



Research in Engineering Education Symposium (REES)
Regional Conference in Engineering Education (RCEE)
& Research in Higher Education (RHEd)
September 29 – October 3, 2025
Johor, Malaysia



Synergy in Mathematical and Computational Thinking for Hydrodynamic Flood Modelling

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ABSTRACT

BACKGROUND & CONTEXT

The integration of mathematical and computational thinking is essential for effective mathematical modelling. This approach involves using mathematical principles and computational techniques to create models that represent real-world problems and predict outcomes. In this paper, we explore how combining these thinking skills enhances the modelling processes, highlighting their importance in engineering problem-solving.

PURPOSE OR GOAL

Our study focuses on the role of thinking to demonstrate how the thoughtful integration of mathematical and computational thinking leads to more accurate and efficient models. We aim to show how mathematical thinking informs computational methods and how computational thinking enhances modelling. This synergy provides valuable insights for transforming teaching practices that connect theory with practical application, ensuring students are equipped with the skills needed to tackle real problems effectively.

APPROACH OR METHODOLOGY

We used a case study on hydrodynamic flood modelling to show how mathematical and computational thinking work together. Analysing these modelling processes reveals the specific thought processes involved in solving real-world phenomena. These insights support proposed curricular changes, aiming to link mathematical theory with practical application more effectively.

OUTCOMES

The hydrodynamic flood modelling case study highlights the power and versatility of mathematical and computational thinking in solving real-world problems. The synergy between both thinking emphasizes the importance and potential of bridging research and practice to transform the mathematical education of engineering undergraduates.

CONCLUSIONS

Integrating mathematical and computational thinking can revolutionize engineering education. Equipping students with skills in both areas will enhance their problem-solving capabilities and prepare them for the complex challenges of the modern world. Bridging these approaches requires a deep understanding of both domains and the ability to effectively translate abstract mathematical concepts into computational models.

KEYWORDS

Mathematical Thinking, Computational Thinking, Mathematical Modelling.

Introduction

Mathematical modelling serves as an essential tool in understanding and predicting complex natural phenomena, such as hydrodynamic floods. It involves creating a mathematical representation of the physical world, typically through equations, algorithms, and simulations to better understand underlying processes and to forecast outcomes. In the case of floods, this means capturing the behaviour of water as it moves across and interacts with various terrains, infrastructures, and environmental conditions. The goal is not to replicate every detail of the real world, but to abstract the most relevant features into a simplified, yet sufficiently accurate, model that can inform planning, risk assessment, and emergency response.

The development of hydrodynamic flood models necessitates a rigorous approach to abstraction and representation. We must identify and incorporate relevant physical variables such as rainfall intensity, terrain elevation, soil permeability, and channel geometry, while simplifying or omitting less critical factors. This selective abstraction ensures that the model remains computationally feasible without sacrificing essential accuracy. The integration of mathematical principles with computational techniques enables the simulation of complex fluid dynamics, providing insights into flood extent, depth, and velocity under various scenarios (Bates & De Roo, 2000).

The cognitive demands of modelling extend beyond initial model construction. As pointed out by Mambretti (2006), throughout the modelling process, continuous engagement with mathematical structures is required to test hypotheses, validate assumptions, and refine model parameters. This iterative process involves logical reasoning, pattern recognition, and critical evaluation to ensure the model's robustness and reliability (Szydłowski, 2006).

In this paper, we explore the synergy between mathematical and computational thinking in the context of hydrodynamic flood modelling. By examining how these cognitive approaches intersect, we aim to highlight their collective role in developing models that are both theoretically sound and practically applicable. Through this integration, we can enhance the accuracy and efficiency of flood predictions, thereby supporting informed decision-making in flood risk management.

Definition of Key Concepts

Mathematical and computational thinking are foundational approaches to understanding and solving complex problems. Together, they offer a powerful framework for developing models that are both theoretically rigorous and practically effective, enabling solutions across fields such as industry, science, and technology (Blum & Leiß, 2007; Wing, 2006).

