].

Lemma 5 (Extrapolation behavior, Lemma 2)**.** *Under the neural tangent kernel (NTK) regime [30], for any in-sample state-action pair $(s, a) \in \mathcal{D}$ and in-neighborhood state-action pair $(s, \tilde{a})$ such that $\|\tilde{a} - a\| \leq \epsilon$, the value difference of the deep Q function can be bounded as:*

$$\|Q_\theta(s, \tilde{a}) - Q_\theta(s, a)\| \leq C(\sqrt{\min(\|s \oplus a\|, \|s \oplus \tilde{a}\|)}\sqrt{\epsilon} + 2\epsilon), \tag{30}$$

where $\oplus$ denotes the vector concatenation operation, and C is a finite constant.

Proof. The lemma follows directly from Theorem 1 or Lemma 4 in [45]. Please refer to [45] for detailed proofs. □

Proposition 1 in the main text assumes that the transition dynamics is K_P-Lipschitz continuous, which is a common assumption in the theoretical RL studies [15, 16, 87, 66]. A small K_P is also empirically supported by observations in physical systems, where small action changes often lead to limited next-state variation [76].

Proposition 2 (Distribution shift, Proposition 1)**.** *Let π_1 be a deterministic policy that satisfies the neighborhood constraint with threshold ϵ. Assume that the transition dynamics P is K_P-Lipschitz continuous: $\forall s \in \mathcal{S}$, $\forall a_1, a_2 \in \mathcal{A}$, $\|P(s'|s, a_1) - P(s'|s, a_2)\| \leq K_P \|a_1 - a_2\|$. Then, there exists a policy π_2 satisfying the sample constraint such that:*

$$\mathrm{D_{TV}}\left(d^{\pi_1}(\cdot), d^{\pi_2}(\cdot)\right) \leq \frac{\gamma K_P \epsilon}{2(1-\gamma)}, \tag{31}$$

where $d^\pi(s) = (1-\gamma)\sum_{t=0}^{\infty} \gamma^t \mathbb{E}_\pi\left[\mathbb{I}\left[s_t = s\right]\right]$ is the state occupancy induced by policy π.

Proof. We analyze the difference in state occupancy measures induced by two deterministic policies π_1 and π_2, leveraging the Lipschitz continuity of the transition dynamics.

The state occupancy measure d^π satisfies the Bellman-flow constraints.:

$$d^\pi(s) = (1-\gamma)d_0(s) + \gamma \sum_{s'} P(s \mid s', \pi(s'))d^\pi(s'), \tag{32}$$

where $d_0(s)$ is the initial state distribution.

For policies π_1 and π_2, their corresponding occupancy measures d^{π_1} and d^{π_2} satisfy:

$$d^{\pi_1}(s) - d^{\pi_2}(s) = \gamma \sum_{s'} \left[P(s \mid s', \pi_1(s'))d^{\pi_1}(s') - P(s \mid s', \pi_2(s'))d^{\pi_2}(s')\right]. \tag{33}$$

We decompose the difference into two terms:

$$\begin{aligned} d^{\pi_1}(s) - d^{\pi_2}(s) =& \gamma \sum_{s'} P(s \mid s', \pi_1(s'))\left(d^{\pi_1}(s') - d^{\pi_2}(s')\right) \\ &+ \gamma \sum_{s'} \left(P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s'))\right) d^{\pi_2}(s'). \end{aligned}$$

Let $\Delta(s) = |d^{\pi_1}(s) - d^{\pi_2}(s)|$ and apply the triangle inequality:

$$\begin{aligned} \Delta(s) &\leq \gamma \left|\sum_{s'} P(s \mid s', \pi_1(s'))\Delta(s')\right| + \gamma \left|\sum_{s'} \left(P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s'))\right) d^{\pi_2}(s')\right| \\ &\leq \gamma \sum_{s'} P(s \mid s', \pi_1(s'))\Delta(s') + \gamma \sum_{s'} \left|P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s'))\right| d^{\pi_2}(s'). \end{aligned} \tag{34}$$

https://doi.org/10.52202/085713-0877

Sum the above inequality over states s:

$$
\begin{aligned}
\|\Delta\|_1 &:= \sum_s \Delta(s) \\
&\leq \gamma \sum_s \sum_{s'} P(s \mid s', \pi_1(s'))\Delta(s') + \gamma \sum_s \sum_{s'} \left| P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s')) \right| d^{\pi_2}(s') \\
&= \gamma \sum_{s'} \Delta(s') + \gamma \sum_{s'} d^{\pi_2}(s') \sum_s \left| P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s')) \right| \\
&= \gamma \|\Delta\|_1 + \gamma \sum_{s'} d^{\pi_2}(s') \sum_s \left| P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s')) \right|. \qquad (35)
\end{aligned}
$$

Using the K_P-Lipschitz property for transition dynamics P:

$$
\sum_s \left| P(s \mid s', \pi_1(s')) - P(s \mid s', \pi_2(s')) \right| \leq K_P \|\pi_1(s') - \pi_2(s')\|, \qquad (36)
$$

