
MTL-KD: Multi-Task Learning Via Knowledge Distillation for Generalizable Neural Vehicle Routing Solver

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Abstract

Multi-Task Learning (MTL) in Neural Combinatorial Optimization (NCO) is a promising approach for training a unified model capable of solving multiple Vehicle Routing Problem (VRP) variants. However, existing Reinforcement Learning (RL)-based multi-task methods can only train light decoder models on small-scale problems, exhibiting limited generalization ability when solving large-scale problems. To overcome this limitation, this work introduces a novel multi-task learning method driven by knowledge distillation (MTL-KD), which enables efficient training of heavy decoder models with strong generalization ability. The proposed MTL-KD method transfers policy knowledge from multiple distinct RL-based single-task models to a single heavy decoder model, facilitating label-free training and effectively improving the model’s generalization ability across diverse tasks. In addition, we introduce a flexible inference strategy termed Random Reordering Re-Construction (R3C), which is specifically adapted for diverse VRP tasks and further boosts the performance of the multi-task model. Experimental results on 6 seen and 10 unseen VRP variants with up to 1,000 nodes indicate that our proposed method consistently achieves superior performance on both uniform and real-world benchmarks, demonstrating robust generalization abilities. The code is available at <https://github.com/CIAM-Group/MTLKD>.

1 Introduction

The Vehicle Routing Problem (VRP) is a classical combinatorial optimization problem (COP) with widespread application across numerous real-world scenarios, including logistics distribution, traffic scheduling, and emergency response [1–5]. Obtaining exact VRP solutions is computationally difficult due to its NP-hard nature [6]. Traditional heuristic algorithms [7–9] exhibit superior performance on many classic VRP tasks. However, they typically necessitate substantial domain-specific knowledge for design. In recent years, neural combinatorial optimization (NCO) [10–20] has emerged as a

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promising approach for tackling VRPs. This approach can automatically learn problem-solving strategies using neural networks and has the potential to minimize reliance on specialized domain knowledge. This field has developed rapidly, with some NCO algorithms approaching or even surpassing the performance of classical heuristic algorithms on some COPs [21, 22]. However, when solving different tasks, many NCO methods require modifying the model components for each individual task and then retraining the model, which significantly restricts their versatility in handling diverse VRP variants.

Recently, considerable research efforts have been devoted to developing unified models capable of solving multiple VRP variants using Multi-Task Learning (MTL) [23–25]. The majority of unified models for VRP variants adopt a heavy encoder-light decoder (HELD) architecture. These models have relatively low computational costs and can be directly trained using RL, demonstrating excellent performance on small-scale problems. However, their generalization ability significantly deteriorates when solving with large-scale instances, primarily because the light decoder struggles to extract sufficient information from the high-density and complex node embeddings [26].

On the other hand, the light encoder-heavy decoder [27–29] architecture demonstrates excellent generalization capabilities on large-scale problems. Its heavy decoder contributes to its strong scale generalization performance, which re-evaluates the relationships between the remaining nodes and first and last nodes during the iterative decoding process. Leveraging the heavy decoder architecture has the potential to achieve good scale generalization using a multi-task unified model. However, the substantial memory and computational demands inherent in heavy decoders render the model training using Reinforcement Learning (RL) impractical. Employing Supervised Learning (SL) is also difficult due to the absence of labeled data for multiple VRP variants. While SIT [29] employs a self-improved Training method using operations like local reconstruction [27] to refine model-generated solutions as training labels, it suffers from the generation of numerous low-quality labels during early and mid-training, leading to a protracted process and an increased training burden. We provide a comprehensive literature review on multi-task neural solvers for VRPs in Appendix A.

To address the limitation associated with the training for the heavy decoder-based model on multiple VRP variants, we introduce a novel multi-task learning method driven by knowledge distillation (MTL-KD). The proposed MTL-KD method transfers policy knowledge from multiple distinct RL-based single-task models to a single heavy decoder model, facilitating label-free training and effectively improving the model’s generalization ability across diverse tasks. Furthermore, during inference, we propose a general Random Reordering Reconstruction strategy for various variants of the VRP. By randomly reordering the external order of subtours, R3C significantly enhances solution sampling diversity, mitigates the risk of getting trapped in local optima, and further improves performance. Our contributions are summarized as follows: 1) We achieve efficient label-free training of heavy decoder models for multi-task VRPs through the proposed MTL-KD method. 2) We propose a novel R3C strategy to further enhance the performance of the multi-task model. 3) The MTL-KD model demonstrates excellent performance on 6 seen training tasks and 10 unseen tasks with up to 1,000 nodes, and real-world datasets, exhibiting good scale generalization ability and significantly outperforming existing multi-task VRP models.

2 Preliminaries

Capacitated Vehicle Routing Problem and Its Variants The Capacitated Vehicle Routing Problem (CVRP) is typically defined as follows: Given a central depot v_0 and N customer nodes v_1 to v_n , all interconnected with distances e_{ij} . Each customer i has a demand d_i , and all vehicles depart from the depot, serve customers, and return to the depot. Each vehicle has a capacity C , satisfying $C > d_i$. All customers must be visited exactly once, and the total load of the vehicle during its route must not exceed its capacity. The objective of CVRP is to minimize the total travel distance of all vehicles.

By introducing additional constraints, CVRP can be extended into various variants. The four types of additional constraints studied in this paper include: **(1) Open Route (O)**: Vehicles are not required to return to the depot after completing their service. **(2) Time Windows (TW)**: Each node i has a service time window $[s_i, l_i]$, within which vehicles must arrive at the customer (early arrivals require waiting until the earliest service start time). **(3) Backhaul (B)**: Customer demands can be positive (linehauls) or negative (backhauls), and there is no restriction on the sequence of visiting linehauls and backhauls customers. **(4) Duration Limit (L)**: The total travel distance of each vehicle must not

exceed a predefined threshold L_{thre} . Combining these constraints with CVRP can form 16 different VRP variants, with specific details provided in Appendix B.

Solution Construction Process by Neural Solver Constructive neural solvers [10, 11, 27] typically employ an encoder-decoder architecture and construct solutions in an autoregressive manner. For a given VRP instance $\mathcal{G} = (V, E)$, where V denotes the set of nodes, including node features such as demand, and E represents the set of edges, often formed by a distance matrix. The encoder is used to extract node information and generate embedding vectors for each node, while the decoder constructs the next node based on the current partial solution state s_{t-1} . The decoder is subject to node constraints during the construction process to avoid generating infeasible solutions. The entire solving process can be represented as:

$$\pi_{\theta}(\tau|\mathcal{G}) = \prod_{t=1}^{\ell} \pi_{\theta}(a_t|s_t, \mathcal{G}),$$

where τ is the complete solution, a_t is the action (node) selected at time step t , ℓ is the total number of actions, and π_{θ} represents the neural solver parameterized by θ . The decoder continuously constructs nodes until all nodes have been visited, forming a complete solution. Through training, the model learns how to effectively select the next node based on the current partial solution, thereby optimizing the quality of the entire solution.

Knowledge distillation Knowledge Distillation [30] is a model compression and knowledge transfer technique aimed at transferring the knowledge learned by a high-performing, pre-trained teacher model to a smaller student model. The core idea is to achieve this transfer by minimizing the discrepancy between the outputs of the teacher and student models. The student model's training loss typically includes a distillation loss ($\mathcal{L}_{KD}$) that measures the difference between the student's predictions and the teacher's soft targets, generally the KL divergence $\mathcal{L}_{KD} = \text{KL}(\pi_T||\pi_S)$. Additionally, depending on the specific task requirements, an original task loss ($\mathcal{L}_{Task}$) can optionally be included, and the total loss ($\mathcal{L}$) for the student model can be expressed as:

$$\mathcal{L} = \alpha\mathcal{L}_{KD} + (1 - \alpha)\mathcal{L}_{Task},$$

where α is a weight parameter. In the field of NCO, knowledge distillation has been used to achieve model lightweighting and can improve the generalization ability of models on unseen problems to a certain extent [31–34].

3 Multi-Task Learning Via Knowledge Distillation

In this section, we propose a multi-task learning framework via knowledge distillation to address the challenge of applying heavy decoder models in the multi-task domain due to their reliance on large amounts of labeled data. The proposed framework effectively enhances the model's generalization capabilities across different tasks and scales.

3.1 Framework Overview

Our proposed multi-task training framework via knowledge distillation aims to enhance the model's generalization capabilities across different tasks and scales. As depicted in Figure 1, the framework first categorizes VRP variants into seen and unseen tasks for model training. Given N seen tasks, we first pre-train N individual teacher models employing a Heavy Encoder-Light Decoder architecture [11]. Subsequently, we construct a multi-task heavy decoder student model and train it using knowledge distillation, leveraging the output distributions of the teacher models as supervision signals.

3.2 Teacher Model Training

For each seen VRP variant task, we first generate the corresponding instance data. Then, we independently train a teacher model for each task, which adopts the POMO [11] structure and serves as a policy network. We optimize the parameters of the teacher models using the policy

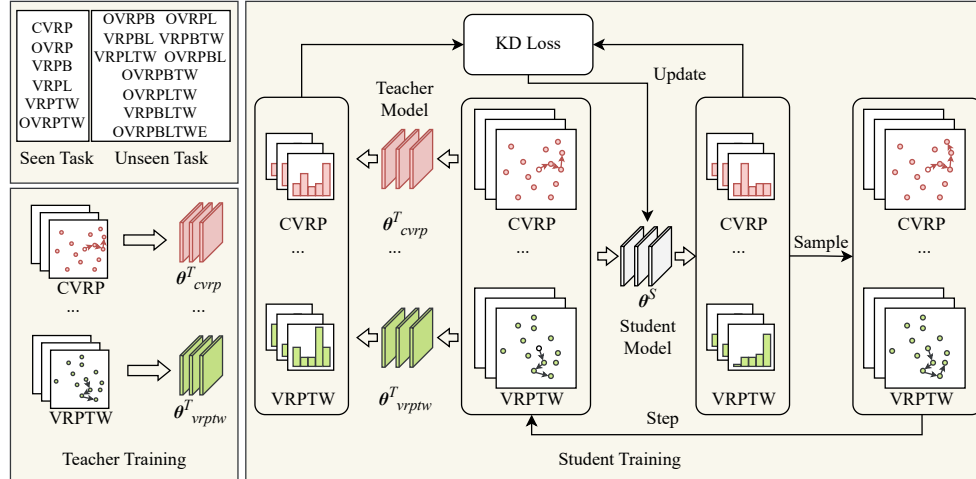


Figure 1: Framework for Multi-Task Training via Knowledge Distillation. Top-left: seen and unseen tasks. Bottom-left: Independent training of teacher models. Right: Student model distillation via teacher output distributions.

gradient method in RL, with the objective of maximizing the reward (negative tour length). The loss function [35] can be expressed as:

$$\mathcal{L}(\theta^T) = -\mathbb{E}_{\tau \sim \pi_{\theta^T}(\cdot|\mathcal{G})}[(r(\tau) - b^T(\mathcal{G})) \log \pi_{\theta^T}(\tau|\mathcal{G})],$$

where $b^T(\mathcal{G})$ is the baseline (average reward of multi-start trajectories for the instance), π_{θ^T} is the teacher model's policy distribution, θ^T represents the parameters of the teacher model, and r is the reward.