Mathematical Thinking

Mason et al. (2010) has described mathematical thinking as a dynamic process in a series of spiralling loops which begins with the manipulation of objects (mental, physical, pictorial, symbolic), moving to getting a sense of pattern or some property of those objects, to articulating that property as an expression of generality. Thus, by increasing the complexity of ideas we can handle, we expand our understanding, ability to manipulate the ideas and concepts and used to seek out further properties. Each successive step assumes that thinking is more complex. Mental blocks, misunderstanding, or conjectures that fail under testing may cause an oscillation within and across the loops.

While Schoenfeld (1992) defines mathematical thinking as (a) developing a mathematical point of view, an appreciation of the processes of mathematization and abstraction and being able to apply them; and (b) developing a competence with tools of mathematics to enhance understanding mathematical sense making. He maintains that fundamental aspects of thinking mathematically include core-knowledge, problem-solving strategies, effective use of one's resources, having a mathematical perspective and engagement in mathematical practices. Tall's theory of the three worlds of mathematics (2013) describes three distinct but interrelated

worlds of mathematical thinking, namely, conceptual embodiment, operational symbolism, and axiomatic formalism. The theory builds on the fundamental human activities of perception, action and reflection. The journey through the worlds of embodiment and perceptual symbolism to the formal world of mathematical proof, allow a global vision of mathematical growth to emerge. The theory provides a way to identify learners' struggle as they go through the process of mathematical knowledge development.

In short, mathematical thinking is essential for analysing and solving complex real-world issues, as it provides structured frameworks that enable us to break down intricate problems into manageable components. Through logical reasoning, abstraction, and quantitative analysis, mathematical thinking transforms real-world phenomena such as floods into simplified models that can be systematically explored and understood. This process often involves identifying relevant variables, recognizing patterns, and establishing relationships, which are then expressed through equations, functions, or other mathematical representations. By applying Mason et al problem-solving strategies (2010) within these frameworks, we gain deeper insights into the underlying mechanics of the issue but also develop practical solutions that are both efficient and scalable.

Computational Thinking

Real-world environmental complexity often requires numerical methods to approximate solutions, making computational thinking essential. It involves applying algorithmic strategies to discretise equations, implement models in code, and manage large datasets such as those related to topography, land use, and hydrology. As Wing (2006) defines it, computational thinking goes beyond programming; it is a problem-solving approach rooted in computer science principles like algorithm design, data structure organization, abstraction, and decomposition, enabling both humans and machines to effectively process and solve complex problems.

The computational thinking process starts with data as the input and seeks to derive meaning and answers from it. In the context of hydrodynamic flood modelling, computational thinking is essential for translating mathematical formulations into executable simulations. Once a mathematical model typically governed by partial differential equations is developed, computational thinking enables its numerical solution through discretization techniques such as finite difference, finite volume, or finite element methods (Toro, 2009). These techniques require algorithmic precision and logical structuring, both characteristics of computational thinking.

Studies such as Wing (2006) and Aho (2012) provide basic perspectives on computational thinking. Wing argues for its importance across disciplines, while Aho (2012) emphasizes the centrality of computational processes in modern scientific inquiry. Building on these works, here we will illustrate how computational thinking not only enhances the practical usability of hydrodynamic models but also fosters innovative solutions by enabling scalable, adaptive, and intelligent modelling systems.

Application in Hydrodynamic Flood Modelling

We will now explore in depth how mathematical and computational thinking are applied specifically in the context of hydrodynamic flood modelling. We shall break down the modelling process into three stages: conceptualization, formulation, and analysis. Here we highlight the key roles that mathematical and computational thinking play in transforming complex hydrodynamic systems into usable predictive tools.