Substitute Eq. (36) into the TV bound in Eq. (35):

$$
\begin{aligned}
\|\Delta\|_1 &\leq \gamma \|\Delta\|_1 + \gamma K_P \sum_{s'} d^{\pi_2}(s') \|\pi_1(s') - \pi_2(s')\| \qquad (37) \\
&\leq \gamma \|\Delta\|_1 + \gamma K_P \max_s \|\pi_1(s) - \pi_2(s)\|. \qquad (38)
\end{aligned}
$$

Rearranging terms to isolate $\|\Delta\|_1$, we obtain the following expression for the total variation distance between the state occupancy measures:

$$
\mathrm{D}_{\mathrm{TV}}(d^{\pi_1}, d^{\pi_2}) := \frac{1}{2}\|\Delta\|_1 \leq \frac{\gamma K_P}{2(1-\gamma)} \max_s \|\pi_1(s) - \pi_2(s)\|. \qquad (39)
$$

Finally, based on the definitions of the neighborhood and sample constraints, we conclude that for any deterministic policy π_1 that satisfies the neighborhood constraint (Definition 4), there exists a deterministic policy π_2 satisfying the sample constraint (Definition 3) such that for any $s \in \mathcal{D}$, $\|\pi_1(s) - \pi_2(s)\| \leq \epsilon$. For $s \notin \mathcal{D}$, We define $\pi_2(s) := \pi_1(s)$. Thus, $\max_s \|\pi_1(s) - \pi_2(s)\| \leq \epsilon$. Therefore, the TV distance between the state occupancies of such policies π_1 and π_2 satisfies:

$$
\mathrm{D}_{\mathrm{TV}}(d^{\pi_1}, d^{\pi_2}) \leq \frac{\gamma K_P \epsilon}{2(1-\gamma)}. \qquad (40)
$$

□

Theorem 1 in the main text uses the standardness assumption, which is common in geometric measure theory [13, 8].

Assumption 3 (Standardness, Assumption 1)**.** *Let $S \subseteq \mathbb{R}^d$ be the support of a probability distribution ν, and $B(x,r)$ be the closed ball of radius r centered at $x \in \mathbb{R}^d$. There exist constants $r_0 > 0$ and $C_0 > 0$ such that:*

$$
\forall x \in S, \;\; \forall r \leq r_0, \;\; \nu(B(x,r)) \geq C_0 \cdot r^d. \qquad (41)
$$

It ensures that the measure ν does not exhibit "holes" at small scales, preventing ν from being too sparse near any $x \in S$. It imposes a lower bound on the measure of small balls but does not explicitly require the existence of a density.

Theorem 2 (Support approximation via neighborhoods, Theorem 1)**.** *Let $S \subseteq \mathbb{R}^d$ be the compact support of a distribution ν, and let $X_1, \ldots, X_n$ be independent and identically distributed samples from ν. Define $U_{n,\epsilon} = \bigcup_{i=1}^n B(X_i, \epsilon)$ as the union of closed balls of radius ϵ centered at the samples. Let $\mathcal{N}(S, \epsilon/2)$ denote the covering number of S, i.e., the minimal number of $\epsilon/2$-balls required to cover S. Under the standardness Assumption 1 with constants $r_0, C_0 > 0$, for any $\delta \in (0,1)$ and $\epsilon \leq 2r_0$, if*

$$
n \geq \frac{1}{C_0(\epsilon/2)^d} \left(\log \mathcal{N}(S, \epsilon/2) + \log(1/\delta) \right), \qquad (42)
$$

then with probability at least $1-\delta$, the Hausdorff distance between S and $U_{n,\epsilon}$ satisfies

$$
d_H(S, U_{n,\epsilon}) := \max\left(\sup_{x \in S} \inf_{u \in U_{n,\epsilon}} d(x,u), \; \sup_{u \in U_{n,\epsilon}} \inf_{x \in S} d(x,u) \right) \leq \epsilon. \qquad (43)
$$

https://doi.org/10.52202/085713-0877

Proof. By the compactness of S, for any $\epsilon > 0$, there exists a finite set of points $\{y_1, \ldots, y_N\} \subset S$ such that:

$$S \subseteq \bigcup_{j=1}^{N} B(y_j, \epsilon/2), \tag{44}$$

where $B(y_j, \epsilon/2)$ denotes the closed ball of radius $\epsilon/2$ centered at y_j, and $N = \mathcal{N}(S, \epsilon/2)$ denotes the covering number of S for a given radius $\epsilon/2$, i.e., the minimal number of $\epsilon/2$-balls required to cover S. The existence of such a finite cover follows directly from the compactness of S.