3.3 Student Model Training

During the training phase of the student model, we employ a multi-task heavy decoder architecture that processes instances from all M seen problem types simultaneously. For each training batch, we generate instances from all M seen problem types and process them together. At each decoding step t , the student model parameterized by θ^S outputs a probability distribution $\pi_{\theta^S}(a_t|s_t, \mathcal{G})$ over the next node to be selected.

For knowledge distillation, we feed each problem instance to its corresponding pre-trained teacher model parameterized by θ^{T_m} , $m \in \{1, \dots, M\}$ to obtain the teacher's output distribution $\pi_{\theta^{T_m}}(a_t|s_t, \mathcal{G})$ at the same decoding step, ensuring that each problem instance is supervised by its corresponding specialized teacher model.

The knowledge distillation loss at training step t is computed as the Kullback-Leibler divergence between the student model's output distribution and the corresponding teacher model's output distribution:

$$\mathcal{L}_{KD}^{(t)} = \sum_{m=1}^M \text{KL}(\pi_{\theta^{T_m}}(a_t|s_t, \mathcal{G}) \parallel \pi_{\theta^S}(a_t|s_t, \mathcal{G})).$$

By minimizing this loss function, the student model learns to mimic the behavior of multiple teacher models simultaneously, thereby acquiring generalized knowledge from different problem types. After each decoding step, the algorithm selects the next node based on the student model's probability distribution, transitions to the new state, and repeats this knowledge distillation process.

4 Architecture of Generalizable Multi-Task Neural Solver

Heavy decoder models have demonstrated remarkable scale generalization performance on single-task VRP. However, existing multi-task models for VRP variants predominantly rely on heavy encoder-light decoder architectures, exhibiting poor scale generalization. To address this limitation, as depicted in Figure 2, we have developed a multi-task heavy decoder model specifically designed for VRP variants, aiming to overcome the inadequate generalization capabilities of current approaches.

4.1 Encoder

Given an instance S of the VRP, which includes N node features $(s_1, \dots, s_n)$, in VRP variants, where each node feature s_i is represented as $(x_i, d_i, \delta_i, s_i, l_i)$, denoting the coordinate information, demand, service time, and the start and end times of the time window, respectively. These node features are then mapped through a linear layer to obtain the initial embedding matrix $H^{init} = (\mathbf{h}_1^{init}, \dots, \mathbf{h}_n^{init})$, where $\mathbf{h}_i^{init} = \text{Linear}(s_i)$, and $\text{Linear}(\cdot)$ denotes the linear layer. Subsequently, these initial embeddings are passed through a Transformer layer to capture node relationships and generate the node embeddings. Therefore, the output of the encoder can be represented as:

$$H^{enc} = \text{TFL}(H^{init}) = \text{TFL}(\mathbf{h}_1^{init}, \dots, \mathbf{h}_n^{init}),$$

where $H^{enc} = (\mathbf{h}_1^{enc}, \dots, \mathbf{h}_n^{enc})$ is the matrix of final node embeddings, $\text{TFL}(\cdot)$ denote the Transformer layer, and $\mathbf{h}_i$ represents the embedding of the i -th node.

4.2 Decoder

At the step t during the decoding phase, we first extract the dynamic features $D = \{l_r, t_c, d_r, o\}$, encompassing the remaining vehicle load, current time, remaining route duration, and a binary flag indicating whether the route is open. These dynamic features are combined with the last visited node $(\mathbf{h}_{last})$ and the depot node $(\mathbf{h}_{depot})$, respectively, and then each combination is passed through its own linear layer. The resulting embeddings, along with the unvisited nodes' embeddings $(\mathbf{h}_{unvisited})$, are processed by an L -layer Transformer network to yield updated node embeddings:

$$\begin{aligned} H^{(0)} &= \text{concat}(\mathbf{h}_{unvisited}^{enc}, \text{Linear}(D \oplus \mathbf{h}_{last}^{enc}), \text{Linear}(D \oplus \mathbf{h}_{depot}^{enc})), \\ H^{(1)} &= \text{TFL}(H^{(0)}; M^{\text{pad}}), \\ &\dots \\ H^{(L)} &= \text{TFL}(H^{(L-1)}; M^{\text{pad}}). \end{aligned} \tag{1}$$

Here, $M^{\text{pad}} \in \{0, -\infty\}^N$ denotes the padding mask, where 0 marks valid positions and $-\infty$ marks padded positions. We introduce this mask to handle different numbers of unvisited nodes across batch instances (which affects efficiency): sequences are padded to a unified length by appending depot tokens to shorter sequences and then masked by M^{pad} .

Next, we compute compatibility scores between the embeddings of all unvisited nodes and a context embedding $\mathbf{h}_q = \text{Linear}(\text{concat}(\mathbf{h}_{last}^{(L)}, \mathbf{h}_{depot}^{(L)}))$ using single-head attention:

$$c(\mathbf{h}_i^{(L)}, \mathbf{h}_q) = \text{SHA}(\mathbf{h}_i^{(L)}, \mathbf{h}_q).$$

Before selection, we add a padding mask and a feasibility mask to the logits to ensure valid routes. Finally, the probability $\pi(i|s_t)$ of selecting the i -th unvisited node is determined by the softmax function applied to the masked compatibility scores:

$$\pi(i|s_t) = \text{softmax}(c(\mathbf{h}_i^{(L)}, \mathbf{h}_q) + M_i^{\text{pad}} + M_i^{\text{feas}}),$$

where $c(\mathbf{h}_i^{(L)}, \mathbf{h}_q)$ is the compatibility score from SHA, $\mathbf{h}_i^{(L)}$ is the embedding of the i -th unvisited node, and s_t is the current decoder state. The masks take values in $\{0, -\infty\}$: M^{pad} excludes padded tokens and M^{feas} excludes currently infeasible moves.

It is noteworthy that we remove layer normalization from all attention layers [27].

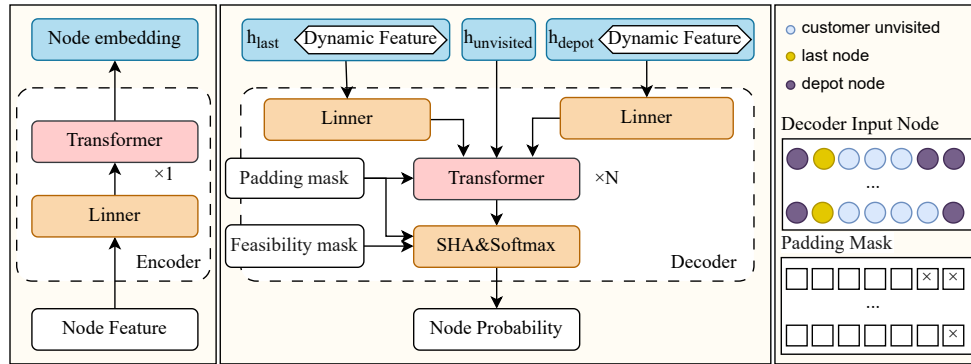


Figure 2: Architecture of the proposed multi-task neural solver. Left to right: Encoder, Decoder, Node Padding & Masking.

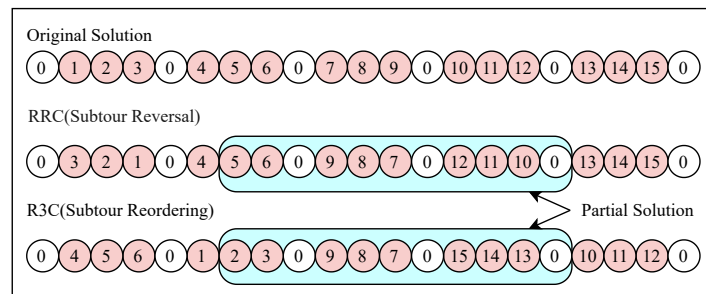


Figure 3: Comparison of RRC and R3C methods. RRC increases the diversity of sampled subproblems by randomly reversing subtours, while R3C enhances this diversity by randomly reordering the external sequence of subtours.

5 Random Reordering Re-Construct

Random Re-Construct (RRC) [27] is an iterative method to improve solution quality by randomly sampling and re-optimizing segments of the initial solution. RRC enhances sampling diversity via random subtour reversals and segment lengths. However, reversals can violate feasibility for some problems (e.g., VRPTW), and sampling solely on the original sequence limits diversity, hindering iterative performance. We thus propose Random Reordering Re-Construct to increase sampling diversity and improve iterative performance.

The R3C strategy is detailed in Figure 3. Given a solution, we first decompose it into subtours, then randomize their external order in the sequence. Subsequently, a random-length contiguous segment is randomly sampled and re-optimized by our model. If the re-optimized segment improves the objective value, it replaces the original segment. By randomizing subtour order, R3C allows for more random combinations of subtours into partial solutions, aiding escape from local optima and improving iterative performance. All sampled segments end at the depot. Additionally, feasible subtour reversals are also incorporated to further enhance search diversity.

6 Experiments

In this section, we present a series of experiments conducted on 16 VRP variants to validate the performance of our model. All experiments are performed using one NVIDIA RTX 4090 GPU.