Role of Mathematical Thinking

In mathematical problem solving, Mason et al (2010) proposed three phases of work, which they named, ENTRY, ATTACK and REVIEW. We are now in our ENTRY phase of the problem in which we are making sense of the problem from the ideas we have gathered about the physical characteristics of a flood such as water velocity and depth, bed roughness, initial and

boundary conditions. Examining the system and identifying the problem have led us to the following resolutions:

- We WANT to model the flood and
- We KNOW that the application of conservation laws is capable of realizing what we want (Toro, 2009).

These statements of ‘what we WANT’ and ‘what we KNOW’ underlies the main aim of our study. To attain this aim, we need to introduce certain concepts, definitions and terminologies. In this conceptualisation stage, it involves identifying key variables, simplifying assumptions, and establishing the physical laws that govern the system through the use of conservation laws of mass and momentum. In achieving our goal to model the flood process, we divide our work accordingly. Each of these divisions signifies the flow of our work and making sense of each particular sub-problem can be explained in the following table 1.

Table 1: Making sense of each particular sub-problems

The Problem: To model the flood		
What do we <i>WANT</i> ?	What do we <i>KNOW</i> ?	What can we <i>INTRODUCE</i> ?
1. To find the relationship between the variables such as the flow discharge, the channel cross-sectional area of the flow, the water depth, and the hydrostatic pressure.	Conservation laws can provide this relationship due to the wide range of its applications.	Conservation laws (a system of partial differential equations PDEs) • Conservation of mass equation (one equation). • Conservation of momentum equations (three equations).
2. To reduce the model spatial multidimensional in sub-problem 1 from 3D to 2D.	The water depth in shallow water flows is significantly smaller compared to horizontal length scales and the vertical acceleration is negligible.	The assumption leads to the vertically averaged variables and reduces the system to three partial differential equations.
3. To reduce the model spatial multidimensional in sub-problem 1 from 2D to 1D.	Over each channel cross section, the flow velocity is uniform, and the free surface is assumed to be horizontal	The assumptions lead a system of two partial differential equations.
4. To construct the lateral discharge representing the flood.	The inflow and outflow lateral discharge variables can be involved in both mass and momentum equations (Bousmar, 2002).	A full system representing the flood.
5. To simplify the dynamic model.	The channel flow can be assumed to have a constant lateral width.	A simplified flood model is introduced with sours term depends only on the bed elevation.

We make a start by first trying to understand the flood process. When we had developed a “sense of meaning” of the problem, we are ready for the ATTACK phase. We specialized by focusing our attention investigating flow variables to look for patterns and to articulate the underlying pattern. Studying the flood flow has led us to embark in the first sub-problem, that is, to formulate a mathematical relation of the situation. To do so, firstly we need to introduce the basic concepts of conservation laws as described below. Our attempt to simplify the original model to a simplified model makes it deemed necessary to provide a brief description of what flood modelling is all about.

The time-dependent hyperbolic conservation laws in three-dimensional geometries can be expressed in a generalized matrix form (Toro, 2009):

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{U}) = \mathbf{S}(\mathbf{U})$$

where $\mathbf{U}$ is the vector of the conservative variable, ∇ is a differential gradient operator, $\mathbf{F}(\mathbf{U})$ is the flux vector and $\mathbf{S}(\mathbf{U})$ is the source term vector.

The use of vector notation and differential operators exemplifies the generalization and formal structure. In the first sub-problem, conservation laws can be imposed to relate the flow variables and the vectors are given as following:

$$\left\{ \begin{array}{l} \mathbf{U} = \begin{pmatrix} 1 \\ u \\ v \\ w \end{pmatrix}, \quad \mathbf{F}(\mathbf{U}) = (\mathbf{F}(\mathbf{U}), \mathbf{G}(\mathbf{U}), \mathbf{H}(\mathbf{U})), \quad \nabla = (\partial_x, \partial_y, \partial_z) \\ \mathbf{F}(\mathbf{U}) = \begin{pmatrix} u \\ u^2 + P \\ uv \\ uw \end{pmatrix}, \quad \mathbf{G}(\mathbf{U}) = \begin{pmatrix} v \\ uv \\ v^2 + P \\ vw \end{pmatrix}, \quad \mathbf{H}(\mathbf{U}) = \begin{pmatrix} w \\ uw \\ vw \\ w^2 + P \end{pmatrix}, \quad \mathbf{S}(\mathbf{U}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mathbf{g} \end{pmatrix} \end{array} \right.$$