For each y_j, the probability that none of the samples $X_1, \ldots, X_n$ lies in $B(y_j, \epsilon/2)$ is:

$$\mathbb{P}\left(B(y_j, \epsilon/2) \cap \{X_1, \ldots, X_n\} = \emptyset\right) = \left(1 - \nu(B(y_j, \epsilon/2))\right)^n. \tag{45}$$

Under the standardness assumption, there exist constants $r_0 > 0$ and $C_0 > 0$ such that

$$\forall x \in S,\ \ \forall r \le r_0,\ \ \nu(B(x, r)) \ge C_0 \cdot r^d. \tag{46}$$

For $\epsilon \le 2r_0$, we have $\epsilon/2 \le r_0$, and thus

$$\nu(B(y_j, \epsilon/2)) \ge C_0 \cdot (\epsilon/2)^d. \tag{47}$$

Using the inequality $1 - x \le e^{-x}$, it follows that

$$\left(1 - \nu(B(y_j, \epsilon/2))\right)^n \le e^{-n\nu(B(y_j, \epsilon/2))} \le e^{-nC_0(\epsilon/2)^d}. \tag{48}$$

Applying the union bound over all N covering balls:

$$\mathbb{P}\left(\exists j \text{ s.t. } B(y_j, \epsilon/2) \cap \{X_1, \ldots, X_n\} = \emptyset\right) \le N\left(1 - \nu(B(y_j, \epsilon/2))\right)^n \le N e^{-nC_0(\epsilon/2)^d}. \tag{49}$$

To ensure this probability is at most δ, we require

$$N e^{-nC_0(\epsilon/2)^d} := \mathcal{N}(S, \epsilon/2) \cdot e^{-nC_0(\epsilon/2)^d} \le \delta. \tag{50}$$

Solving for n, we obtain

$$n \ge \frac{\log \mathcal{N}(S, \epsilon/2) + \log(1/\delta)}{C_0(\epsilon/2)^d}, \tag{51}$$

which guarantees that, with probability at least $1 - \delta$, every $B(y_j, \epsilon/2)$ contains at least one sample X_i.

The Hausdorff distance is a metric used to quantify the discrepancy between two sets and is widely applied in shape comparison and analysis across various domains [29, 74]. It captures the maximum mismatch between the sets by measuring how far one must travel from a point in one set to reach the nearest point in the other. Mathematically, for two non-empty sets A and B in a metric space (e.g., Euclidean space) with distance function d, the Hausdorff distance $d_H(A, B)$ is defined as:

$$d_H(A, B) := \max\left(\sup_{a \in A} \inf_{b \in B} d(a, b), \sup_{b \in B} \inf_{a \in A} d(a, b)\right) \tag{52}$$

That is, for each point in set A, we find the closest point in set B, and take the maximum of these minimum distances. Similarly, for each point in set B, we find the closest point in set A, and take the maximum of these minimum distances. The Hausdorff distance is the larger of these two values.

This distance is sensitive to outliers, as even a single distant point can significantly increase the value of the distance. This property renders it particularly appropriate for evaluating approximation effects in our setting, where we aim to optimize the Q function over an approximated constraint set. Outliers in such a set can exert a substantial influence on the Q function's value.

The Hausdorff distance between S and $U_{n,\epsilon} := \bigcup_{i=1}^{n} B(X_i, \epsilon)$ is:

$$d_H(S, U_{n,\epsilon}) := \max\left(\sup_{x \in S} \inf_{u \in U_{n,\epsilon}} d(x, u), \sup_{u \in U_{n,\epsilon}} \inf_{x \in S} d(x, u)\right) \tag{53}$$

where d is the Euclidean distance.

We now analyze each term separately.

(1) Bounding the term $\sup_{x \in S} \inf_{u \in U_{n,\epsilon}} d(x, u)$:

By Eq. (44), for any $x \in S$, there exists y_j such that $x \in B(y_j, \epsilon/2)$. By the coverage guarantee under Eq. (51) (with probability at least $1 - \delta$, every $B(y_j, \epsilon/2)$ contains at least one sample X_i), it follows that $B(y_j, \epsilon/2)$ contains some X_i. Thus,

$$d(x, X_i) \le d(x, y_j) + d(y_j, X_i) \le \epsilon/2 + \epsilon/2 = \epsilon. \tag{54}$$

This implies $x \in B(X_i, \epsilon) \subseteq U_{n,\epsilon}$. Since $x \in S$ is arbitrary, it holds that $S \subseteq U_{n,\epsilon}$, yielding $\sup_{x \in S} \inf_{u \in U_{n,\epsilon}} d(x, u) = 0$.

(2) Bounding the term $\sup_{u \in U_{n,\epsilon}} \inf_{x \in S} d(x, u)$:

By definition, any $u \in U_{n,\epsilon}$ belongs to $B(X_i, \epsilon)$ for some $X_i \in S$. Therefore, for any $u \in U_{n,\epsilon}$, the following holds:

$$\inf_{x \in S} d(x, u) \le d(X_i, u) \le \epsilon. \tag{55}$$

Thus, $\sup_{u \in U_{n,\epsilon}} \inf_{x \in S} d(x, u) \le \epsilon$.