Baselines The baseline methods are categorized into two groups: traditional heuristic solvers and multi-task neural solvers. **(1) Traditional Solvers:** PyVRP [36] is an open-source Hybrid Genetic

Search (HGS)-based solver supporting multiple complex VRP variants. It represents the current state-of-the-art in heuristic approaches for the problems considered in this study. OR-Tools [9] is Google's open-source optimization toolkit, applicable to all VRP variants examined in this study.

(2) Multi-task Neural Solvers: MT-POMO [23] is a multi-task extension of the POMO framework, enabling simultaneous learning across different VRP variants. MVMoE [24] is an architecture that enhances model capacity and improves performance over MT-POMO by incorporating Mixture-of-Experts (MoE) modules. RouteFinder [25] is a multi-task training framework that enhances training efficiency through mixed batch training and enables fast adaptation to unseen tasks via lightweight adapter layers. The framework includes three model variants: RF-Transformer, RF-POMO, and RF-MVMoE. CaDA [37] incorporates constraint awareness and a dual-branch structure to better handle diverse tasks within a multi-task learning framework.

Datasets We adopt the same problem settings as MVMoE [24]. We generate a test set comprising 16 VRP problems across four scales: 100, 200, 500, and 1000. The dataset at the 100-scale contains 1,000 instances, while the others contain 128 instances each. In these settings, the vehicle capacity is uniformly set to 50, the duration limit is set to 3, the time window for the depot node in Time Window problems is $[0, 3.0]$, and for backhaul problems, the proportion of backhaul nodes is set to 20%. Our datasets are generated based on the MVMoE codebase to evaluate the performance of various baselines.

Training Configuration and Model Parameters We follow a similar setup to MVMoE, training our model on 6 VRPs and evaluating its zero-shot generalization performance on 10 unseen tasks (refer to Figure 1). The training process consists of two phases: teacher model pre-training and student model training. **(1) Teacher Model Pre-training:** We employ the POMO model as our teacher model. For each of the 6 visible tasks, we independently train a single-task model using reinforcement learning. All teacher models share the same parameter configuration: 6 encoder layers, 1 decoder layer, an embedding dimension of 128, 8 attention heads, and a Feedforward hidden dimension of 512. The teacher models are trained for 4000 epochs on random instances of scale 100, with 20,000 training instances per epoch, a batch size of 128, and an initial learning rate of 0.0001. **(2) Student Model Training:** The student model consists of 1 encoder layer and 6 decoder layers, with embedding dimensions set to 128 and 96, employing 8 multi-head attention heads and a Feedforward hidden dimension of 512. The student model simultaneously learns from the 6 pre-trained teacher models on datasets with a problem scale of 100. The training batch size is set to 1500 (250×6), with 24,000 training instances per epoch, an initial learning rate of 0.0001, and a total of 850 training epochs. After the 300th epoch, the learning rate is halved every 100 epochs.

Inference and Metrics We adopt the Average Objective Value and Gap as evaluation metrics, where smaller values indicate better performance. The Objective Value represents the total distance of the solution in the Vehicle Routing Problem, while the Gap evaluates the performance difference of each method compared to a traditional baseline method (such as HGS-PyVRP).

We test all methods using Single Trajectory Greedy Search (S.T.). Furthermore, to compare with the results of other solvers exploring multiple trajectories under data augmentation (denoted as $M \times \text{aug}8$, where M is the number of nodes and $\text{aug}8$ represents 8 types of data augmentation [11]), our method employs the R3C strategy with 200 iterations under a single trajectory.

6.1 Main Results

Performance on seen Tasks We evaluate the performance of our model on the training tasks, and the results are shown in Table 1. The results demonstrate that our proposed MTL-KD model exhibits excellent performance on the seen training tasks, particularly showcasing a more pronounced advantage when dealing with larger-scale problems. This validates the effectiveness of our proposed knowledge distillation training framework and model architecture.

Performance on Unseen Tasks The zero-shot generalization experimental results on 10 unseen tasks (as shown in Table 2) indicate that our MTL-KD model outperforms the compared models in most cases, demonstrating remarkable cross-task generalization ability, especially with a significant advantage on large-scale problems. This validates that multi-task knowledge distillation training endows the model with strong generalization capabilities across different VRP variants.

Table 1: Performance on seen VRPs across three scales. * represents the baseline used for gap calculation.

Method	Pro.	Problem Size			Pro.	Problem Size		
		n=100	n=500	n=1k		n=100	n=500	n=1k
HGS-PyVRP	CVRP	15.53(*)	62.07(*)	119.54(*)	VRPTW	24.35(*)	90.61(*)	166.47(*)
OR-Tools		15.94(2.63%)	66.51(7.15%)	125.91(5.32%)		25.21(3.54%)	98.45(8.65%)	178.47(7.21%)
MT-POMO(S.T.)		16.25(4.64%)	70.60(13.74%)	146.92(22.90%)		26.66(10.40%)	122.61(35.50%)	247.44(66.47%)
MT-POMO(M+aug8)		15.79(1.69%)	67.99(9.54%)	136.62(14.28%)		25.61(5.18%)	115.43(27.39%)	229.82(38.06%)
MVMOE(S.T.)		16.30(5.00%)	77.44(24.77%)	191.07(59.83%)		26.88(10.44%)	122.78(35.26%)	267.33(60.59%)
MVMOE(M+aug8)		15.76(1.50%)	73.61(18.59%)	176.40(47.57%)		25.51(4.78%)	116.67(28.76%)	253.35(52.19%)
RF-Transformer(M+aug8)		15.82(1.88%)	67.71(9.08%)	132.79(11.08%)		27.39(12.49%)	108.70(19.97%)	222.92(33.91%)
RF-MTPOMO(M+aug8)		15.87(2.20%)	67.42(8.62%)	132.82(11.11%)		26.29(7.98%)	102.77(13.42%)	193.57(16.28%)
RF-MVMOE(M+aug8)		15.84(2.01%)	67.36(8.52%)	134.85(12.80%)		26.29(7.98%)	100.78(11.22%)	187.87(12.86%)
CaDa(M+aug8)		15.84(2.00%)	175.65(182.99%)	542.56(353.87%)		30.16(23.87%)	300.11(231.21%)	693.61(316.66%)
MTL-KD ₉₆		16.06(3.41%)	64.57(4.02%)	123.88(3.63%)		26.18(7.52%)	100.17(10.55%)	188.94(13.50%)
MTL-KD ₁₂₈		16.04(3.30%)	64.61(4.09%)	124.44(4.09%)		26.13(7.32%)	99.05(9.31%)	184.92(11.09%)
MTL-KD(R3C200) ₁₂₈		15.76(1.48%)	63.63(2.51%)	122.06(2.10%)		25.31(3.93%)	96.43(6.42%)	181.85(9.24%)
HGS-PyVRP	VRPL	15.58(*)	63.55(*)	122.68(*)	OVRP	9.71(*)	35.30(*)	66.10(*)
OR-Tools		16.00(2.69%)	67.53(6.26%)	128.16(4.46%)		9.84(1.38%)	37.81(7.11%)	70.38(6.48%)
MT-POMO(S.T.)		16.29(4.52%)	71.46(12.66%)	149.55(21.90%)		10.66(9.83%)	44.62(26.41%)	92.78(40.36%)
MT-POMO(M+aug8)		15.85(1.67%)	68.95(8.49%)	138.97(13.27%)		10.17(4.74%)	41.93(18.78%)	85.27(29.00%)
MVMOE(S.T.)		16.36(4.94%)	78.79(23.97%)	192.29(56.74%)		10.77(10.94%)	49.35(39.81%)	135.30(104.70%)
MVMOE(M+aug8)		15.81(1.45%)	74.67(17.50%)	177.89(45.00%)		10.14(4.42%)	46.23(30.97%)	116.82(76.74%)
RF-Transformer(M+aug8)		15.88(1.93%)	68.59(7.93%)	135.15(10.17%)		10.11(4.11%)	42.88(21.48%)	84.40(27.68%)
RF-MTPOMO(M+aug8)		15.93(2.23%)	68.33(7.52%)	134.53(9.66%)		10.17(4.70%)	41.60(17.86%)	82.14(24.26%)
RF-MVMOE(M+aug8)		15.90(2.06%)	68.39(7.62%)	138.60(12.98%)		10.13(4.29%)	41.18(16.65%)	81.82(23.78%)
CaDa(M+aug8)		15.89(2.00%)	176.04(177.01%)	536.71(337.49%)		10.10(4.04%)	187.72(431.78%)	442.57(569.55%)
MTL-KD ₉₆		16.14(3.58%)	65.45(2.99%)	126.11(2.79%)		10.41(7.21%)	38.73(9.72%)	72.73(10.03%)
MTL-KD ₁₂₈		16.12(3.45%)	65.48(3.03%)	126.66(3.25%)		10.46(7.19%)	38.89(10.17%)	72.70(9.98%)
MTL-KD(R3C200) ₁₂₈		15.82(1.50%)	64.5246(1.53%)	124.59(1.56%)		10.05(3.53%)	37.7934(7.07%)	71.40(8.03%)
HGS-PyVRP	VRPB	-	-	-	OVRPTW	13.95(*)	48.15(*)	82.98(*)
OR-Tools		11.97(*)	47.76(*)	88.57(*)		14.38(3.06%)	52.48(9.00%)	90.03(8.49%)
MT-POMO(S.T.)		12.47(4.21%)	50.08(4.85%)	99.56(12.41%)		15.56(11.53%)	71.63(48.76%)	147.84(78.16%)
MT-POMO(M+aug8)		12.04(0.63%)	48.49(1.52%)	95.35(7.64%)		14.85(6.43%)	66.74(38.61%)	135.99(63.87%)
MVMOE(S.T.)		12.42(3.80%)	71.99(50.75%)	186.49(110.54%)		15.70(12.54%)	82.81(71.99%)	215.21(159.34%)
MVMOE(M+aug8)		12.01(0.31%)	66.26(38.73%)	167.26(88.84%)		14.78(5.90%)	76.28(58.43%)	195.64(135.76%)
MTL-KD ₉₆		12.39(3.51%)	46.12(-3.44%)	86.90(-1.89%)		15.11(8.32%)	53.74(11.61%)	94.12(13.42%)
MTL-KD ₁₂₈		12.38(3.41%)	45.99(-3.70%)	86.99(-1.78%)		15.08(8.07%)	53.89(11.91%)	93.96(13.22%)
MTL-KD(R3C200) ₁₂₈		12.02(0.43%)	44.58(-6.66%)	83.66(-5.55%)		14.53(4.12%)	52.08(8.15%)	92.40(11.35%)

Performance on Real-World Instances We also evaluate the performance of our model on real-world datasets, including Set-X (medium and large scale) from CVRPLIB [38] for CVRP and the Solomon dataset [39] for VRPTW. We compare our model against several single-task and multi-task models, and the experimental results are presented in Tables 6, 7, and 8, AppendixD. Our model consistently outperforms other single-task and multi-task models across all three real-world datasets, demonstrating particularly strong performance on the large-scale Set-X dataset, where the gap is significantly smaller than other baselines. This indicates the strong applicability of our approach to real-world scenarios.