where u, v, w are the partial velocity in x, y, z direction, respectively, P is the pressure force, and $\mathbf{g}$ is the body force in z direction. The system of partial differential equations presented above is known as the Free Surface Euler Equations (FSEEs). We use abstraction and simplification to make the system more tractable for specific flow conditions. For shallow flows, we observed that the vertical acceleration component is negligible. This approximation allows us to assume hydrostatic pressure, meaning pressure depends only on the gravitational force and not on vertical motion. Through generalization, we apply a vertical averaging process to the full 3D equations, transforming System 1 into a simplified form suitable for shallow water conditions (Nwaigwe, 2016).

$$\left\{ \begin{array}{l} \mathbf{U} = \begin{pmatrix} h \\ q_x \\ q_y \end{pmatrix}, \quad \mathbf{F}(\mathbf{U}) = (\mathbf{F}(\mathbf{U}), \mathbf{G}(\mathbf{U})), \quad \nabla = (\partial_x, \partial_y) \\ \mathbf{F}(\mathbf{U}) = \begin{pmatrix} q_x \\ \frac{q_x^2}{h} + \frac{1}{2}gh^2 \\ \frac{q_x q_y}{h} \end{pmatrix}, \quad \mathbf{G}(\mathbf{U}) = \begin{pmatrix} q_y \\ \frac{q_x q_y}{h} \\ \frac{q_y^2}{h} + \frac{1}{2}gh^2 \end{pmatrix}, \quad \mathbf{S}(\mathbf{U}) = \begin{pmatrix} 0 \\ -gh\partial_x z_b \\ -gh\partial_y z_b \end{pmatrix} \end{array} \right.$$

where h is the water depth, q_x and q_y are the horizontal and lateral discharge respectively, and z_b is the flow bed elevation. The system is called 2D shallow water equations (SWEs). They can be reduced to be 1-dimensional by assuming that the flow velocity is uniform across

the channel width and the free surface is horizontally constant. The resulting system is called 1D Saint Venant Equations (SVEs) and is given as:

$$\begin{cases} \mathbf{U} = \begin{pmatrix} A \\ Q \end{pmatrix}, & \mathbf{F}(\mathbf{U}) = \mathbf{F}(\mathbf{U}) \quad , \quad \nabla = \partial_x \\ \mathbf{F}(\mathbf{U}) = \begin{pmatrix} \frac{Q^2}{A} \\ \frac{1}{2}g\frac{A^2}{B} \end{pmatrix}, & \mathbf{S}(\mathbf{U}) = \begin{pmatrix} 0 \\ \frac{1}{2}g\frac{A^2}{B^2} + \partial_x B - gA\partial_x Z_b \end{pmatrix} \end{cases}$$

where, A is the cross sectional area of the channel, Q is the longitudinal discharge, B is the channel width and Z_b is the bottom elevation of the cross section.

According to Bousmar (2002), additional terms account for lateral inflows and outflows can be added to represents the flood flow whenever the water depth exceed the maximum wall elevation of the channel. The suggested terms incorporate lateral discharge variables into both mass and momentum equations. The resulting system defer from system only on the source term which is given as:

$$\mathbf{S}(\mathbf{U}) = \mathbf{S}_{nf}(\mathbf{U}) + \mathbf{S}_f(\mathbf{U}) = \begin{pmatrix} 0 \\ \frac{1}{2}g\frac{A^2}{B^2} + \partial_x B - gA\partial_x Z_b \end{pmatrix} + \begin{pmatrix} q_{in} - q_{out} \\ q_{in}u_L - q_{out}\bar{u} \end{pmatrix}$$