Combining both terms, we conclude

$$d_H(S, U_{n,\epsilon}) \le \max(0, \epsilon) \le \epsilon. \tag{56}$$

Therefore, when the sample size n satisfies Eq. (51), it follows that, with probability at least $1 - \delta$, the Hausdorff distance between S and $U_{n,\epsilon}$ satisfies $d_H(S, U_{n,\epsilon}) \le \epsilon$. $\square$

C Experimental Details

C.1 Experimental Details on D4RL Benchmarks

We adopt evaluation criteria consistent with those employed in prior studies. For the Gym locomotion tasks, performance is assessed by averaging returns over 10 evaluation trajectories and 5 random seeds. In the AntMaze suite, returns are averaged over 100 evaluation trajectories across the same number of seeds. Following the D4RL benchmark guidelines [20], we subtract 1 from the rewards in the AntMaze datasets. Additionally, in line with established practices [21, 40, 82, 88], we normalize the states in all Gym locomotion datasets. Our implementation builds upon TD3 [22], optimizing a deterministic policy. Consequently, we employ mean squared error in the weighted behavior cloning objective in Eq. (17), instead of a log-likelihood term. This approach is commonly used in RL algorithms, where a maximum likelihood problem is transformed into a regression problem when dealing with Gaussians with a fixed variance [21]. The architectures of the auxiliary policy and the final policy are identical, with the former's output range (i.e., max action) set to twice that of the latter to account for the most extreme cases. Performance is reported using normalized scores provided by the D4RL benchmark [20], which quantify the quality of the learned policy relative to both random and expert baselines:

$$\text{D4RL score} = 100 \times \frac{\text{learned policy return} - \text{random policy return}}{\text{expert policy return} - \text{random policy return}} \tag{57}$$

ANQ introduces two key hyperparameters: the Lagrange multiplier λ, which controls the overall neighborhood radius, and the inverse temperature α, which controls the adaptiveness of neighborhood radius to advantage values. For the inverse temperature, we fix $\alpha = 1$ across all tasks, which yields consistently strong performance. For the Lagrange multiplier, we tune λ within a small set of values $\{0.1, 5.0\}$ for all tasks. Given that part of our algorithm incorporates techniques from IQL [40], such as expectile regression and weighted behavior cloning, we adopt the hyperparameters suggested in their work: $\tau = 0.7$ and $\beta = 3$ for Gym locomotion tasks, and $\tau = 0.9$ and $\beta = 10$ for AntMaze tasks. To ensure that the constrained neighborhood is neither too large nor too small, we clip the exponentiated advantage weight $\exp(\alpha(Q_{\theta'}(s, a) - V_\psi(s)))$ in Eq. (13) to $[0.01, 30]$ in the Gym locomotion domains and $[0.01, 10]$ in the Antmaze domains. Following prior practices, we also clip the exponentiated weight $\exp(\beta(Q_{\theta'}(s, a + \mu_\omega(s, a)) - V_\psi(s)))$ in Eq. (17) to $[0, 3]$ in the Gym locomotion domains and $[0, 100]$ in the Antmaze domains to avoid instability. A comprehensive list of hyperparameter settings for ANQ is provided in Table 3.

 https://doi.org/10.52202/085713-0877

Insights into hyperparameter selection. For selecting the Lagrange multiplier λ, a general principle is as follows: for narrow-distribution datasets (typically expert or demonstration data), a larger λ can be used to induce smaller neighborhoods, thereby helping to more effectively control extrapolation error. For more dispersed datasets (often containing noisy or mixed-quality data), the impact of extrapolation error is relatively weaker; hence, a smaller λ (i.e., larger neighborhoods) can be adopted to promote broader optimization over the action space and mitigate the impacts of suboptimal actions. Additionally, using a larger inverse temperature α in such cases can further enhance this effect by downweighting low-advantage actions more aggressively.

Table 3: Hyperparameters of ANQ.

	Hyperparameter	Value
ANQ	Optimizer	Adam [37]
	Critic learning rate	3×10^{-4}
	Actor learning rate	3×10^{-4} with cosine schedule
	Discount factor	0.99 for Gym, 0.995 for Antmaze
	Target update rate	0.005
	Policy update frequency	2
	Number of Critics	4
	Batch size	256
	Number of iterations	10^6
	Lagrange multiplier λ	{0.1, 5.0}
	Inverse temperature α	1
IQL Specific	Expectile τ	0.7 for Gym, 0.9 for Antmaze
	Inverse temperature β	3.0 for Gym, 10.0 for Antmaze
Architecture	Actor	input-256-256-output
	Critic	input-256-256-1

C.2 Experimental Details of Noisy Data Experiments

In the noisy data experiments, we construct noisy datasets by mixing the random and expert datasets in D4RL at various expert ratios, thereby simulating real-world scenarios such as suboptimal data collection in autonomous systems or imperfect demonstrations in robotics. The total size of the combined dataset is fixed at 1×10^6. In certain environments, the sizes of the random or expert datasets in D4RL are slightly smaller than 1×10^6, so we directly use the corresponding D4RL dataset when the expert ratio is 0 or 1.