6.2 Ablation Study

Scale Generalization Comparison: Student vs. Teacher While heavy decoders are generally considered to have significant potential for large-scale generalization, the teacher model, POMO, primarily excels at its specific training scale and lacks cross-scale generalization capabilities. This naturally raises a crucial question: Will the student model also be limited to learning knowledge specific to a particular scale, thereby restricting its generalization ability? To investigate this, we compare the performance of several single-task teacher models with our MTL-KD model on the same dataset, as shown in Figure 4. In contrast to the teacher models, the student model demonstrates a notable capacity for scale generalization. This observation indicates that the MTL-KD model, during the learning process, does not simply replicate the teacher’s knowledge but rather effectively adapts to scale variations, consequently achieving robust generalization performance.

Performance Comparison: KD vs. RL To highlight the effectiveness of our knowledge distillation approach, we train the proposed multi-task LEHD model under two distinct training paradigms: knowledge distillation (MTL-KD) and reinforcement learning (MTL-RL). Given the high computational cost associated with training the LEHD model with RL, we are only able to train it at a scale of 20. Subsequently, we evaluate the average gap of both models in all tasks seen and unseen, with the experimental results presented in Table 3. The results indicate that the RL method, due to its limitation to training on small scales, struggles to leverage the full potential of the LEHD model,

Table 2: Performance on unseen VRPs across three scales. * represents the baseline used for gap calculation.

Method	Pro.	Problem Size			Pro.	Problem Size		
		n=100	n=500	n=1k		n=100	n=500	n=1k
HGS-PyVRP		9.67(*)	34.70(*)	65.38(*)		24.44(*)	91.86(*)	174.79(*)
OR-Tools		9.79(1.25%)	37.09(6.89%)	69.64(6.51%)		25.65(4.96%)	99.50(8.31%)	186.38(6.63%)
MT-POMO(S.T.)		10.61(9.69%)	43.77(26.13%)	92.49(41.45%)		26.76(9.52%)	123.97(34.95%)	252.82(44.65%)
MT-POMO(M+aug8)		10.13(4.72%)	41.28(18.95%)	84.40(29.08%)		25.71(5.21%)	116.62(26.95%)	235.94(34.99%)
MVMoE(S.T.)		10.76(11.25%)	48.60(40.05%)	135.08(106.58%)		27.00(10.48%)	124.25(35.25%)	274.68(57.15%)
MVMoE(M+aug8)		10.10(4.43%)	45.58(31.36%)	116.71(78.48%)		25.62(4.84%)	118.00(28.45%)	260.07(48.79%)
MTL-KD(S.T.) ₉₆		10.42(7.73%)	38.23(10.17%)	72.32(10.61%)		26.37(7.89%)	101.39(10.36%)	195.65(11.94%)
MTL-KD(S.T.) ₁₂₈		10.44(7.95%)	38.43(10.76%)	72.78(11.31%)		26.35(7.81%)	100.40(9.30%)	192.41(10.08%)
MTL-KD(R3C200) ₁₂₈		10.08(4.29%)	37.27(7.41%)	71.35(9.12%)		25.55(4.54%)	97.85(6.51%)	189.02(8.14%)
HGS-PyVRP		14.00(*)	47.97(*)	83.68(*)		-	-	-
OR-Tools		14.28(1.98%)	52.94(10.36%)	91.69(9.57%)		8.37(*)	29.98(*)	54.87(*)
MT-POMO(S.T.)		15.61(11.46%)	71.24(48.52%)	147.97(76.82%)		9.52(13.82%)	35.43(18.17%)	74.59(35.95%)
MT-POMO(M+aug8)		14.90(6.39%)	66.65(38.95%)	137.13(63.87%)		8.98(7.34%)	32.76(9.28%)	66.91(21.95%)
MVMoE(S.T.)		15.74(12.39%)	82.49(71.97%)	219.21(161.95%)		9.74(16.41%)	45.80(52.76%)	129.38(135.81%)
MVMoE(M+aug8)		14.83(5.90%)	76.12(58.68%)	197.84(136.41%)		8.96(7.10%)	40.41(34.77%)	109.08(98.81%)
MTL-KD(S.T.) ₉₆		15.28(9.12%)	53.38(11.28%)	94.65(13.11%)		9.25(10.58%)	30.90(3.08%)	58.57(6.76%)
MTL-KD(S.T.) ₁₂₈		15.26(8.95%)	53.56(11.65%)	95.33(13.92%)		9.27(10.81%)	30.45(1.58%)	56.92(3.73%)
MTL-KD(R3C200) ₁₂₈		14.71(5.02%)	51.97(8.34%)	93.91(12.22%)		8.78(4.95%)	28.73(4.16%)	52.41(4.48%)
OR-Tools		12.02(*)	47.93(*)	89.82(*)		25.41(*)	97.77(*)	194.69(*)
MT-POMO(S.T.)		12.71(5.79%)	50.70(5.77%)	99.51(10.78%)		28.64(12.70%)	126.00(28.87%)	270.68(39.03%)
MT-POMO(M+aug8)		12.10(0.66%)	48.14(0.45%)	94.26(4.94%)		26.94(6.04%)	118.20(20.89%)	251.58(29.22%)
MVMoE(S.T.)		12.86(7.06%)	70.03(46.11%)	159.91(78.03%)		28.88(13.67%)	125.89(28.76%)	294.23(51.13%)
MVMoE(M+aug8)		12.05(0.28%)	63.57(32.63%)	145.61(62.10%)		26.89(5.82%)	119.44(22.16%)	276.17(41.85%)
MTL-KD(S.T.) ₉₆		12.56(4.54%)	45.90(-4.23%)	86.45(-3.75%)		28.48(12.11%)	105.43(7.83%)	215.94(10.92%)
MTL-KD(S.T.) ₁₂₈		12.54(4.32%)	45.67(-4.72%)	86.28(-3.95%)		28.49(12.12%)	103.85(6.21%)	209.82(7.77%)
MTL-KD(R3C200) ₁₂₈		12.08(0.55%)	44.27(-7.64%)	82.56(-8.08%)		26.64(4.84%)	100.25(2.53%)	205.13(5.37%)
OR-Tools		8.35(*)	29.60(*)	54.30(*)		14.38(*)	51.88(*)	90.86(*)
MT-POMO(S.T.)		9.50(13.79%)	35.34(19.37%)	74.83(37.80%)		16.92(17.64%)	72.91(40.53%)	152.63(67.98%)
MT-POMO(M+aug8)		8.96(7.35%)	32.60(10.13%)	66.87(23.13%)		15.88(10.39%)	67.95(30.96%)	140.72(54.87%)
MVMoE(S.T.)		9.75(16.75%)	45.46(53.57%)	127.34(134.49%)		17.07(18.68%)	81.57(57.22%)	214.76(136.36%)
MVMoE(M+aug8)		8.94(7.12%)	40.35(36.29%)	108.53(99.86%)		15.81(9.90%)	74.43(43.46%)	193.33(112.77%)
MTL-KD(S.T.) ₉₆		9.29(11.27%)	31.96(7.96%)	61.37(13.01%)		16.70(16.10%)	56.02(7.97%)	99.85(9.89%)
MTL-KD(S.T.) ₁₂₈		9.41(12.75%)	31.42(6.14%)	59.25(9.11%)		16.74(16.39%)	56.02(7.98%)	99.20(9.18%)
MTL-KD(R3C200) ₁₂₈		8.84(5.95%)	29.23(-1.26%)	53.77(-0.98%)		15.58(8.30%)	53.75(3.62%)	97.37(7.17%)
OR-Tools		25.34(*)	103.17(*)	189.31(*)		14.25(*)	52.37(*)	92.16(*)
MT-POMO(S.T.)		28.92(14.11%)	131.68(27.64%)	264.39(39.66%)		16.78(17.79%)	73.26(39.90%)	152.61(65.60%)
MT-POMO(M+aug8)		27.25(7.52%)	123.71(19.91%)	245.61(29.74%)		15.74(10.45%)	68.08(30.01%)	141.27(53.28%)
MVMoE(S.T.)		29.19(15.18%)	132.23(28.17%)	286.27(51.21%)		16.95(18.94%)	82.36(57.28%)	215.69(134.03%)
MVMoE(M+aug8)		27.14(7.10%)	125.27(21.43%)	269.13(42.16%)		15.67(9.97%)	74.83(42.90%)	195.45(112.07%)
MTL-KD(S.T.) ₉₆		29.01(14.4703%)	111.33(7.92%)	210.45(11.17%)		16.78(17.74%)	56.34(7.59%)	101.43(10.06%)
MTL-KD(S.T.) ₁₂₈		29.13(14.96%)	110.33(6.94%)	205.52(8.56%)		16.95(18.93%)	57.15(9.13%)	101.64(10.29%)
MTL-KD(R3C200) ₁₂₈		27.12(7.03%)	106.52(3.26%)	201.29(6.33%)		15.65(9.84%)	54.68(4.41%)	99.64(8.11%)

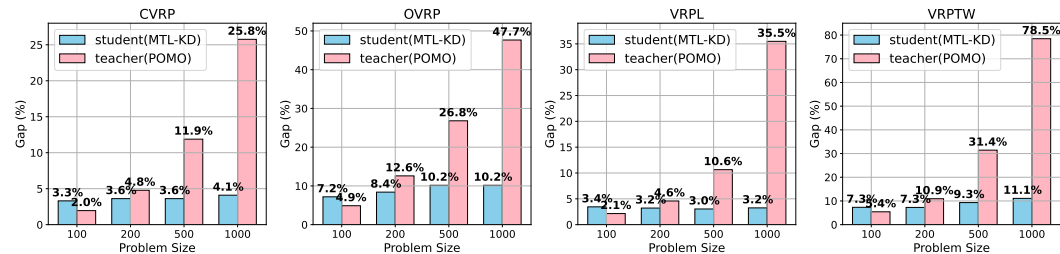


Figure 4: Performance Comparison between Teacher and Student Models, both trained on instances of scale 100.

leading to significantly poorer performance. In contrast, our distillation-based MTL-KD model demonstrates a clear performance advantage.