where q_{in} and q_{out} represent inflow and outflow, respectively, u_L is the velocity of the lateral inflow in the direction of the main flow, and $\bar{u} = Q/A$ is the mean velocity in the considered section. Finally, in case of the channel width variation is very small, the differential term $\partial_x B$ is vanish in & , the system can be rewritten as:

$$\begin{cases} \mathbf{U} = \begin{pmatrix} \bar{h} \\ \bar{q} \end{pmatrix}, & \mathbf{F}(\mathbf{U}) = \mathbf{F}(\mathbf{U}) \quad , \quad \nabla = \partial_x \\ \mathbf{F}(\mathbf{U}) = \begin{pmatrix} \bar{q} \\ \frac{\bar{q}^2}{\bar{h}^2} + \frac{1}{2}g\bar{h}^2 \end{pmatrix}, & \mathbf{S}(\mathbf{U}) = \begin{pmatrix} 0 \\ -g\bar{h}\partial_x Z_b \end{pmatrix} + \begin{pmatrix} q_{in} - q_{out} \\ q_{in}u_L - q_{out}\frac{\bar{q}}{\bar{h}} \end{pmatrix} \end{cases}$$

where $\bar{q} = \bar{u}\bar{h}$.

In modelling the situation mathematically, we used logical assumptions and structured transformations to reduce the complex systems into solvable forms, enabling further analysis and numerical implementation. Throughout the journey, we make use of prompts and questions such as “what is this an example of...? what needs to be changed so that..., what is the same and different about...? what happens in general? how can we be sure that...” to invoke our mathematical thinking processes in going through the journey of conceptualization and the mathematical formulation. On many occasions when we got STUCK, we had to re-enter the ENTRY phase, re-assed the problem, specialize further and reflect on what we were doing.

Role of Computational Thinking

The nonlinear, time-dependent hyperbolic system of equations used in flood modelling is too complex to solve analytically. Computational thinking provides a structured framework to

address this challenge through four core steps: decomposition, pattern recognition, abstraction, and algorithmic thinking. This framework enables systematic translation of mathematical models into executable simulations. Like mathematical thinking, computational thinking involves breaking the problems into smaller parts, analysing the data to identify similarities and connections among the parts, and abstracting relevant information. Algorithmic thinking is then use seeking to automate the problem-solving process by creating a series of systematic logical steps. The developed algorithm allows the work to be replicable by humans or computers. We continue to use the prompts and questions to bring about the awareness and be reflective of our thinking during the problem-solving process. Figure 1 proposed the generalized framework of computational thinking, which will be demonstrated in the example of solving a particular set of partial differential equations (PDEs).

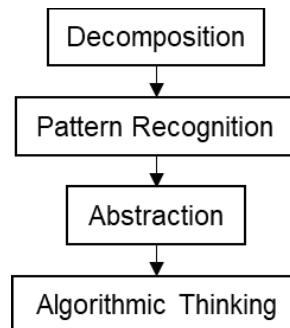


Figure 1: General framework of computational thinking

Guided by computational thinking, we start with decomposition, breaking down the continuous problem represented by PDEs into a finite set of discrete elements. This enables the use of numerical methods, such as finite difference, finite volume, and finite element methods, which have been extensively developed over time. We begin by breaking down the continuous problem—represented by partial differential equations (PDEs)—into discrete, manageable elements. For flood modelling, this involves dividing the spatial and temporal domains into finite control volumes and time steps. In the dam break case study, we further decompose the problem into a simplified test scenario: simulating a sluice gate release between two static water bodies in a uniform channel.