We evaluate the performance of several representative algorithms with different constraint types, including CQL (density constraint), IQL (sample constraint), SPOT (support constraint), and ANQ (neighborhood constraint). The hyperparameters of ANQ follow those used in the default benchmark datasets, as detailed in Table 3, and we fix $\lambda = 5$ for all the mixed datasets. In addition, we clip the exponentiated advantage weight in Eq. (13) to $[0.1, 100]$ for Hopper and Walker2d, and to $[0.1, 30]$ for Halfcheetah.

Hyperparameters of baseline methods. The hyperparameter configurations for the baseline methods are as follows. For CQL, we tune its regularization coefficient within the set $\{5, 10, 20, 30\}$, as it performs relatively well in this range, and report the best results obtained for each dataset. For IQL, we adopt the hyperparameters suggested in their work: expectile $\tau = 0.7$ and inverse temperature $\beta = 3$ on Gym locomotion tasks. For SPOT, we follow the implementation details provided in their paper, tuning its regularization coefficient within $\{0.05, 0.1, 0.2, 0.5, 1.0, 2.0\}$ on Gym locomotion tasks, and report the best results obtained for each dataset.

C.3 Experimental Details of Limited Data Experiments

In the limited data experiments, we generate reduced datasets by randomly discarding some portion of transitions from the default AntMaze datasets. This setup simulates practical scenarios where data is sparse or partially missing, such as in healthcare applications.

We evaluate the performance of IQL (sample constraint), SPOT (support constraint), and ANQ (neighborhood constraint), excluding CQL (density constraint) due to its consistently inferior performance on Antmaze tasks, as reported in Table 2. The hyperparameters for ANQ follow those used in the default benchmark datasets, as detailed in Table 3, except that the Lagrange multiplier λ is set to 2 to enable a sufficiently large neighborhood.

Hyperparameters of baseline methods. The hyperparameter configurations for the baseline methods are as follows. For CQL, we tune its regularization coefficient within the set $\{5, 10, 20, 30\}$, as it performs relatively well in this range, and report the best results obtained for each dataset. For IQL, we adopt the hyperparameters suggested in their work: expectile $\tau = 0.9$ and inverse temperature $\beta = 10$ on Antmaze tasks. For SPOT, we follow the implementation details provided in their paper, tuning its regularization coefficient within $\{0.025, 0.05, 0.1, 0.25, 0.5, 1.0\}$ on Antmaze tasks, and report the best results obtained for each dataset.

D Additional Experimental Results

D.1 Computational Cost

We assess the runtime of offline RL algorithms on the halfcheetah-medium-replay-v2 dataset using a GeForce RTX 3090. The training time for ANQ and the baseline methods are presented in Figure 5. ANQ completes the task in approximately two hours, achieving competitive efficiency with other fast offline RL algorithms such as AWAC, IQL, and TD3BC,

Several factors contribute to this efficiency: (i) ANQ is model-free and ensemble-free, making it more efficient than model-based (e.g., MOPO [90]) and ensemble-based methods (e.g., EDAC [2]); (ii) ANQ uses simple three-layer MLPs for policy and value networks (as in TD3 [22]), which are more lightweight than complex architectures used in methods like Decision Transformer [10] or Diffusion Q-learning [80]; (iii) ANQ optimizes deterministic policies with a policy update frequency of 2 (as in TD3 [22]), offering slightly better efficiency than algorithms with stochastic policies; (iv) ANQ solves the bi-level objective via alternating updates, requiring only one extra lightweight optimization step for the auxiliary policy compared to IQL [40]. As a result, ANQ achieves competitive training efficiency while maintaining strong performance.

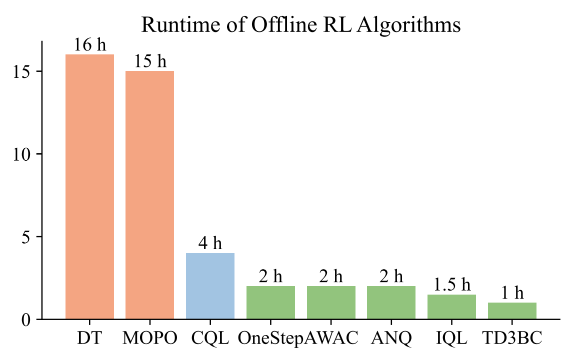


Figure 5: Runtime of algorithms on halfcheetah-medium-replay-v2 on a GeForce RTX 3090.

D.2 Additional Benchmark Comparisons

This section extends our evaluation by comparing ANQ with additional recent SOTA algorithms of other constraint types on the D4RL benchmark [20]. For support constraints, we compare against STR [53], CPI [50], CPED [93], and SVR [54]. For sample constraints, we compare against IAC [92], SQL [88], and EQL [88]. As shown in Table 4, ANQ achieves the best overall score in the Antmaze domain and demonstrates highly competitive performance in the Gym locomotion domain. From a methodological design perspective, the proposed neighborhood constraint may be particularly beneficial in high-dimensional action spaces such as those in Antmaze. In such settings, support constraint methods often struggle to accurately model the behavior policy, while sample constraint

 https://doi.org/10.52202/085713-0877

methods risk being overly conservative due to limited coverage of near-optimal actions. ANQ avoids both issues by enabling controlled generalization without explicit behavior policy modeling.