Effectiveness Analysis of R3C To validate the effectiveness of our proposed R3C method, we conduct experiments on CVRP, VRPL, OVRP, and VRPTW at scale 100. We perform an ablation study on the random reordering of the subtour external order component, with the following comparative experiments: random sampling on the original solution only (RS); random reordering of the subtour external order followed by sampling (RS+Ro); and for CVRP and VRPL, we also analyze the impact of adding a random flipping operation (F) since it does not affect the legality of the solution. The experimental results are shown in Figure 5. Incorporating the random reordering operation significantly improves the iterative performance during reconstruction, which benefits from the ability of random reordering to enhance the diversity of sampled subproblems. Furthermore, the random flipping operation can moderately increase the diversity of sampled solutions when

Table 3: Heavy Decoder Performance (RL vs. KD) on Seen/Unseen Tasks (Average Gap).

Problem Size		100	200	500	1000
Training task	MTL-RL	21.7834%	27.2457%	38.2808%	50.0606%
	MTL-KD	5.4569%	5.2775%	5.8028%	6.6413%
unseen task	MTL-RL	31.2513%	37.1404%	48.9074%	62.2226%
	MTL-KD	19.1658%	12.9145%	10.8275%	13.3322%

only initial solution random sampling is performed; however, its impact is minimal with random reordering, further confirming the effectiveness of the random reordering operation.

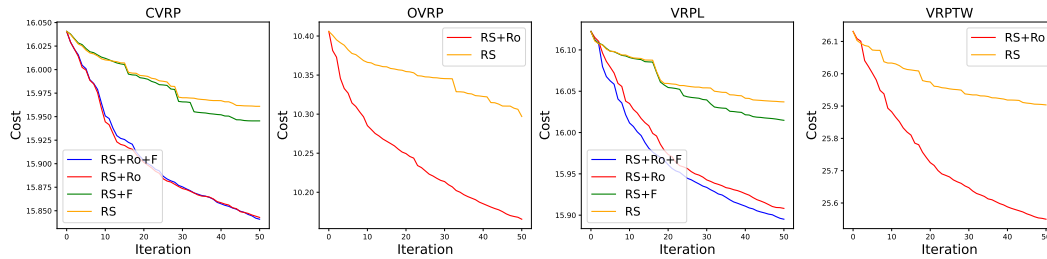


Figure 5: Impact of Different Components in R3C.

7 Conclusion

This paper presents a heavy decoder model for the multi-task VRP domain and achieves label-free training of this model through a multi-task knowledge distillation method. The proposed approach has demonstrated excellent performance on both seen and unseen tasks, as well as real-world datasets, exhibiting significant large-scale generalization ability and thereby validating the effectiveness of our method. Furthermore, the proposed R3C method has further enhanced the model's performance. However, the structure of the heavy decoder results in high computational complexity. Future work will explore how to implement more efficient and robust model architectures in the multi-task domain.

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References

- [1] Diego Cattaruzza, Nabil Absi, Dominique Feillet, and Jesús González-Feliu. Vehicle routing problems for city logistics. *EURO Journal on Transportation and Logistics*, 6(1):51–79, 2017.
- [2] Mouhcine Elgarej, Mansouri Khalifa, and Mohamed Youssfi. Optimized path planning for electric vehicle routing and charging station navigation systems. In *Research Anthology on Architectures, Frameworks, and Integration Strategies for Distributed and Cloud Computing*, pages 1945–1967. IGI Global, 2021.
- [3] Jianan Zhou, Yaoxin Wu, Zhiguang Cao, Wen Song, Jie Zhang, and Zhenghua Chen. Learning large neighborhood search for vehicle routing in airport ground handling. *IEEE Transactions on knowledge and data engineering*, 35(9):9769–9782, 2023.
- [4] Kai Li, Fei Liu, Zhenkun Wang, Xialiang Tong, Xiongwei Han, Mingxuan Yuan, and Qingfu Zhang. Ars: Automatic routing solver with large language models. *arXiv preprint arXiv:2502.15359*, 2025.

- [5] Kai Li, Ruihao Zheng, Xinye Hao, and Zhenkun Wang. Multi-objective infeasibility diagnosis for routing problems using large language models. *arXiv preprint arXiv:2508.03406*, 2025.
- [6] Giorgio Ausiello, Pierluigi Crescenzi, Giorgio Gambosi, Viggo Kann, Alberto Marchetti-Spaccamela, and Marco Protasi. *Complexity and approximation: Combinatorial optimization problems and their approximability properties*. Springer Science & Business Media, 2012.
- [7] Keld Helsgaun. An extension of the lin-kernighan-helsgaun tsp solver for constrained traveling salesman and vehicle routing problems. *Roskilde: Roskilde University*, 12:966–980, 2017.
- [8] Thibaut Vidal. Hybrid genetic search for the cvrp: Open-source implementation and swap* neighborhood. *Computers & Operations Research*, 140:105643, 2022.
- [9] Vincent Furnon and Laurent Perron. Or-tools routing library, 2023. URL <https://developers.google.com/optimization/routing>.
- [10] Wouter Kool, Herke van Hoof, and Max Welling. Attention, learn to solve routing problems! In *International Conference on Learning Representations*, 2019.
- [11] Yeong-Dae Kwon, Jinho Choo, Byoungjip Kim, Iljoo Yoon, Youngjune Gwon, and Seungjai Min. Pomo: Policy optimization with multiple optima for reinforcement learning. *Advances in Neural Information Processing Systems*, 33:21188–21198, 2020.
- [12] Minsu Kim, Junyoung Park, and Jinkyoo Park. Sym-nco: Leveraging symmetry for neural combinatorial optimization. *Advances in Neural Information Processing Systems*, 35:1936–1949, 2022.
- [13] Yan Jin, Yuandong Ding, Xuanhao Pan, Kun He, Li Zhao, Tao Qin, Lei Song, and Jiang Bian. Pointerformer: Deep reinforced multi-pointer transformer for the traveling salesman problem. *Proceedings of the AAAI Conference on Artificial Intelligence*, 37(7):8132–8140, 2023.
- [14] Zhi Zheng, Zhuoliang Xie, Zhenkun Wang, and Bryan Hooi. Monte carlo tree search for comprehensive exploration in LLM-based automatic heuristic design. In *International Conference on Machine Learning*, 2025. URL <https://openreview.net/forum?id=Do10dZzYHr>.
- [15] Rui Sun, Zhi Zheng, and Zhenkun Wang. Learning encodings for constructive neural combinatorial optimization needs to regret. *Proceedings of the AAAI Conference on Artificial Intelligence*, 38(18):20803–20811, Mar. 2024. doi: 10.1609/aaai.v38i18.30069. URL <https://ojs.aaai.org/index.php/AAAI/article/view/30069>.
- [16] Zhi Zheng, Shunyu Yao, Genghui Li, Linxi Han, and Zhenkun Wang. Pareto improver: Learning improvement heuristics for multi-objective route planning. *IEEE Transactions on Intelligent Transportation Systems*, 25(1):1033–1043, 2024. doi: 10.1109/TITS.2023.3313688.
- [17] Han Li, Fei Liu, Zhi Zheng, Yu Zhang, and Zhenkun Wang. CaDA: Cross-problem routing solver with constraint-aware dual-attention. In *International Conference on Machine Learning*, 2025. URL <https://openreview.net/forum?id=CS4RyQuTig>.
- [18] Fu Luo, Yaoxin Wu, Zhi Zheng, and Zhenkun Wang. Rethinking neural combinatorial optimization for vehicle routing problems with different constraint tightness degrees, 2025. URL <https://arxiv.org/abs/2505.24627>.
- [19] Changliang Zhou, Xi Lin, Zhenkun Wang, and Qingfu Zhang. Learning to reduce search space for generalizable neural routing solver. *arXiv preprint arXiv:2503.03137*, 2025.
- [20] Changliang Zhou, Canhong Yu, Shunyu Yao, Xi Lin, Zhenkun Wang, Yu Zhou, and Qingfu Zhang. Urs: A unified neural routing solver for cross-problem zero-shot generalization. *arXiv preprint arXiv:2509.23413*, 2025.
- [21] Liang Xin, Wen Song, Zhiguang Cao, and Jie Zhang. Neurolkh: Combining deep learning model with lin-kernighan-helsgaun heuristic for solving the traveling salesman problem. In *Advances in Neural Information Processing Systems*, volume 34, 2021.
- [22] André Hottung, Paula Wong-Chung, and Kevin Tierney. Neural deconstruction search for vehicle routing problems, 2025. URL <https://arxiv.org/abs/2501.03715>.
- [23] Fei Liu, Xi Lin, Qingfu Zhang, Xialiang Tong, and Mingxuan Yuan. Multi-task learning for routing problem with cross-problem zero-shot generalization. *International Conference on Knowledge Discovery and Data Mining (KDD)*, 2024.