Analysis of the decomposed system reveals recurring challenges, such as discontinuities (e.g., shock waves during dam breaks). Pattern recognition identifies these as weak solutions that invalidate the differential form of conservation laws within finite time (Toro, 2009). This insight directs us toward numerical methods capable of handling discontinuities. To address this, we apply abstraction, moving from the differential to the integral form of the conservation laws, which captures the system's behavior over control volumes and their boundaries. Algorithmic thinking then drives the implementation of this approach in computational models. By discretizing the domain and enforcing conservation locally, we arrive at the Finite Volume Method (FVM), which is well-suited for handling complex physical phenomena in flood modelling (Toro, 2009).

Finite Volume Method (FVM) is implemented to solve the dam break problem. The problem is decomposed into a simplified test case: the sudden opening of a sluice gate separating two still water bodies of differing depths in a 500-meter-long, uniformly wide channel. Reflective boundary conditions are applied at both ends of the channel, allowing controlled simulation of wave reflections.

Abstraction is applied by representing the continuous physical system with discrete control volumes, enabling numerical approximation of the governing conservation laws. The Random Choice Method (RCM), known for its ability to capture complex wave dynamics and recognize patterns such as discontinuities (e.g., shock waves and material interfaces), is used to enhance the solution's accuracy. More details on the RCM can be found in Xue et al (2023).

Finally, we design step-by-step procedures to automate solutions. Algorithmic thinking drives the implementation of the Finite Volume Method (FVM), specifying workflows for flux computation, state updates, and boundary handling. In the dam break simulation, this translates to codifying iterative steps—from initializing conditions to updating wave propagation—ensuring physical fidelity through repeatable logic. As an illustration, the setup begins with the following initial conditions:

$$\begin{cases} h = 8.0 \text{ m and } u = 0.0 \text{ m/s at } x \leq 400 \text{ m} \\ h = 2.0 \text{ m and } u = 0.0 \text{ m/s at } x > 400 \text{ m} \end{cases}$$

Figure 2 presents a comparison between the simulation results of the current model and the Godunov-type scheme (GTS), alongside the exact solution for a given Courant number (0.8)

and grid resolution $\Delta x = 10$. The findings demonstrate that the current model effectively captures discontinuities, especially shock waves, with greater clarity than the GTS. It also exhibits strong accuracy in representing rarefaction waves, although some oscillations are observed at the tail of the rarefaction region.

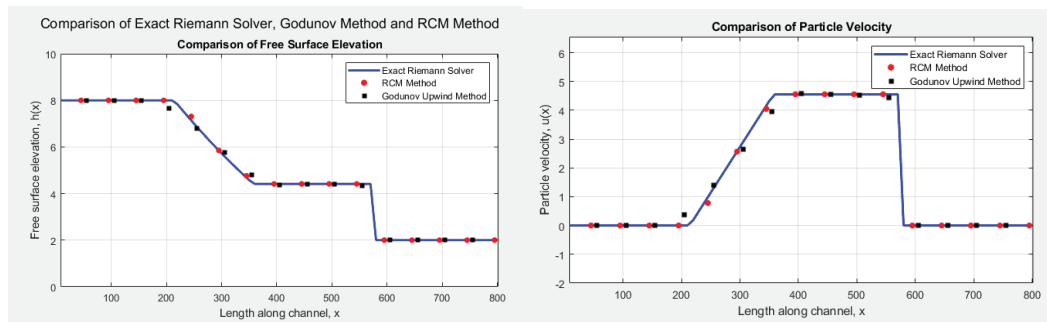


Figure 2: Water depth and particle velocity profile simulations compared with the exact solution at $t = 21\text{s}$, $\Delta x = 10$.

In summary, computational thinking is fundamental to flood modelling, enabling the effective management and interpretation of complex datasets such as digital elevation models (DEMs), land-use patterns, rainfall records, and real-time sensor inputs. By supporting structured data models, algorithmic logic, and efficient simulation workflows, it facilitates accurate and timely analysis. Moreover, through intuitive visualization tools and scenario-based simulations, helps to transforms abstract mathematical outputs into actionable insights that are crucial for early warning systems, infrastructure planning, and emergency response. Its integration not only enhances technical performance but also strengthens decision-making in the face of increasingly dynamic and data-driven flood risks.