Table 4: Averaged normalized scores on Gym locomotion and Antmaze tasks over five random seeds. m = medium, m-r = medium-replay, m-e = medium-expert, e = expert, r = random; u = umaze, u-d = umaze-diverse, m-p = medium-play, m-d = medium-diverse, l-p= large-play, l-d = large-diverse.

Dataset-v2	IAC	SQL	EQL	STR	CPI	CPED	SVR	ANQ (Ours)
halfcheetah-m	51.6±0.3	48.3±0.2	47.2±0.3	51.8±0.3	**64.4±1.3**	61.8±1.6	60.5±1.2	61.8±1.4
hopper-m	74.6±11.5	75.5±3.4	70.6±2.6	**101.3±0.4**	98.5±3.0	**100.1±2.8**	**103.5±0.4**	**100.9±0.6**
walker2d-m	85.2±0.4	84.2±4.6	83.2±4.4	85.9±1.1	85.8±0.8	90.2±1.7	**92.4±1.2**	82.9±1.5
halfcheetah-m-r	47.2±0.3	44.8±0.7	44.5±0.5	47.5±0.2	54.6±1.3	**55.8±2.9**	52.5±3.0	**55.5±1.4**
hopper-m-r	**103.2±1.0**	**101.7±3.3**	98.1±3.6	100.0±1.2	**101.7±1.6**	98.1±2.1	**103.7±1.3**	**101.5±2.7**
walker2d-m-r	**93.2±1.8**	77.2±3.8	81.6±4.2	85.7±2.2	91.8±2.9	91.9±0.9	**95.6±2.5**	92.7±3.8
halfcheetah-m-e	92.9±0.7	**94.0±0.4**	**94.6±0.5**	**94.9±1.6**	**94.7±1.1**	85.4±10.9	**94.2±2.2**	**94.2±0.8**
hopper-m-e	109.3±4.0	**111.8±2.2**	**111.5±2.1**	**111.9±0.6**	106.4±4.3	95.3±13.5	**111.2±0.9**	107.0±4.9
walker2d-m-e	110.1±0.1	110.0±0.8	110.2±0.8	110.2±0.1	110.9±0.4	**113.0±1.4**	109.3±0.2	**111.7±0.2**
halfcheetah-e	94.5±0.5	-	-	95.2±0.3	**96.5±0.2**	-	**96.1±0.7**	**95.9±0.4**
hopper-e	110.6±1.9	-	-	**111.2±0.3**	**112.2±0.5**	-	**111.1±0.4**	**111.4±2.5**
walker2d-e	**114.8±1.2**	-	-	110.1±0.1	110.6±0.1	-	110.0±0.2	111.8±0.1
halfcheetah-r	20.9±1.2	-	-	20.6±1.1	**29.7±1.1**	-	27.2±1.2	24.9±1.0
hopper-r	**31.3±0.3**	-	-	**31.3±0.3**	29.5±3.7	-	**31.0±0.3**	**31.1±0.2**
walker2d-r	3.0±1.3	-	-	4.7±3.8	5.9±1.7	-	2.2±1.5	**11.2±9.5**
locomotion total	1142.4	-	-	1162.2	1193.2	-	**1200.5**	1194.5
antmaze-u	77.6±3.8	92.2±1.4	93.2±2.2	93.6±4.0	**98.8±1.1**	**96.8±2.6**	-	**96.0±1.6**
antmaze-u-d	71.2±8.6	74.0±2.3	70.4±2.7	77.4±7.2	**88.6±5.7**	55.6±2.2	-	80.2±1.8
antmaze-m-p	72.0±7.6	80.2±3.7	77.5±4.3	82.6±5.4	82.4±5.8	**85.1±3.4**	-	76.2±3.3
antmaze-m-d	74.2±4.1	75.1±4.2	74.0±3.7	**87.0±4.2**	80.4±8.9	72.1±2.9	-	77.2±6.1
antmaze-l-p	**57.0±7.4**	50.2±4.8	45.6±4.2	42.8±8.7	20.6±16.3	34.9±5.3	-	**56.2±4.9**
antmaze-l-d	47.2±9.4	**52.3±5.2**	49.5±4.7	46.8±7.6	45.2±6.9	32.3±7.4	-	**55.8±4.0**
antmaze total	399.2	424.0	410.2	430.2	416.0	376.8	-	**441.6**

D.3 Effect of the Auxiliary Policy

This section conducts an ablation study that replaces the auxiliary policy μ_ω in ANQ with a random Gaussian noise.