- [24] Jianan Zhou, Zhiguang Cao, Yaoxin Wu, Wen Song, Yining Ma, Jie Zhang, and Chi Xu. Mvmoe: multi-task vehicle routing solver with mixture-of-experts. In *International Conference on Machine Learning*, pages 61804–61824, 2024.
- [25] Federico Berto, Chuanbo Hua, Nayeli Gast Zepeda, André Hottung, Niels Wouda, Leon Lan, Junyoung Park, Kevin Tierney, and Jinkyoo Park. Routefinder: Towards foundation models for vehicle routing problems. *arXiv preprint arXiv:2406.15007*, 2024.
- [26] Ziwei Huang, Jianan Zhou, Zhiguang Cao, and Yixin XU. Rethinking light decoder-based solvers for vehicle routing problems. In *International Conference on Learning Representations*, 2025.
- [27] Fu Luo, Xi Lin, Fei Liu, Qingfu Zhang, and Zhenkun Wang. Neural combinatorial optimization with heavy decoder: Toward large scale generalization. In A. Oh, T. Naumann, A. Globerson, K. Saenko, M. Hardt, and S. Levine, editors, *Advances in Neural Information Processing Systems*, volume 36, pages 8845–8864. Curran Associates, Inc., 2023.
- [28] Darko Drakulic, Sofia Michel, Florian Mai, Arnaud Sors, and Jean-Marc Andreoli. Bq-nco: Bisimulation quotienting for efficient neural combinatorial optimization. In A. Oh, T. Naumann, A. Globerson, K. Saenko, M. Hardt, and S. Levine, editors, *Advances in Neural Information Processing Systems*, volume 36, pages 77416–77429. Curran Associates, Inc., 2023.
- [29] Fu Luo, Xi Lin, Yaoxin Wu, Zhenkun Wang, Tong Xialiang, Mingxuan Yuan, and Qingfu Zhang. Boosting neural combinatorial optimization for large-scale vehicle routing problems. In *International Conference on Learning Representations*, 2025.
- [30] GeoffreyE. Hinton, Oriol Vinyals, and J.Michael Dean. Distilling the knowledge in a neural network. *arXiv: Machine Learning, arXiv: Machine Learning*, Mar 2015.
- [31] Jieyi Bi, Yining Ma, Jiahai Wang, Zhiguang Cao, Jinbiao Chen, Yuan Sun, and Yeow Meng Chee. Learning generalizable models for vehicle routing problems via knowledge distillation. *Advances in Neural Information Processing Systems*, 35:31226–31238, 2022.
- [32] Yubin Xiao, Di Wang, Boyang Li, Mingzhao Wang, Xuan Wu, Changliang Zhou, and You Zhou. Distilling autoregressive models to obtain high-performance non-autoregressive solvers for vehicle routing problems with faster inference speed. *Proceedings of the AAAI Conference on Artificial Intelligence*, 38(18): 20274–20283, 2024.
- [33] Qidong Liu, Xin Shen, Chaoyue Liu, Dong Chen, Xin Zhou, and Mingliang Xu. Enhancing the generalization capability of 2d array pointer networks through multiple teacher-forcing knowledge distillation. *Journal of Automation and Intelligence*, 2025.
- [34] Sisi Zheng and Rongye Ye. Aspdd: An adaptive knowledge distillation framework for tsp generalization problems. *IEEE Access*, 2024.
- [35] Ronald J Williams. Simple statistical gradient-following algorithms for connectionist reinforcement learning. *Machine learning*, 8:229–256, 1992.
- [36] Niels A Wouda, Leon Lan, and Wouter Kool. PyVRP: A high-performance VRP solver package. *INFORMS Journal on Computing*, 2024.
- [37] Han Li, Fei Liu, Zhi Zheng, Yu Zhang, and Zhenkun Wang. Cada: Cross-problem routing solver with constraint-aware dual-attention. *arXiv preprint arXiv:2412.00346*, 2024.
- [38] Eduardo Uchoa, Diego Pecin, Artur Pessoa, Marcus Poggi, Thibaut Vidal, and Anand Subramanian. New benchmark instances for the capacitated vehicle routing problem. *European Journal of Operational Research*, 257(3):845–858, 2017.
- [39] Marius M Solomon. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations research*, 35(2):254–265, 1987.
- [40] Oriol Vinyals, Meire Fortunato, and Navdeep Jaitly. Pointer networks. *Advances in neural information processing systems*, 28, 2015.
- [41] Irwan Bello, Hieu Pham, Quoc V. Le, Mohammad Norouzi, and Samy Bengio. Neural combinatorial optimization with reinforcement learning. In *International Conference on Learning Representations Workshop Track*, 2017. URL https://openreview.net/forum?id=HyG_D4v9xx.
- [42] Mohammadreza Nazari, Afshin Oroojlooy, Lawrence Snyder, and Martin Takác. Reinforcement learning for solving the vehicle routing problem. *Advances in neural information processing systems*, 31, 2018.

- [43] Yeong-Dae Kwon, Jinho Choo, Iljoo Yoon, Minah Park, Duwon Park, and Youngjune Gwon. Matrix encoding networks for neural combinatorial optimization. *Advances in Neural Information Processing Systems*, 34:5138–5149, 2021.
- [44] Liang Xin, Wen Song, Zhiguang Cao, and Jie Zhang. Multi-decoder attention model with embedding glimpse for solving vehicle routing problems. *Proceedings of the AAAI Conference on Artificial Intelligence*, 35(13):12042–12049, 2021.
- [45] Jianan Zhou, Yaoxin Wu, Wen Song, Zhiguang Cao, and Jie Zhang. Towards omni-generalizable neural methods for vehicle routing problems. In *International Conference on Machine Learning*, pages 42769–42789. PMLR, 2023.
- [46] Rui Sun, Zhi Zheng, and Zhenkun Wang. Learning encodings for constructive neural combinatorial optimization needs to regret. *Proceedings of the AAAI Conference on Artificial Intelligence*, 38(18):20803–20811, 2024.
- [47] Zhi Zheng, Shunyu Yao, Zhenkun Wang, Tong Xialiang, Mingxuan Yuan, and Ke Tang. DPN: Decoupling partition and navigation for neural solvers of min-max vehicle routing problems. In *International Conference on Machine Learning*, 2024. URL <https://openreview.net/forum?id=ar174skI9u>.
- [48] Chengrui Gao, Haopu Shang, Ke Xue, Dong Li, and Chao Qian. Towards generalizable neural solvers for vehicle routing problems via ensemble with transferrable local policy. In *Proceedings of the Thirty-Third International Joint Conference on Artificial Intelligence*, pages 6914–6922, 2024.
- [49] Han Fang, Zhihao Song, Paul Weng, and Yutong Ban. Invt: a generalizable routing problem solver with invariant nested view transformer. In *International Conference on Machine Learning*, pages 12973–12992, 2024.
- [50] Changliang Zhou, Xi Lin, Zhenkun Wang, Xialiang Tong, Mingxuan Yuan, and Qingfu Zhang. Instance-conditioned adaptation for large-scale generalization of neural combinatorial optimization. *arXiv preprint arXiv:2405.01906*, 2024.
- [51] Qingchun Hou, Jingwei Yang, Yiqiang Su, Xiaoqing Wang, and Yuming Deng. Generalize learned heuristics to solve large-scale vehicle routing problems in real-time. In *International Conference on Learning Representations*, 2023.
- [52] Haoran Ye, Jiarui Wang, Helan Liang, Zhiguang Cao, Yong Li, and Fanzhang Li. Glop: Learning global partition and local construction for solving large-scale routing problems in real-time. *Proceedings of the AAAI Conference on Artificial Intelligence*, 38(18):20284–20292, 2024.
- [53] Zhi Zheng, Changliang Zhou, Xialiang Tong, Mingxuan Yuan, and Zhenkun Wang. Udc: A unified neural divide-and-conquer framework for large-scale combinatorial optimization problems. In A. Globerson, L. Mackey, D. Belgrave, A. Fan, U. Paquet, J. Tomczak, and C. Zhang, editors, *Advances in Neural Information Processing Systems*, volume 37, pages 6081–6125. Curran Associates, Inc., 2024.
- [54] Chenguang Wang and Tianshu Yu. Efficient training of multi-task combinatorial neural solver with multi-armed bandits. *arXiv preprint arXiv:2305.06361*, 2023.
- [55] Zhuoyi Lin, Yaoxin Wu, Bangjian Zhou, Zhiguang Cao, Wen Song, Yingqian Zhang, and Jayavelu Senthilnath. Cross-problem learning for solving vehicle routing problems. In *The 33rd International Joint Conference on Artificial Intelligence (IJCAI-24)*, 2024.

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A Related work

A.1 Single-task Autoregressive Neural Solver

Research on high-performance autoregressive neural solvers for VRP is a hot topic in NCO. It is typically based on the encoder-decoder structure, where the encoder generates node embeddings, and the decoder is used to dynamically generate node sequences. Early works are based on Pointer Networks [40], gradually evolving from supervised learning to reinforcement learning training [41, 42]. Subsequently, the Transformer framework is introduced and becomes a mainstream paradigm [10]. POMO [11] further leverages the symmetry of the VRP, optimizing the baseline calculation in RL training and enhancing the exploration diversity of the model. Subsequent research largely focuses on improving the Transformer framework [43, 44, 13, 12, 45–47]. However, these approaches predominantly adopt the Heavy Encoder and Light Decoder (HELD) structure, which poses challenges in handling large-scale problems. To enhance the capability of solving large instances, some methods continue to build upon HELD. Examples include ELG [48], which integrates local search strategies; INVIT [49], which utilizes K-nearest neighbor multi-view embeddings to enhance local information; and ICAM [50], which introduces instance-conditional adaptation to better perceive problem scales. On the other hand, some studies [27, 28] explore heavy decoder architectures. By dynamically capturing the relationships between remaining nodes during the decoding process, these methods demonstrate superior generalization performance on large-scale problems. However, their high computational demands, often necessitating training on labeled data, represent a significant limitation. Furthermore, some methods [51–53, 29] decompose large-scale problems into multiple smaller subproblems for solving, and autoregressive neural solvers can be employed at the lower level to address these subproblems.

A.2 Multi-Task Autoregressive Neural Solver

To enhance the generalizability of neural solvers across diverse VRP variants, several existing methods have proposed unified multi-task models. For example, Wang and Yu [54] employ a multi-armed bandit approach to achieve efficient training across multiple combinatorial optimization problems. Lin et al. [55] pre-train a Transformer backbone on the Traveling Salesman Problem and fine-tune it for specific VRP variants, thereby extending its applicability to a subset of VRP problems. These methods represent initial attempts at cross-problem learning but are limited in scope, focusing primarily on a small number of problems. Building on these foundations, MT-POMO [23] views VRP variants as combinations of distinct attributes, enabling unified representation and zero-shot generalization across multiple VRP variants, achieving notable performance on ten tasks. MVMoE [24] further boosts model capacity and multi-task performance by introducing a mixture-of-experts model. RouteFinder [25] enhances multi-task training efficiency and performance through mixed batch training and enables rapid adaptation to unseen tasks via efficient adapter layers. CaDA [37] improves the model's ability to handle diverse tasks within a multi-task framework by incorporating constraint awareness and a dual-branch structure. However, these studies have predominantly focused on small-scale problems, lacking extensive exploration of large-scale generalization capabilities.