Challenges and Future Directions

The integration of computational and mathematical thinking remains a significant challenge. Bridging the gap between abstract mathematical formulations such as differential equations and their computational implementation requires deep conceptual understanding, interdisciplinary skills, and robust educational frameworks (Weintrop et al., 2016; Rich et al., 2019). Current shortcomings include the steep learning curve associated with mathematical modelling, the need for more intuitive tools for algorithm design, and a lack of cohesion between theory and practice in many learning environments (Tabach, 2021; Asempapa & Sturgill, 2019).

Looking ahead, future directions include developing more accessible modelling platforms, enhancing data literacy, and incorporating real-world, computationally rich problems into mathematics education (Broley et al, 2023). Advancements in artificial intelligence, real-time data assimilation, and cloud-based simulation environments offer exciting opportunities to

deepen the integration of mathematical reasoning with computational practice (Rehan, 2023). By embedding these elements into curricula and professional development, we can better prepare learners and practitioners to engage with complex, data-driven challenges such as flood risk and climate resilience through both rigorous mathematical analysis and effective computational strategies. This evolving landscape holds great promise for innovation, not only in applied fields like hydrology and environmental science but also in how we teach, learn, and apply mathematics in a digitally connected world.

Significance for Engineering Mathematics Curriculum

Modern engineering demands a curriculum that goes beyond traditional mathematics to include computational thinking. As engineering problems grow in complexity and scale, students need not only a solid foundation in mathematical reasoning but also the ability to model, simulate, and solve problems using computational tools. Integrating mathematical and computational thinking offers a transformative approach to engineering education.

Mathematical thinking develops essential skills in abstraction, logic, and quantitative reasoning which are cornerstones of engineering problem-solving. However, on its own, it often lacks the tools to handle real-world systems that involve dynamic processes, large data sets, or iterative modelling. As illustrated by the modelling of hydrodynamic flood, computational thinking enables us to break down complex problems, design algorithms, and implement them using programming and software tools. It bridges the gap between abstract theory and practical application, enhancing learners' capacity to explore, simulate, and optimize engineering systems (Wing, 2006; Learning.com, 2021).

Incorporating computational thinking equips students with problem-solving strategies aligned with industry needs such as system modelling, data analysis, and automation. Studies show that this integration enhances student engagement, deepens conceptual understanding, and builds readiness for careers in modern engineering fields (EdUHK Repository, 2021). Embedding computational thinking within the engineering mathematics curriculum marks a necessary advancement. It promotes deeper learning, fosters innovation, and prepares students for complex, data-rich challenges. This shift is essential for developing engineers who are not only mathematically competent but also computationally fluent, ready to lead in a rapidly changing technological landscape.

To demonstrate how mathematical and computational thinking integration for engineering mathematics curriculum, coupled with several intervention to address curriculum gaps in engineering mathematics, Table 2 is presented to link modelling stages (conceptualization, formulation, analysis) to missing skills in current curriculum and Figure 3 is introduced to demonstrate an example of curriculum reform for industry-ready graduates.

Table 2: Modelling stages linked towards the gap of skills in current engineering mathematics curriculum

Modelling Stage	Mathematical Thinking	Computational Thinking	Curriculum Gap	Proposed Intervention
1. Conceptualization	Abstraction (conservation laws) Simplification (2D/1D reduction)	Decomposition (system breakdown)	Lack of real-world problem framing	Integrate case studies (e.g., flood scenarios) in calculus/PDE courses
2. Formulation	Symbolic representation (PDEs)	Algorithm design (discretization) Pattern recognition (shock waves)	Minimal numerical methods training	Add labs on FVM/FDM implementation in <i>Differential Equations</i>