Discussion on exploiting vs. smoothing. Defining the auxiliary policy μ as Gaussian noise is reminiscent of the target policy noise trick introduced in TD3 [22]. Specifically, in TD3, Gaussian noise is added to the policy actions in the Bellman target. Because the trained policy is prone to overfitting to local optima in the Q-function landscape, such noise injection can smooth the value estimates and reduce accumulated errors in Q-function training. By contrast, in our method, μ_ω is applied to fixed in-dataset actions. Therefore, there is no need for Q-function estimate smoothing as in TD3. On the contrary, our method exploits, rather than smooths, the local Q landscape surrounding dataset actions, using it as a guide for the inner optimization. In our algorithm, μ_ω is optimized to seek better actions within the neighborhood of dataset actions. Replacing μ_ω with Gaussian noise thus turns this directed search into undirected random perturbations.

Empirical results. We compared different choices of the auxiliary policy μ on the Gym locomotion tasks, with results shown in Table 5. The tested variants include: (i) ANQ-Gaussian noise μ: μ is replaced with Gaussian noise, i.e., $\mu \sim \text{clip}(\mathcal{N}(0, 0.04), -0.5, 0.5)$ as used in TD3 and TD3BC; (ii) IQL: correspond to ANQ with μ set to zero; (iii) ANQ-default: our default ANQ algorithm where μ_ω is optimized. The results show that using an optimized μ (i.e., default ANQ) significantly outperforms the other two variants, especially on low-quality datasets, clearly demonstrating the benefit of our design. Additionally, ANQ with Gaussian noise μ performs slightly worse than IQL, likely because the Q values of dataset actions do not require smoothing, and the randomness introduced by Gaussian noise may cause the Bellman target to select actions inferior to the original dataset actions.

https://doi.org/10.52202/085713-0877

Table 5: Averaged normalized scores on Gym locomotion tasks over five random seeds.

Dataset-v2	IQL	ANQ-Gaussian noise μ	ANQ-stochastic μ&π	ANQ-default
halfcheetah-m	47.4±0.2	46.9±0.2	59.8±3.6	61.8±1.4
hopper-m	66.2±5.7	63.2±5.4	94.5±7.9	100.9±0.6
walker2d-m	78.3±8.7	82.9±2.2	84.3±0.9	82.9±1.5
halfcheetah-m-r	44.2±1.2	43.1±0.2	54.3±1.9	55.5±1.4
hopper-m-r	94.7±8.6	66.1±9.8	103.3±0.4	101.5±2.7
walker2d-m-r	73.8±7.1	81.2±4.5	93.6±2.9	92.7±3.8
halfcheetah-m-e	86.7±5.3	85.1±6.2	94.6±0.5	94.2±0.8
hopper-m-e	91.5±14.3	60.8±38.4	106.0±5.2	107.0±4.9
walker2d-m-e	109.6±1.0	111.9±0.5	112.1±0.2	111.7±0.2
halfcheetah-r	13.1±1.3	2.3±0.0	26.5±1.7	24.9±1.0
hopper-r	7.9±0.2	7.2±0.2	31.0±0.2	31.1±0.2
walker2d-r	5.4±1.2	6.0±0.2	8.3±7.6	11.2±9.5
total	718.8	656.7	868.2	875.4

D.4 Stochastic Policies

In practice, our algorithm is compatible with stochastic policies. Both policies introduced in the method (the auxiliary policy μ and the final policy π) can be replaced by stochastic counterparts without modification to the overall framework.

For optimizing a stochastic auxiliary policy μ_ω (typically Gaussian), Eq. (13) can be directly optimized using the reparameterization trick:

$$\max_{\mu_\omega} \mathbb{E}_{(s,a)\sim\mathcal{D},\epsilon\sim\mathcal{N}(0,1)} \left[Q_\theta(s, a+\mu_\omega(\epsilon; s,a)) - \lambda \exp(\alpha(Q_{\theta'}(s,a) - V_\psi(s)))\|\mu_\omega(\epsilon; s,a)\|\right].$$

For extracting a stochastic final policy π_ϕ (typically Gaussian), the regression problem in Eq. (17) can be reformulated as a maximum likelihood objective:

$$\min_{\pi_\phi} \mathbb{E}_{(s,a)\sim\mathcal{D},\epsilon\sim\mathcal{N}(0,1)} \exp(\beta(Q_{\theta'}(s, a+\mu_\omega(\epsilon; s,a)) - V_\psi(s))) \log \pi_\phi(a+\mu_\omega(\epsilon; s,a)|s)$$

Empirically, we tested a stochastic version of the algorithm, denoted as ANQ-stochastic μ&π, where the both policies are Gaussian. The results, reported in Table 5, show that it performs comparably to the default ANQ across most Gym locomotion tasks, though slightly worse on a few.

D.5 Learning Curves

Learning curves of ANQ on Gym-MuJoCo locomotion tasks and Antmaze tasks are presented in Figure 6 and Figure 7, respectively. The curves are averaged over 5 random seeds, with the shaded area representing the standard deviation across seeds.