B Problem Definition and Settings

Our VRP variant problem settings follow the work of Zhou et al. [24]. The detailed settings are as follows:

- **Coordinates:** All node coordinates are uniformly sampled from the 2D space $[0, 1) \times [0, 1)$.
- **Demands:** For all customer nodes, demands are randomly sampled from the set $\{1, 2, \dots, 9\}$.
- **Vehicle Capacity:** Each vehicle has a fixed transportation capacity, which we set to 50 across all problem scales. The sum of demands for nodes visited by a vehicle must be less than or equal to its current capacity.
- **Open Routes:** In open vehicle routing problems, vehicles are not required to return to the depot. We use a binary flag vector to distinguish whether a path is open; setting it to 1 indicates an open path.

Table 4: 16 VRP variants with five constraints.

Constraint	Capacity	Open Route	Backhaul	Duration Limit	Time Window
CVRP	✓				
OVRP	✓	✓			
VRPB	✓		✓		
VRPL	✓			✓	
VRPTW	✓				✓
OVRPTW	✓	✓			✓
OVRPB	✓	✓	✓		
OVRPL	✓	✓		✓	
VRPBL	✓		✓	✓	
VRPBTW	✓		✓		✓
VRPLTW	✓			✓	✓
OVRPBL	✓	✓	✓	✓	
OVRPBTW	✓	✓	✓		✓
OVRPLTW	✓	✓		✓	✓
VRPBLTW	✓		✓	✓	✓
OVRPBLTW	✓	✓	✓	✓	✓

- **Backhauls:** In problems involving backhauls, a portion of the nodes have negative demands, referred to as backhaul nodes. We designate 20% of the customer nodes as backhaul nodes, whose demands are randomly sampled from $\{-1, -2, \dots, -9\}$.
- **Duration Limit:** Each vehicle has a maximum travel distance limit, which we set to 3.
- **Time Windows:** Customer node time windows are uniformly sampled, and service time is uniformly set to 0.2. Vehicles must visit nodes within their time windows; if a vehicle arrives early, it must wait until the earliest start time. The depot's time window is set to $[0, 3]$, with a service time of 0. Vehicle speed is uniformly set to 1, meaning the time spent traveling on a path equals its distance divided by speed. Notably, for non-open problems, it is crucial to ensure that after visiting the last customer node, the vehicle can return to the depot within its latest allowable time window; this consideration is not necessary for open problems.

By combining different constraints, 16 VRP variants can be formed, as detailed in Table 4.

C Sensitivity Analysis on R3C Re-optimization Segment Length

To confirm the efficacy of our random segment length selection in the R3C strategy, we compare it against fixed segment lengths ($k \in \{10, 20, 30, 40, 50\}$) on 100-node instances for four VRP variants (CVRP, OVRP, VRPL, VRPTW). As shown in Table 5, the random length consistently yields the best performance across all problems. We attribute this advantage to the increased solution diversity provided by varying segment lengths, which effectively prevents the search from getting trapped in local optima, a common issue with fixed segment sampling.

Table 5: Results after 50 iterations of R3C with different Re-optimization Segment Lengths on 100-node instances.

Problem	segment=10	segment=20	segment=30	segment=40	segment=50	Random
CVRP	16.01	15.96	15.94	15.91	15.91	15.88
OVRP	10.34	10.25	10.21	10.17	10.20	10.17
VRPL	16.09	16.03	16.00	15.97	15.98	15.90
VRPTW	26.02	25.86	25.77	25.65	25.69	25.55

D Performance on Real-World Instances

Our test results on real-world instance datasets are presented in Tables 6, 7, and 8. All models are trained on instances of size $N = 100$. During inference, data augmentation is applied, and a

greedy rollout strategy is adopted by default. Some results for comparative models are sourced from MVMoE [24] and RouteFinder [25].

Table 6: Results on small-scale CVRPLIB instances (Set-X) [38]. All models are trained on instances of size $N = 100$, and inference utilizes data augmentation [11] and greedy rollout by default.

Set-X		POMO		POMO-MTL		MVMoE/4E		MVMOE/4E-L		MTL-KD	
Instance	Opt.	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap
X-n101-k25	27591	30138	9.231%	32482	17.727%	29361	6.415%	29015	5.161%	29058	5.317%
X-n106-k14	26362	39322	49.162%	27369	3.820%	27278	3.475%	27242	3.338%	26918	2.109%
X-n110-k13	14971	15223	1.683%	15151	1.202%	15089	0.788%	15196	1.503%	15407	2.912%
X-n115-k10	12747	16113	26.406%	14785	15.988%	13847	8.629%	13325	4.534%	13513	6.009%
X-n120-k6	13332	14085	5.648%	13931	4.493%	14089	5.678%	13833	3.758%	13657	2.438%
X-n125-k30	55539	58513	5.355%	60687	9.269%	58944	6.131%	58603	5.517%	57615	3.738%
X-n129-k18	28940	29246	1.057%	30332	4.810%	29802	2.979%	29457	1.786%	29309	1.275%
X-n134-k13	10916	11302	3.536%	11581	6.092%	11353	4.003%	11398	4.416%	11363	4.095%
X-n139-k10	13590	14035	3.274%	13911	2.362%	13825	1.729%	13800	1.545%	13911	2.362%
X-n143-k7	15700	16131	2.745%	16660	6.115%	16125	2.707%	16147	2.847%	15955	1.624%
X-n148-k46	43448	49328	13.533%	50782	16.880%	46758	7.618%	45599	4.951%	45463	4.638%
X-n153-k22	21220	32476	53.040%	26237	23.643%	23793	12.125%	23316	9.877%	23340	9.991%
X-n157-k13	16876	17660	4.646%	17510	3.757%	17650	4.586%	17410	3.164%	17161	1.689%
X-n162-k11	14138	14889	5.312%	14720	4.117%	14654	3.650%	14662	3.706%	14487	2.469%
X-n167-k10	20557	21822	6.154%	21399	4.096%	21340	3.809%	21275	3.493%	21053	2.413%
X-n172-k51	45607	49556	8.659%	56385	23.632%	51292	12.465%	49073	7.600%	47850	4.918%
X-n176-k26	47812	54197	13.354%	57637	20.549%	55520	16.121%	52727	10.280%	52476	9.755%
X-n181-k23	25569	37311	45.923%	26219	2.542%	26258	2.695%	26241	2.628%	25919	1.369%
X-n186-k15	24145	25222	4.461%	25000	3.541%	25182	4.295%	24836	2.862%	24711	2.344%
X-n190-k8	16980	18315	7.862%	18113	6.673%	18327	7.933%	18113	6.673%	17539	3.292%
X-n195-k51	44225	49158	11.154%	54090	22.306%	49984	13.022%	48185	8.954%	46301	4.694%
X-n200-k36	58578	64618	10.311%	61654	5.251%	61530	5.039%	61483	4.959%	60978	4.097%
X-n209-k16	30656	32212	5.076%	32011	4.420%	32033	4.492%	32055	4.564%	31536	2.871%
X-n219-k73	117595	133545	13.564%	119887	1.949%	121046	2.935%	120421	2.403%	118499	0.769%
X-n228-k23	25742	48689	89.142%	33091	28.549%	31054	20.636%	28561	10.951%	28156	9.378%
X-n237-k14	27042	29893	10.543%	28472	5.288%	28550	5.577%	28486	5.340%	27789	2.762%
X-n247-k50	37274	56167	50.687%	45065	20.902%	43673	17.167%	41800	12.143%	41106	10.281%
X-n251-k28	38684	40263	4.082%	40614	4.989%	41022	6.044%	40822	5.527%	39877	3.084%
Avg. Gap		16.629%		9.820%		6.884%		5.160%		4.025%	

E Licenses for Code and Datasets

The licenses for the codes and the datasets used in this work are listed in Table 9.

F Broader Impacts

This research contributes to the field of neural combinatorial optimization by employing advanced machine learning techniques to address large-scale and multi-variant VRP problems. We believe that the proposed multi-task knowledge distillation training framework, heavy decoder model, and Random Reordering Reconstruction (R3C) strategy can provide valuable insights and inspire subsequent work to explore more efficient and effective neural methods for solving large-scale and diverse VRP problems. As a general learning-based approach to solving the VRP, the proposed multi-task knowledge distillation training framework and R3C strategy do not inherently possess any specific potential negative social impacts.

Table 7: Results on VRPTW instances (Set-Solomon) [39]. All models are trained on instances of size $N = 100$, and inference utilized data augmentation [11] and greedy rollout by default.