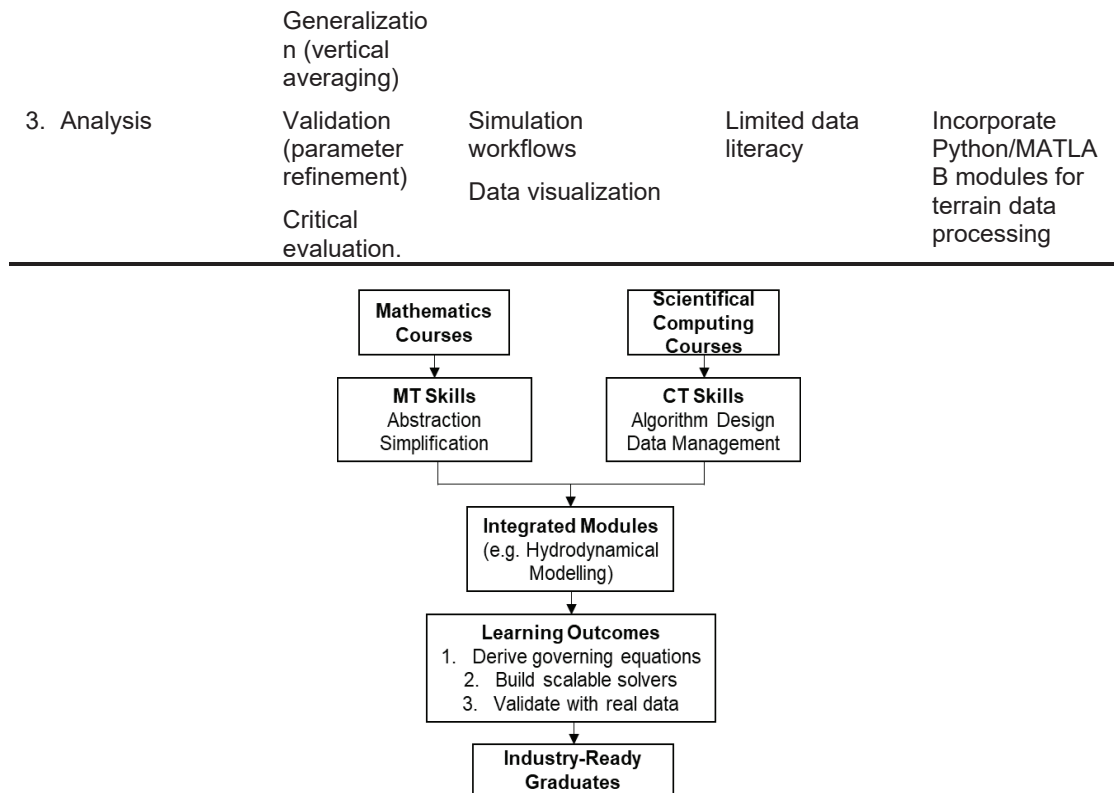


Figure 3: General example of engineering mathematics curriculum reform

Conclusion

The modelling of hydrodynamic flood has shown the integration of mathematical and computational thinking is essential for engineering education. Mathematical thinking provides the basis for understanding fluid behaviour and building models using equations that describe water flow and terrain. However, solving these equations analytically is impractical for real-world scenarios, especially when dealing with large-scale or dynamic environments. Computational thinking enables us to simulate these complex systems, analyse changing variables, and visualize flood behaviour over time. Together, mathematical and computational approaches allow us to build accurate, adaptable models that support better decisions.

Integrating mathematical and computational thinking has the potential to transform engineering education by better preparing students for complex, data-driven, and interdisciplinary challenges. Mathematical thinking supports abstract reasoning and modelling, while computational thinking enables the implementation, simulation, and visualization of these models in real-world contexts. When combined, they create a powerful foundation for innovation and practical problem-solving. To fully realize this potential, educational programs should connect these domains intentionally through integrated curricula and active learning strategies. Students need opportunities to practice translating mathematical models into computational solutions in applied, interdisciplinary contexts. Further works are needed to refine teaching strategies, develop assessments that capture integrated thinking, and understand how this approach supports long-term success in engineering practice.

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