E Broader Impact

Offline reinforcement learning (RL) holds significant potential for expanding RL's real-world applications in fields like robotics, recommendation systems, healthcare, and education, particularly in scenarios where data collection is expensive or risky. Nevertheless, it is essential to be aware of the potential negative societal impacts that could arise from the deployment of offline RL systems. One concern is that the data used for training may contain inherent biases, which could then be reflected in the resulting policies, potentially exacerbating existing inequalities or reinforcing harmful stereotypes. Another issue is the potential impact of offline RL on employment, as it may lead to the automation of jobs that are currently performed by humans, such as in manufacturing or autonomous driving. To ensure the responsible use of offline RL, it is crucial to address these challenges, striking a balance between fostering innovation and mitigating adverse societal consequences.

https://doi.org/10.52202/085713-0877

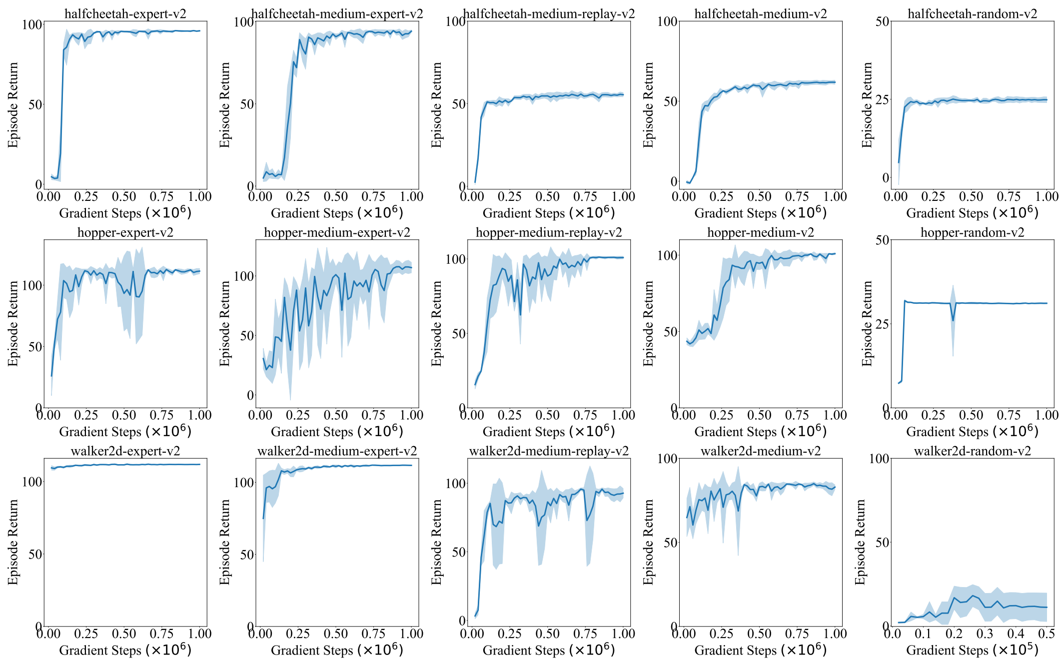


Figure 6: Learning curves of ANQ on Gym locomotion tasks during offline training. The curves are averaged over 5 random seeds, with the shaded area representing the standard deviation across seeds.

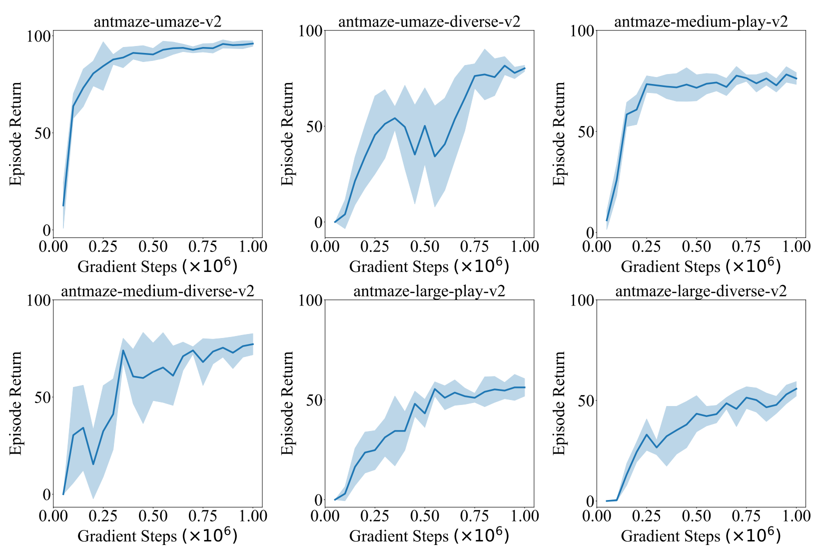


Figure 7: Learning curves of ANQ on Antmaze tasks during offline training. The curves are averaged over 5 random seeds, with the shaded area representing the standard deviation across seeds.