Set-Solomon		POMO		POMO-MTL		MVMOE/4E		MVMOE/4E-L		MTL-KD	
Instance	Opt.	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap
R101	1637.7	1805.6	10.252%	1821.2	11.205%	1798.1	9.794%	1730.1	5.641%	1768.0	7.956%
R102	1466.6	1556.7	6.143%	1596.0	8.823%	1572.0	7.187%	1574.3	7.345%	1564.4	6.668%
R103	1208.7	1341.4	10.979%	1327.3	9.812%	1328.2	9.887%	1359.4	12.470%	1379.0	14.090%
R104	971.5	1118.6	15.142%	1120.7	15.358%	1124.8	15.780%	1098.8	13.100%	1074.1	10.561%
R105	1355.3	1506.4	11.149%	1514.6	11.754%	1479.4	9.157%	1456.0	7.433%	1465.2	8.109%
R106	1234.6	1365.2	10.578%	1380.5	11.818%	1362.4	10.352%	1353.5	9.627%	1346.1	9.031%
R107	1064.6	1214.2	14.052%	1209.3	13.592%	1182.1	11.037%	1196.5	12.391%	1193.9	12.145%
R108	932.1	1058.9	13.604%	1061.8	13.915%	1023.2	9.774%	1039.1	11.481%	1035.4	11.082%
R109	1146.9	1249.0	8.902%	1265.7	10.358%	1255.6	9.478%	1224.3	6.750%	1260.0	9.861%
R110	1068.0	1180.4	10.524%	1171.4	9.682%	1185.7	11.021%	1160.2	8.635%	1199.5	12.313%
R111	1048.7	1177.2	12.253%	1211.5	15.524%	1176.1	12.148%	1197.8	14.220%	1178.7	12.396%
R112	948.6	1063.1	12.070%	1057.0	11.427%	1045.2	10.183%	1044.2	10.082%	1057.2	11.448%
RC101	1619.8	2643.0	63.168%	1833.3	13.181%	1774.4	9.544%	1749.2	7.988%	1774.0	9.520%
RC102	1457.4	1534.8	5.311%	1546.1	6.086%	1544.5	5.976%	1556.1	6.771%	1588.4	8.989%
RC103	1258.0	1407.5	11.884%	1396.2	10.986%	1402.5	11.486%	1415.3	12.502%	1451.8	15.405%
RC104	1132.3	1261.8	11.437%	1271.7	12.311%	1265.4	11.755%	1264.2	11.649%	1241.6	9.653%
RC105	1513.7	1612.9	6.553%	1644.9	8.668%	1635.5	8.047%	1619.4	6.980%	1688.3	11.535%
RC106	1372.7	1539.3	12.137%	1552.8	13.120%	1505.0	9.638%	1509.5	9.968%	1468.8	7.001%
RC107	1207.8	1347.7	11.583%	1384.8	14.655%	1351.6	11.906%	1324.1	9.625%	1367.6	13.231%
RC108	1114.2	1305.5	17.169%	1274.4	14.378%	1254.2	12.565%	1247.2	11.939%	1232.3	10.600%
RC201	1261.8	2045.6	62.118%	1761.1	39.570%	1577.3	25.004%	1517.8	20.285%	1490.8	18.149%
RC202	1092.3	1805.1	65.257%	1486.2	36.062%	1616.5	47.990%	1480.3	35.520%	1336.5	22.356%
RC203	923.7	1470.4	59.186%	1360.4	47.277%	1473.5	59.521%	1479.6	60.182%	1237.8	34.005%
RC204	783.5	1323.9	68.973%	1331.7	69.968%	1286.6	64.212%	1232.8	57.342%	1066.0	36.056%
RC205	1154.0	1568.4	35.910%	1539.2	33.380%	1537.7	33.250%	1440.8	24.850%	1448.3	25.503%
RC206	1051.1	1707.5	62.449%	1472.6	40.101%	1468.9	39.749%	1394.5	32.671%	1354.7	28.884%
RC207	962.9	1567.2	62.758%	1375.7	42.870%	1442.0	49.756%	1346.4	39.831%	1235.9	28.352%
RC208	776.1	1505.4	93.970%	1185.6	52.764%	1107.4	42.688%	1167.5	50.437%	1064.0	37.096%
Avg. Gap		28.054%		21.380%		20.317%		18.490%		15.786%	

Table 8: Results on large-scale CVRPLIB instances (Set-X) [38]. All models are trained on instances of size $N = 100$, and inference utilizes data augmentation [11] and greedy rollout by default.

Set-X		POMO		LEHD		POMO-MTL		MVMoE/4E		MVMoE/4E-L		RF-MVMoE		RF-TE		MTL-KD	
Instance	Opt.	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap	Obj.	Gap
X-n502-k39	69226	75617	9.232%	71438	3.195%	77284	11.640%	73533	6.222%	74429	7.516%	76338	10.274%	71791	3.705%	71124	2.742%
X-n513-k21	24201	30518	26.102%	25624	5.880%	28510	17.805%	32102	32.647%	31231	29.048%	32639	34.866%	28465	17.619%	25947	7.215%
X-n524-k153	154593	201877	30.586%	280556	81.480%	192249	24.358%	186540	20.665%	182392	17.982%	170999	10.612%	174381	12.800%	171306	10.811%
X-n536-k96	94846	106073	11.837%	103785	9.425%	106514	12.302%	109581	15.536%	108543	14.441%	105847	11.599%	103272	8.884%	101893	7.430%
X-n548-k50	86700	103093	18.908%	90644	4.549%	94562	9.068%	95894	10.604%	95917	10.631%	104289	20.287%	100956	16.443%	89169	2.848%
X-n561-k42	42717	49370	15.575%	44728	4.708%	47846	12.007%	56008	31.114%	51810	21.287%	53383	24.969%	49454	15.771%	45467	6.438%
X-n573-k30	50673	83545	64.871%	53482	5.543%	60913	20.208%	59473	17.366%	57042	12.569%	61524	21.414%	55952	10.418%	53466	5.512%
X-n586-k159	190316	229887	20.792%	232867	22.358%	208893	9.761%	215668	13.321%	214577	12.748%	212151	11.473%	205575	8.018%	200863	5.542%
X-n599-k92	108451	150572	38.839%	115377	6.386%	120333	10.956%	128949	18.901%	125279	15.517%	126578	16.714%	116560	7.477%	113513	4.668%
X-n613-k62	59535	68451	14.976%	62484	4.953%	67984	14.192%	82586	38.718%	74945	25.884%	73456	23.383%	67267	12.987%	63035	5.879%
X-n627-k43	62164	84434	35.825%	67568	8.693%	73060	17.528%	70987	14.193%	70905	14.061%	70414	13.271%	67572	8.700%	65755	5.777%
X-n641-k35	63682	75573	18.672%	68249	7.172%	72643	14.071%	75329	18.289%	72655	14.090%	71975	13.023%	70831	11.226%	67593	6.141%
X-n655-k131	106780	127211	19.134%	117532	10.069%	116988	9.560%	117678	10.206%	118475	10.952%	119057	11.497%	112202	5.078%	109748	2.780%
X-n670-k130	146332	208079	42.197%	220927	50.977%	190118	29.922%	197695	35.100%	183447	25.364%	168226	14.962%	168999	15.490%	161076	10.076%
X-n685-k75	68205	79482	16.534%	72946	6.951%	80892	18.601%	97388	42.787%	89441	31.136%	82269	20.620%	77847	14.137%	73249	7.395%
X-n701-k44	81923	97843	19.433%	86327	5.376%	92075	12.392%	98469	20.197%	94924	15.870%	90189	10.090%	89932	9.776%	85967	4.936%
X-n716-k35	43373	51381	18.463%	46502	7.214%	52709	21.525%	56773	30.895%	52305	20.593%	52250	20.467%	49669	14.516%	47012	8.390%
X-n733-k159	136187	159098	16.823%	149115	9.493%	161961	18.925%	178322	30.939%	167477	22.976%	156387	14.833%	148463	9.014%	142712	4.791%
X-n749-k98	77269	87786	13.611%	83439	7.985%	90582	17.229%	100438	29.985%	94497	22.296%	92147	19.255%	85171	10.227%	82295	6.505%
X-n766-k71	114417	135464	18.395%	131487	14.919%	144041	25.891%	152352	33.155%	136255	19.086%	130505	14.061%	129935	13.563%	123310	7.772%
X-n783-k48	72386	90289	24.733%	76766	6.051%	83169	14.897%	100383	38.677%	92960	28.423%	96336	33.087%	83185	14.919%	77332	6.833%
X-n801-k40	73305	124278	69.536%	77546	5.785%	85077	16.059%	91560	24.903%	87662	19.585%	87118	18.843%	86164	17.542%	78041	6.460%
X-n819-k171	158121	193451	22.344%	178558	12.925%	177157	12.039%	183599	16.113%	185832	17.525%	179596	13.581%	174441	10.321%	170672	7.938%
X-n837-k142	193737	237884	22.787%	207709	7.212%	214207	10.566%	229526	18.473%	221286	14.220%	230362	18.904%	208528	7.635%	203165	4.866%
X-n856-k95	88965	152528	71.447%	92936	4.464%	101774	14.398%	99129	11.425%	106816	20.065%	105801	18.924%	98291	10.483%	94079	5.748%
X-n876-k59	99299	119764	20.609%	104183	4.918%	116617	17.440%	119619	20.463%	114333	15.140%	114016	14.821%	107416	8.174%	105873	6.620%
X-n895-k37	53860	70245	30.421%	58028	7.739%	65587	21.773%	79018	46.710%	64310	19.402%	69099	28.294%	64871	20.444%	58606	8.812%
X-n916-k207	329179	399372	21.324%	385208	17.021%	361719	9.885%	383681	16.557%	374016	13.621%	373600	13.494%	352998	7.236%	346940	5.396%
X-n936-k151	132715	237625	79.049%	196547	48.097%	186262	40.347%	220926	66.466%	190407	43.471%	161343	21.571%	163162	22.942%	152094	14.602%
X-n957-k87	85465	130850	53.104%	90295	5.651%	98198	14.898%	113882	33.250%	105629	23.593%	123633	44.659%	102689	20.153%	90407	5.782%
X-n979-k58	118976	147687	24.132%	127972	7.561%	138092	16.067%	146347	23.005%	139682	17.404%	131754	10.740%	129952	9.225%	127650	7.291%
X-n1001-k43	72355	100399	38.759%	76689	5.990%	87660	21.153%	114448	58.176%	94734	30.929%	88969	22.962%	85929	18.760%	78833	8.953%
Avg. Gap		29.658%		12.836%		16.796%		26.408%		19.607%		18.795%		12.303%		6.655%	

Table 9: Licenses of Used Codes and Datasets

Type	Resource	License Type	URL / Reference
Code	MVMoE [24]	MIT License	https://github.com/RoyalSkye/Routing-MVMoE
Code	MT-POMO [23]	MIT License	https://github.com/FeiLiu36/MTNCO
Code	HGS-PyVRP [36]	MIT License	https://github.com/PyVRP/PyVRP
Code	OR-Tools [9]	Apache License	https://github.com/google/or-tools
Dataset	CVRPLIB(Set-X) [38]	Available for academic research use	http://vrp.galgos.inf.puc-rio.br/index.php/en/
Dataset	Solomon [39]	Available for academic research use	https://www.sintef.no/projectweb/top/vrptw/solomon-benchmark/